Abstract. More than we realize it, knowledge is often constructed through interactions among people in small groups. The Internet, by allowing people to communicate globally in limitless combinations, has opened enormous opportunities for the creation of knowledge and understanding. A major barrier today is the poverty of adequate groupware. To design more powerful software that can facilitate the building of collaborative knowledge, we need to better understand the nature of group cognition—the processes whereby ideas are developed by small groups. We need to analyze interaction at both the individual and the group unit of analysis in order to understand the variety of processes that groupware should be supporting. This paper will look closely at an empirical example of an online group problem-solving experience and suggest implications for groupware design.

1. Individual Learning in Groups

Groupware is software that is specifically designed to support the work of groups. Most software in the past, in contrast, has been designed to support the work of individuals. The most popular applications—such as word processors, Internet browsers and spreadsheets—are structured for use by one individual at a time. Software for communication among people—like an email program—assumes a model of communication as transmission of messages from one person to other individuals. Building on these examples, one could design groupware to support groups conceived of as sets of individuals. Such software would allow individuals to express their mental ideas, transmit these expressions to other people, receive expressions transmitted from other people and make sense of received messages as expressions of the ideas in the heads of the other people. Possibilities for improving
these designs might be conceived in terms of “increasing the bandwidth” of the
transmissions, possibly taking face-to-face communication as the “gold standard” of
communication with a wide bandwidth of many channels (words, intonation, gaze,
facial expression, gesture, body language).

Until recently, most research about groups has focused on the individual people in
the group as the cognitive agents. For instance, research on cooperative learning in
the 1970s, 2 assumed that knowledge resided in the individuals, and that group
interaction was most useful as a way of transferring knowledge from one individual
to another or as a way of motivating individuals to perform better. Educational
research on groups typically measured learning in terms of individual test outcomes
and tried to study what is going on in the minds of the individuals through surveys,
interviews and talk-aloud protocols. Similarly, research in social psychology about
small groups conceptualized the groups as sets of rationally calculating individuals
seeking to maximize their own advantages. This broad tradition looks to the
individual as the unit of analysis, both to understand what takes place in group
behavior and to measure quantitative learning or knowledge-building outcomes.

In the 1990s, the individualistic approach was thoroughly critiqued by theories of
situated cognition, 3 distributed cognition, 4 socio-cultural activity theory 5 and
ethnomethodology, 6 building on the philosophies of phenomenology, 7 mediation 8 and
dialog. 9 These new approaches rejected the view that cognition or the construction of
knowledge took place exclusively in the isolated minds of individuals, and showed
how it emerged from concrete situations and interpersonal interactions. One
consequence that could be drawn from this would be to analyze cognition at the
small-group unit of analysis, as in many cases a product of social interaction within
the context of culturally-defined rules or habits of behavior.

An alternative approach to designing groupware based on a group conception of
cognition would provide functionality to support the working of a group as an
organic whole, rather than just supporting the group members as individuals and
treating the group as the sum of its parts. In the past, a number of researchers have
tried to develop groupware that supports the functioning of the group itself, such as
the formation of groups, 10 intertwining of perspectives 11 and negotiation of group
decisions. 12, 13

Here I would like to further develop the approach focused on the group that I
presented in Group Cognition 14 and that is being investigated in the Virtual Math
Teams (VMT) project at the Math Forum at Drexel University. In part I of the book, I
present my own attempts to design software to support small-group interactions
(building, of course, on previous work by others), and conclude that we need to better
understand how groups work before we can effectively design groupware. In part II
of the book, I then discuss how to analyze the methods that are used in groups to
construct meaning and knowledge. Then I develop a concept of group cognition in
part III to talk about what takes place at the group unit of analysis.

In this paper, I report on a group of students working on a set of math problems in
an online chat room. In the VMT project, we were interested in seeing how well
groups work together using a minimal chat system so we could see what forms of
interaction might be supported by groupware with special functionality designed to
increase the effectiveness of collaboration.

In order to capture both the individual and the group contributions to discourse
and to compare their results, we arranged an experiment with a combination of
individual and group work. It consisted of an individual phase where the knowledge
of the individuals can be objectively assessed, followed by a group phase in which
the references and proposals can be analyzed at both the individual and the group
units of analysis. By seeing what the individuals knew before they participated in the
group phase, it was possible to see what the group interaction added.

In previous work at the VMT project, we have characterized two different general
patterns of chat discourse: expository narrative and exploratory inquiry.15 These are
two common methods of conducting online discourse that embody different
relationships of the group to its individual members. We view online chat as a form
of text-based interaction, where short texts respond to each other.16 We analyze the
chat discourse with a variation of conversation analysis—a scientific methodology
based on ethnomethodological principles for analyzing everyday verbal conversation.
In the VMT project, we have begun to adapt conversation analysis to chat by taking
into account the consequences introduced by the textual medium, the math content,
the physical separation and other differences from everyday conversation.

Expository narrative involves one person dominating the interchange by
contributing more and longer texts.17 Basically, the normal turn-taking procedures in
which group members take roughly equal and alternating turns is transformed in
order to let one person narrate an extended story or explanation. For instance, if a
student has already solved a math problem that the group is working on, that student
might propose their solution or indicate that they have a solution and the others might
request an explanation of the proposed solution. There would still be some forms of
interaction, with members of an audience asking questions, encouraging
continuation, indicating understanding, raising questions, etc. But in general, the
proposer would be allowed to provide most of the discourse. In conversation, this
kind of pattern is typical where one member narrates a story or talks in detail about
some events or opinions.18 Exposition in math has its own characteristics, such as
providing mathematical warrants for claims, calculating values, addressing issues of
formal logic, etc. But it follows a turn-taking profile similar to that of conversational
narrative.

Exploratory inquiry has a different structure. Here, the group members work
together to explore a topic. Their texts contribute from different perspectives to
construct some insight, knowledge, position or solution that cannot be attributed to
any one source but that emerges from the “inter-animation of perspectives.”9 19
Exploratory inquiries tend to take on the appearance of group cognition. They
contrast with expository narratives in a way that is analogous to the broad distinction
between collaboration and cooperation.20 Collaboration involves a group of people
working on something together, whereas cooperation involves people dividing the
work up, each working by themselves on their own part and then joining their partial
solutions together for the group solution. Expository narratives tend to take on the
appearance of cooperation, where individuals contribute their own solutions and
narrate an account of how they arrived at them. In a rough way, then, exploratory and
expository forms of discourse seem to reflect group versus individual contributions to
constructing shared knowledge.

I will now analyze our experiment involving a group of students in an online chat
discussing a series of math problems. I will try to tease apart the individual and the
group contributions to meaning making, knowledge building and problem solving.
We conducted the experiment using a set of well-defined math problems for which it
is clear when an individual or a group arrives at the correct answer. We gave the
individuals an opportunity to solve the problems on their own and submit their
results. We then had them interact in an online chat room and decide as a group on
the correct answers. By collecting their individual solutions and logging the chat, we
obtained data about the individual and the group knowledge, which we can
objectively evaluate and compare.
The students were given 11 problems. The problems were a variety of algebra and geometry problems, some stated as word problems. Most required some insight. They came from the Scholastic Aptitude Tests (SAT), which are taken by high school students in order to apply to colleges in the United States. They are primarily multiple choice questions with five possible answers, only one of which is correct.*

For the individual phase of the experiment, the students had 15 minutes to complete the problems individually and submit their answers. The students then worked in randomly-assigned groups to solve the same problems online. They worked together in chat rooms for 40 minutes.

In this paper, I analyze the results of one group of five students who worked together in one chat room. None of the students in this group did impressively well on the test as an individual. They each got 2 or 3 question right out of the 11 (see table 1) for a score of 18% or 27%.

Table 1. Problems answered correctly by individuals and the group.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hal</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>27%</td>
</tr>
<tr>
<td>Dan</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18%</td>
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<tr>
<td>Cosi</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>27%</td>
</tr>
<tr>
<td>Mic</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18%</td>
</tr>
<tr>
<td>Ben</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18%</td>
</tr>
<tr>
<td>Group</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>82%</td>
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</table>

For the experiment’s group phase, the students worked in a chat room using a generic group chat facility without a shared whiteboard. The software is simple and familiar to the students. The students did not know each other and did not have any information about each other. They had not worked together before and had not participated in a chat like this before. The result of the group work was that the group decided upon the correct answers to 9 of the 11 problems, for a group score of 82%. Thus, the group did considerably better than any of the individual students.

However, it seems that each of the correct group answers can be attributed to one of the students. Although each student got only 2 or 3 answers right, together at least one of them correctly answered questions 2, 3, 4, 5, 7, 8, 9. No one understood question 1, and the group did not get this answer either. Question 2 was correctly answered by Hal, who persuaded the group. Question 3 was correctly answered by everyone except Mic. Question 4 was correctly answered by Dan. Question 5 gave the group a lot of frustration because no one could figure it out (although Mic had gotten it right on his own); they eventually accepted the correct answer from someone outside the group. No one understood question 6, and the group got it wrong. They got question 7 right (following Cosi and Mic). Only Hal got question 8, but he persuaded the others. (Ben also got it on his own, but did not participate in the group discussion.) Cosi got the answer to question 9. No one got questions 10 or 11, so the group had to work on these together. The discussion of question 10 was particularly interesting. As we will see, it looks like Cosi got the answer to question 10 and explained it to the others (although she had not gotten it on her own). Hal got


The analysis in this paper is indebted to conversation analysis data sessions at the VMT project, led by Alan Zemel, and comments from Stephen Weimar and Martin Wessner.
question 11 right and the others accepted it (although he had not gotten it on his own).

So it appears as though the math problems were actually solved by individuals. The group responded to proposed answers. In instances where there were competing answers or other issues, the group required the proposer to give an account, defense or explanation. This resulted in an expository form of discourse where one member proposed an answer and explained why it was right. Although the group was not experienced in working together, they succeeded in selecting the best answers that their members could come up with. The result of the group cooperation was to achieve a sum of their best individual results.

It is particularly interesting to observe how the group negotiated their group answers given proposals from various members. In some cases, everyone proposed the same answer and it was easy to establish a consensus. In certain other cases, only one person proposed an answer and the others simply went along with it. In more interesting cases, when someone proposed an answer that contradicted other people’s opinions or was questionable for some other reason, the proposer was required to give an explanation, justification or accounting of their proposal. We do not have space here to analyze each of the negotiations: how they were begun, how people contributed, how the discussion was continued, how decisions were made and how the group decided to move on to a new problem. In particular, we cannot go into the integration of social chatter and math reasoning or fun making and decision making. Rather, we will take a look at the discussion of question 10, which was particularly interesting because no one had already solved this problem and because we can see the solution emerging in the discourse.

Question 10 is a difficult algebra word problem. It would take considerable effort and expertise for a student to set up and solve equations for it. The group manages to finesse the complete algebraic solution and to identify the correct multiple-choice answer through some insightful reasoning. Question 10 is:

Three years ago, men made up two out of every three internet users in America. Today the ratio of male to female users is about 1 to 1. In that time the number of American females using the internet has grown by 30,000,000, while the number of males who use the internet has grown by 100%. By how much has the total internet-user population increased in America in the past three years?
(A) 50,000,000 (B) 60,000,000 (C) 80,000,000 (D) 100,000,000 (E) 200,000,000

The core discussion of this question takes place in the chat excerpts shown in Table 2.

<table>
<thead>
<tr>
<th>Line</th>
<th>Time</th>
<th>Name</th>
<th>Message</th>
<th>Interval</th>
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<tbody>
<tr>
<td>350</td>
<td>4:31:55</td>
<td>Mic</td>
<td>how do we do this...</td>
<td></td>
</tr>
<tr>
<td>351</td>
<td>4:31:59</td>
<td>Mic</td>
<td>without knowing the total number</td>
<td>0:00:04</td>
</tr>
<tr>
<td>352</td>
<td>4:32:01</td>
<td>Mic</td>
<td>of internet users?</td>
<td>0:00:02</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>....</td>
<td></td>
</tr>
<tr>
<td>357</td>
<td>4:32:23</td>
<td>Dan</td>
<td>it all comes from the 300000000</td>
<td></td>
</tr>
<tr>
<td>358</td>
<td>4:32:23</td>
<td>Mic</td>
<td>did u get something for 107</td>
<td>0:00:00</td>
</tr>
<tr>
<td>359</td>
<td>4:32:26</td>
<td>Dan</td>
<td>we already know</td>
<td>0:00:03</td>
</tr>
</tbody>
</table>
360 4:32:44 Mic 30000000 is the number of increase in american females 0:00:18
361 4:33:00 Mic and since the ratio of male to female 0:00:16
362 4:33:02 Mic is 1 to 1 0:00:02
363 4:33:09 Mic thats all i got to give. someone finish it 0:00:07
364 4:33:10 Mic haha 0:00:01
365 4:33:18 Cosi haha you jackass 0:00:08
366 4:33:20 Mic haha 0:00:02
367 4:33:21 Dan hahaha 0:00:01
368 4:33:26 Mic u all thought i was gonna figure it out didnt 0:00:05
369 4:33:27 Mic u 0:00:01
370 4:33:28 Mic huh? 0:00:01
371 4:33:28 Hal it would be 60,000,000 0:00:00
372 4:33:30 Mic hal 0:00:02
373 4:33:31 Mic its all u 0:00:01
374 4:33:33 Mic see 0:00:02
375 4:33:34 Mic i helped 0:00:01
376 4:33:54 Cosi ok, so what’s 11 – just guess on 10 0:00:20
377 4:34:12 Mic lets get back to 5 0:00:01
378 4:34:45 Mic way to complicate things 0:00:10
379 4:35:03 Cosi haha sorry 0:00:06
380 4:35:05 Mic life was good until you said that 0:00:02
381 4:35:07 Mic :(
382 4:35:18 Cosi they cant get higher equally and even out to a 1 to 1 ratio 0:00:11
383 4:35:27 Cosi oh, no wait, less than that 0:00:09
384 4:35:32 Cosi 50000000 0:00:05
385 4:35:34 Cosi yeah, it's that 0:00:02
386 4:35:36 Cosi im pretty sure 0:00:02
387 4:35:37 Mic i helped 0:00:01
388 4:35:38 Mic how? 0:00:01
389 4:35:57 Cosi because the women pop had to grow more than the men in order to even out 0:00:19
390 4:36:17 Dan very smart 0:00:03
391 4:36:21 Cosi Damn im good 0:00:02

We can see here that the group is meandering somewhat in trying to solve problem 10. Mic raises the question of how to solve it (lines 350-352). Dan suggests that the 30,000,000 figure is key, and Mic tries to build on this suggestion. But Mic ends his attempt with a laugh, clowning around that he was only pretending to figure out the
Hal proposes that the answer is 60,000,000 (line 371), but then Cosi complicates matters by questioning this answer (line 387).

Having rejected Hal’s proposal, Cosi proceeds to solve the problem by herself. She reasons that the male and female population cannot grow by the same amount from uneven numbers to arrive at equal numbers (line 392). From this, she concludes that the answer is 50,000,000. She announces that she is “pretty sure” of this answer (line 396). At this point, it seems that Cosi has solved the problem on her own.

Mic responds to the statement that Cosi is only “pretty sure” and not positive by requesting an explanation of how Cosi arrived at her opinion that the answer is 50,000,000—and not the 60,000,000 that Hal proposed (line 398).

In the following lines (399, 400, 403), Cosi provides an account of her reasoning. If the females grew by 30,000,000 then the males must have grown by less than that. Therefore, the total growth must have been less than 60,000,000. The only answer listed that meets this condition is 50,000,000—so that must be the correct answer.

Cosi’s extended turn providing an exposition of her thinking is interrupted only by Mic (lines 401, 402), who simultaneously affirms Cosi’s approach, provides an excuse for not having solved the problem himself, and admits to not having read the problem carefully in the first place. In this way, Mic continues to move the group toward making good decisions about which proposed answers to accept while himself playing the fool. Dan speaks on behalf of the group (line 404), accepting Cosi’s answer and proof by praising her as “very smart,” to which she responds (line 405), “Damn, I’m good.” In the subsequent discussion, both Hal and Mic agree with Cosi’s solution. Cosi is anxious to move on to another problem and finally says (line 419), “ok great, im smart, lets move on.”

From our analysis, we can see the advantages that have long been claimed by other researchers for collaborative learning. A number of students each contributed their best ideas. Some students knew some answers, some others, and together they arrived at a position where they effectively shared the whole set of best answers that any of them had to start with. In addition, the group work sustained their time-on-task beyond what any one student was willing to do, arriving at correct answers for the final two problems.

According to the foregoing analysis, the actual mathematical reasoning was done by individual minds. The group was able to take the results of these individual achievements and gather them together in a particularly effective way. In the end, all members of the group had the opportunity to know more correct answers than they could arrive at on their own. It may not be obvious that every student could then solve all the problems on their own, but there were a number of indications in the chat that students gained insights into aspects of the problem solving that we can assume would stay with them as individual learning outcomes.

In this experiment, we were able to see how the group took good advantage of the knowledge of its members, even though the group had not had any previous experience working together and had no external scaffolding from a teacher or the software in how to collaborate. As researchers, we know which students were able to solve which problems on their own and we could then observe how they interacted to solve the problems in the group context. We had a simple, objective measure of mathematical skill based on correct answers to standardized SAT problems. We observe that a group of students who individually scored 18-27% was able to score 87% when working together. Furthermore, this impressive result can be understood in terms of simply making good decisions about which proposals to listen to on each problem and then spending more engaged time-on-task on the two final problems.
2. Group Cognition in Online Math

In the previous section, the work of the student group was interpreted primarily at the individual unit of analysis. The problem solving was discussed as the accomplishment of individuals. The group decisions were discussed as a form of voting among people who largely made up their minds individually. In many cases, individuals did not hold strong opinions about the answers to the problems and therefore left the group decision up to other individuals—who might have a higher likelihood of knowing the correct answer—by remaining silent.

However, it is possible to analyze the chat differently, taking the group as the unit of analysis.

The central point of this alternative approach is that the meaning constructed in a group discourse is often the result of the subtle ways in which utterances of different speakers or writers interact, rather than the result of a simple addition of ideas expressed or represented in the individual utterances.

Perhaps the greatest problem in understanding how groups work is to clarify the relation of individual to trans-individual contributions to the group meaning making. Clearly, individual group members may have ideas of their own that they introduce into the discourse. Their utterances may have to wait for the right moment in the conversational flow and they might have to design their contributions to fit into the discourse context in order to be accepted as useful proposals with a chance of being taken up, but they also may bring with them some premeditated meaning constructed by their proposer. Individuals also play a necessary role as the interpreters of the group meaning in an on-going way as they respond to the discourse.14, chapter 16 On the other hand, the formative roles of adjacency pairs (explained below) and other relationships among utterances underline the importance of analyzing meaning making at the group unit of analysis, not just interpreting the utterances of individuals.

A more detailed analysis than can be presented here of the negotiations of the answers for questions 1 through 9 in the experiment shows that the group had methods for interacting that were quite effective in making good decisions. They had subtle ways of coalescing the individual group members into a collectivity that could work through the set of math problems, discover solutions and decide which solutions to adopt as the group’s answers. This suggests that the problem-solving methods used by the group of students is qualitatively different from the methods they use individually. Another way of putting it is that the group collaboration brings additional methods at the group unit of analysis that supplement the individual cognitive methods of problem solving. It may be important to distinguish these different classes of methods at the different levels of analysis, as well as to see subsequently how they work together.

In defining his concept of the zone of proximal development, Vygotsky strongly distinguished between what a student could accomplish individually and what that student could accomplish when working with others.5 “It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p.86). Based on psychological experiments, Vygotsky argued that what children “could do only under guidance, in collaboration, and in groups at the age of three-to-five years they could do independently when they reached the age of five-to-seven years” (p. 87). In the chat, we can see that older students can also achieve significantly more in collaborative groups than independently—and we can see the methods of group interaction that one particular group adopted in one case study to accomplish this.
We now revisit the solving of problem 10 as a group achievement. Of course, the sequence of recorded events—the lines in the chat log—are the same. But now we no longer attribute the source of the messages to the individuals as the “expression” of internal mental ideas that they have worked out in advance. Rather, we look for evidence in the details of the log of how messages are responses to each other.

Mic’s opening question (lines 350-352) is a reaction to the problem statement. The problem asks how much the population has increased. A straight-forward calculation of this increase might involve subtracting from the current total number of Internet users the corresponding figure for three years ago. But the two numbers needed for such a calculation are missing from the problem statement. The problem only gives indirect clues. The problem statement thereby calls for a less direct strategy. Mic’s messages respond to this implicit requirement by making it explicit.

Dan responds to Mic’s question by proposing an approach for coming up with a strategy. He says (lines 357 and 359), “It all comes from the 30,000,000 we already know.” In other words, the strategic key is to start with the clue about the number of females having grown by 30,000,000.

(Note that to analyze the log we must disentangle line 358 from the middle of the two fragments of Dan’s text and re-join Dan’s text. Mic’s question (line 358) is posted at the same time as Dan’s proposal, and as a consequence it is ignored and left as a failed proposal.14 chapter 21)

Mic’s next turn (lines 360-364) picks up on the 30,000,000 figure from Dan and tries to take it further by adding the fact that came before that figure in the problem statement, namely that “Today the ratio of male to female users is about 1 to 1.” Mic puts this forward and asks for the group to continue to develop the strategy.

Mic’s contribution is not the expression of some rational problem solving that we might speculate took place in Mic’s mind. In fact, his contribution—if considered as an individual proposal with math content—only vaguely suggests a mathematical logic. It was primarily an interactive move to keep the group effort going. Following Dan’s posting to the chat, there was an unusually long pause of 18 seconds. In face-to-face conversation, a pause of a few seconds is embarrassingly long and exerts considerable pressure on the participants to make another contribution; in chat, 18 seconds can have a similar effect. So Mic repeats Dan’s reference to 30,000,000. Following another long pause of 16 seconds, Mic adds the reference to the 1-to-1 ratio. He then explicitly calls on the other group members to join in. He admits that he cannot take it further himself, and he laughs.

Cosi, Dan and Mic have a good laugh at Mic’s expense, taking his contribution as a practical joke, as an attempt to look like he was making a significant mathematical contribution and then stopping short of delivering. This fills in an otherwise discouraging silence during which no one knows how to advance mathematically with the problem. The laughter lightens up the interaction, allowing people to throw ideas into the mix without worrying that they will necessarily be taken too seriously if they are only partial, or even wrong. After Mic’s jackass-like behavior, any other contribution would seem an improvement. In fact, Mic’s proposal and request are taken up.

Hal then proposes that the answer “would be 60,000,000” (line 371). This is a direct consequence of finishing Mic’s partial proposal. If there are 30,000,000 females (line 360) and the ratio of males to females is 1 to 1 (lines 361-362) and you want to know the total number (line 351), then the conclusion that “it would be 60,000,000” is at hand. Mic takes this to be the answer to problem 10 and tries to take partial credit for it by pointing out, “u see I helped” (lines 373-375).

At that point, Cosi suggests the group should go on to problem 11 and “just guess” on 10 (line 376). This declines to affirm Mic’s acceptance of 60,000,000 as the
answer to question 10, but does so without raising this as a topic for further group
discussion. Without making a decision about 10, the group goes on to all decide that
the answer to problem 11 is C (lines 378-385, spanning just half a minute), as already
stated by Hal in line 353.

Mic then summarizes the group’s status as: “So we got B for 10 and c for 11; lets
get back to 5” (lines 384-386). At this point, Cosi objects to Mic’s continued
assumption that Hal’s 60,000,000 is the answer to problem 10. Mic and Cosi joke
about their disagreement. Again, the group’s light-hearted attitude avoids the
potential of disagreements within the group becoming disruptive of smooth group
functioning.

Cosi then formulates an argument (line 392) why the answer cannot be
60,000,000. The male and female populations cannot get higher equally (i.e., by
30,000,000 each) because they have to even out from unequal numbers according to
the problem statement. After formulating this text, Cosi checks and then corrects her
previous claim that “I think it’s more than 60,000,000” (line 387): “Oh, no wait, less
than that: 50,000,000” (lines 393-394).

Cosi is somewhat hesitant about her revised claim. First she checks it and says,
“Yeah, it’s that” (line 395), followed by the hedge, “I’m pretty sure” (line 396). Mic
continues the laughter and then requests an account of how Cosi is pretty sure that
the answer should be 50,000,000.

After a 19 second pause, Cosi takes the extended expository turn that Mic had
offered her and the others had left open. She lays out a concise proof of her claim.
Her argument concerns the increase in the number of females and the ratios of male
to female users—the issues raised at the beginning of the group discussion by Dan
and Mic. It is plausible that Cosi used the 19 second pause to reflect upon the
solution that the group had come to and that her contributions had completed. Thus,
her well worked out retrospective account seems like the expression of her mental
work in constructing the narrative explanation, although her earlier contributions to
solving the math of the problem seemed more like spontaneous reactions to the flow
of the group discourse.

A solution to problem 10 carried out from scratch using algebraic methods that
translated the word problem into a set of equations to be solved for unknown values
would have looked very different than Cosi’s argument. Her contributions to the chat
did not express an independent, individual approach to the problem. Rather, they
were responses to preceding contributions. Cosi’s texts performed checks on the
previous texts and extended their arguments in directions opened up and called for by
those previous contributions. Although Dan, Mic and Hal did not carry out the
further steps that their own contributions required, they succeeded in starting a
discourse that Cosi was able to repair and complete.

This analysis of the log excerpt gives a more group-centered view of the
collaborative solving of the math problem by the group. Of course, at the level of
individual postings, each contribution was that of an individual. But it is not
necessary to see those contributions as expressions of prior private mental activities.
Rather, they can be seen as responses to the previous texts, the context of the
problem-solving task (e.g., the elements of the problem 10 text) and elicitations of
contributions to come. These ties of the individual postings to the sequentially
unfolding group discourse can be seen in the form of the postings themselves: single
utterances do not stand on their own, but make elliptical references to previous
mentionings, indexical references to matters in the physical and discourse situation
and projective references to anticipated future responses or actions of other people.14,
chapter 12 The references weave a temporal fabric of discourse that defines the meaning
of each text within its narrative context. Thus, the individual contributions are
incorporated into a problem solving dialog at the group unit of analysis, which is where the meaning of the log is constructed.

In weaving the discourse fabric, groups use different methods. We have discussed two methods of group discourse used in math problem solving in this chat: exploratory inquiry and expository narrative. In the excerpt concerning problem 10, we have seen that the group first explores a solution path by different students making small contributions that build on each other sequentially. When a candidate answer is reached that someone is “pretty sure” about, that person is asked to provide an extended account or proof of the answer. Thus, Cosi participates first in the joint exploratory inquiry and then provides an expository narrative. Both these methods are interactive discourse methods that involve responding to requests, structuring texts to be read by other group members and eliciting comments, questions and uptake.

Conversation analysts have identified *adjacency pairs* as a powerful way in which meaning is interactively constructed. An adjacency pair is a set of utterances by different people that forms a smallest meaningful unit. For instance, a greeting or a question cannot meaningfully stand alone. You cannot meaningfully express a greeting or a question without someone else being there in the discourse to respond with a return greeting or an answer. The other speaker may ignore, decline or respond to your greeting or question, but your utterance cannot be a greeting or a question without it addressing itself to a potential respondent. The respondent may just be an imaginary dialog partner if you are carrying out the dialog in your mind. Adjacency pairs are fundamental mechanisms of social interaction; even very young speakers and cognitively disabled speakers (e.g., advanced Alzheimer sufferers) often respond appropriately to greetings and questions. Adjacency pairs are important elements for weaving together contributions from different participants into a group discourse.

When I analyzed a different online chat of mathematics problem solving, I defined an adjacency pair that seemed to play a prominent role. I called it the *math proposal adjacency pair*. In that chat, a math proposal adjacency pair consisted of a problem solving proposal by one person followed by a response. The proposal addressed the other students as a group and required one or more of them to respond to the proposal on behalf of the group. The proposal might be a tactical suggestion, like “I think we should start with the 30,000,000 figure.” Alternatively, it might be a next step in the mathematical solution, like “They can’t get higher equally and even out to a 1 to 1 ratio.” The response might simply be “k”—“okay, that’s interesting, what’s next?” The pattern was that progress in problem solving would not continue after someone made a proposal until the group responded to that proposal. If they responded affirmatively, a next step could then be proposed. If they responded with a question or an objection, then that response would have to be resolved before a next proposal could be put forward. It was important to the group that there be some kind of explicit uptake by the group to each proposal. A counter-example proved the rule. One participant made a failed proposal. This was an attempt to suggest a strategy involving proportions. But the proposer failed to formulate his contribution as an effective first part of a math proposal adjacency pair, and the rest of the group failed to take it up with the necessary second pair-part response.

In the chat we are analyzing now, the math proposal adjacency pairs have a somewhat different appearance. We can identify proposals in, for instance, lines 352, 357, 360, 362, 371, 387, 392 and 394. None of these is followed by a simple, explicit response, like “ok.” Rather, each is eventually followed by the next proposal that builds on the first, thereby implicitly affirming it. This is an interesting variation on the math proposal adjacency pair method of problem solving. It illustrates how
different groups develop and follow different group methods of doing what they are
doing, such as deciding upon answers to math problems.

If we combine the proposals from Mic, Dan, Hal and Cosi, they read like a
coherent cognitive process of an individual problem solver:

How can I figure out the increase in users without knowing the
total number of internet users? (Mic) It seems to all come from the
30,000,000 figure. (Dan) 30,000,000 is the number of increase in
American females. Since the ratio of male to female is 1 to 1, (Mic)
the total of male and female combined would be 60,000,000. (Hal)
No, I think it must be more than 60,000,000 because the male and
female user populations can’t get higher at equal rates and still
even out to a 1 to 1 ratio after starting uneven. No, I made a
mistake, the total must be less than 60,000,000. It could be
50,000,000, which is the only multiple choice option less than
60,000,000. (Cosi)

Mathematical problem solving is a paradigm case of human cognition. It is
common to say of someone who can solve math problems that he or she is smart. In
fact, we see that taking place in line 404. Here, the group has solved the problem by
constructing an argument much like what an individual might construct. So we can
attribute group cognition or intelligence to the group.14, esp. chapter 19

Unfortunately, the group of students in the chat log does not seem to attribute the
problem solving intelligence to itself, but only to one of its members, Cosi. Because
she takes the final step and arrives at the answer and because she provides the
narrative account or proof, Dan says of her, “very smart” (line 404). Later (line 419),
Cosi agrees, downgrading the self-praise by using it to close the discussion of
problem 10 and of her role in solving it by proposing that the group move on to a
remaining problem: “Ok great, im smart, lets move on.” Casting Cosi as the smart
one who solves problems leaves Mic cast as the jackass or class clown when in fact
Mic is very skilled at facilitating the chat so that the whole group solves problems
that neither Mic nor the others solved independently.

There is an ideology of individualism at work here that encourages both
educational researchers and student participants to view problem solving as an
accomplishment of individuals rather than groups. This has serious consequences for
the design and adoption of groupware to support problem solving, as well as for
research methodology and student learning. If groupware designers tried to support
collaborative interactions, then they might design more than just generic
communication platforms for the transmission of expressions of personal ideas. If
researchers studying the use of groupware focused on processes of collaboration and
the methods that groups used to solve problems—as opposed to treating exclusively
individuals as cognitive agents—then research methods might focus more on
conversation analysis,17 video analysis24 and their application to discourse logs than
on surveys and interviews of individual opinions. If students using groupware
conceived of their work as interactively achieving a group solution, they might take
more advantage of groupware collaboration features and might structure their textual
contributions more explicitly as parts of an interwoven fabric of collaborative
knowledge-building group discourse.
In the previous section we saw a small group solve a difficult math problem which none of the group members had managed to solve individually beforehand. Attributing the solution to the group rather than to the sum of the individuals in the group was motivated by seeing that the construction of mathematical meaning in the solution process was done across individuals. That is, meaning was created by means of interactions among individual contributions (postings) to the chat—such as through adjacency pairs—more than by individual postings construed as expressing a series of personal mental representations.

The group has collectively produced a stream of utterances that follow each other to form a problem-solving narrative. This narrative is virtually indistinguishable from the problem-solving narrative earlier attributed to an individual, Cosi. Psychologists like Williams James and novelists like Jack Kerouac have described narratives that we tell ourselves silently about what we are doing or observing as our “stream of consciousness.” This “inner voice” rattles on even as we sleep, making connections that Sigmund Freud found significant (if somewhat shocking in his day). In what sense might online chats—with their meanderings, flaming, associative referencing, unpredictable meaning making and unexpected images—deserve equal status as streams of (group) consciousness? Perhaps group cognition can also be considered conscious in this sense.

Our sense of time and the rhythms of life are largely reliant upon the narratives we tell ourselves. We know that we have already lived through a certain part of the day or of our life because we place the present in the nexus of its ties to our memories of the past or our hopes for the future. In similar ways, the web of references in a chat that connects postings to prior postings to which they respond and to future postings that they elicit defines a temporality of the chat. This is a lived sense of time that is shared by the group in the chat. Like our individual internal clocks, the group temporality must be attuned to the larger world outside—the world of family life that calls the students away from the chat for dinner or the world of school that interrupts a chat with class changes or homework pressures. The temporality that is constructed as a dimension of the collaborative experience is constrained by the nature of its social situation and technological environment.

The fact that meaning is created at the group unit of analysis rather than by particular individuals suggests the notion of group cognition. The traditional view of cognition, particularly in Western philosophy since Descartes, is that meaning, ideas and thoughts are created in individual minds. Recent theories in cognitive science formulate this in terms of mental representations in the heads of individuals—an approach that has been critiqued by more recent theories of situated and distributed cognition. The mental contents of individual cognition—in the traditional view—can subsequently be expressed in language and communicated through the external world, to be then interpreted in the minds of other individuals. Meaning, in this view, exists only in individual minds, and cognition is always personal.

Whether or not one accepts some version of the cognitivist view in general, it seems that in situations of collaboration notions of shared meaning and group cognition are useful and important. Here, “shared meaning” has a deeper significance than what seems to underlie Clark’s analysis of common ground, where sharing is reduced to coordination among the individual mental contents of several minds. In chat, shared meaning is constructed across pairs or triplets of postings by more than one participant. It is not that an answer to a question implies that the answerer has in mind the same thing as the questioner, but that the answer and the question by
themselves are fragmentary; they have meaning only as part of the question-answer interaction. The unit of meaning is the interaction itself, and this is a group phenomenon not an individual one. Moreover, with adequate capture of collaborative interactions, it is possible to see the construction of meaning in the traces of interaction; it is not necessary to hypothesize about hidden mental operations or contents.

Of course, in some sense it is easier to visualize individuals than groups as cognitive agents. We are used to identifying other individuals as meaning-expressing agents. Given our perceptual orientation to a primarily visual world, it is more natural for us to assign agency to physical objects like human bodies than to more abstract entities like online groups. What, we may well wonder, is a collaborative group?

Groups constitute themselves. We can see how they do this in the chat logs. At one level the VMT service brings several students together and locates them in a chat room together. It may supply a math problem for them to work on and it may provide a facilitator who introduces them to the environment. At this point, they are a potential group with a provisionally defined membership. The facilitator might say something like, “Welcome to our first session of Virtual Math Teams! I am the facilitator for your session. . . . As a group, decide which question you would like to work on.” Here we can see that the facilitator has defined the group (“as a group . . . you”) and distinguished her own role as outside the group (“I am the facilitator . . . your session”). The potential group projected by the facilitator need not necessarily materialize. Individual students may come to the setting, look around, decide it is lame, and leave as individuals. However, this rarely happens. Sometimes an individual will leave without ever interacting, but as long as there are enough students there, a group emerges.

Apparently, students come to the chat environment with motivations, expectations and experiences that are generally sufficient to get a group started. One can see the group form itself. This is often reflected in the shift from singular to plural pronouns: “How do we do this?” We saw this in Mic’s admonition to the group, “u all thought i was gonna figure it out didn’t u, huh?” He is shifting the burden of the math work from himself as an individual to the group as a collectivity. Through his use of “u all,” Mic constitutes the group. Through his statement, he posits the group as a recipient of the statement and elicits a response from them. Someone other than Mic must now respond to him on behalf of the group. When Hal says, “it would be 60,000,000,” he is accepting Mic’s statement as a task for the group to work on and in so doing he makes a proposal about how the group should go about solving this task. In this interchange, the group (a) is projected as an agent in the math work, and (b) actually becomes an agent of meaning making because the meaning of Mic’s statement is defined by the interaction within the group (e.g., through its elicitation of Hal’s response).

If the group experience is a positive one for the participants, they may want to return. Some chats end with people making plans to get together again. In some experiments, the same groups attended multiple sessions. We would like to see a community of users form, with teams re-forming repeatedly and with old-timers helping new groups to form and learn how to collaborate effectively.

The recognition that collaborative groups constitute themselves interactionally and that their sense making takes place at the group unit of analysis has fundamental methodological implications for the study of collaboration. The field of computer-supported collaborative learning (CSCL) was founded a decade ago to pursue the analysis of group meaning making. We view the research described here as a contribution to the CSCL tradition.
4. Designing Groupware to Support Group Cognition

We are designers. Our goal is to design an exciting mathematical group experience for students. We want to design an online collaborative service, with strong pedagogical direction and effective computer support. We approach this by trying to understand how groups of students construct their experience in such settings. Because we are designing a computer-supported experience that has never before existed and because we want our design to be based on detailed study of how students actually create their collaborative experience in the environment we are designing, we follow a highly iterative try-analyze-redesign cycle of design-based research.

When students enter our website, they are confronted by a densely designed environment. The lobby to our chat rooms is configured to help students find their way to a room that will meet their needs. In the room, there is a bewildering array of software functionality for posting and displaying chat notes, drawing geometric forms and annotating them, keeping track of who is doing what and configuring the space to suit oneself. There may be a statement of a math problem to solve or an imaginary world to explore mathematically. The service, problems and software are all designed to enhance the user’s experience. But how can a student who is new to all this understand the meanings of the many features and affordances that have been built into the environment?

Groups of students spontaneously develop methods for exploring and responding to their environments. They try things out and discuss what happens. A new group may doodle on the whiteboard and then joke about the results. They bring with them knowledge of paint and draw programs and skills from video games, SMS and IM. The individuals may have considerable experience with single-user apps, but react when someone else erases their drawing; they must learn to integrate coordination and communication into their actions. The math problems they find in the chat rooms may be quite different from the drill-and-practice problems they are used to in traditional math textbooks. It may take the group a while to get started in productive problem solving, so the group has to find ways to keep the group together and interacting in the meantime. There may be various forms of socializing, interspersed with attempts to approach the math. As unaccustomed as the math may be, the students always have some knowledge and experience that they can bring to bear. They may apply numerical computations to given values; try to define unknowns and set up equations; graph relationships; put successive cases in a table; use trigonometric relationships or geometric figures; draw graphical representations or add lines to an existing drawing. Mainly, they put proposals out in the chat stream and respond to them. Sometimes the flow of ideas wanders without strong mathematical reflection. Other times, one individual can contribute substantial progress and engage in expository narrative to share her contribution with the group.

Chat is a highly constrained medium. Participants feel various pressures to get their individual points of view out there. In a system like we are designing, there is a lot to keep track of: new postings, changes to the whiteboard, signs that people are joining, leaving, typing, drawing. Small details in how something is written, drawn or referenced may have manifold implications through references to present, past or future circumstances. Students learn to track these details; apply them creatively; acknowledge to the group that they have been recognized; check, critique and repair them. Each group responds to the environment in its own way, giving group meaning to the features of the collaborative world and thereby putting their unique stamp on their group experience.
In the process, they create a group experience that they share. This experience is held together with myriad sorts of references and ties among the chat postings and drawings. Often, what is not said is as significant as what is. Individual postings are fragmentary, wildly ambiguous, and frequently confusing. In lively chats, much of what happens remains confusing for most participants. Clarity comes only through explicit reflections, up-takes, appreciations, or probing. The interactions among postings, at many levels, coheres into a stream of group consciousness, a flow of collaboration, a shared lived temporality and, with luck, an experience of mathematical group cognition.

As we have seen in this paper, when students enter into one of our chats they enter into a complex social world. They typically quickly constitute a working group and begin to engage in activities that configure a group experience. This experience is conditioned by a social, cultural, technological and pedagogical environment that has been designed for them. Within this environment, they adopt, adapt and create methods of social practice for interacting together with the other students who they find in the chat environment. Over time, they explore their situation together, create shared meaning, decide what they will do and how they will behave, engage in some form of mathematical discourse, socialize and eventually decide to end their session. Then our job begins: to analyze what has happened and how the environment we are designing both permits and conditions the collaborative experiences that groups construct there.

The first step in thinking about the design of groupware today is to understand the methods that groups use to accomplish problem solving, scientific inquiry, decision making, argumentation and the other tasks that they want to do. Generic communication platforms developed to meet the needs of corporations will continue to make new technologies available in response to market pressures. Within education, course management systems to support the administration of distance education will proliferate under their own economic drives. But those developments are almost exclusively guided by a philosophy of individual cognition and the transfer of representations of mental contents.

The preceding analysis of a case study of group cognition suggests a variety of new design principles. Clearly, one or two case studies is not enough to inform a new approach to groupware design. This paper has only suggested the kind of analysis that is needed to investigate and characterize the methods that groups of students might use to do their work collaboratively. Different age groups, tasks, cultures and environments will introduce considerable variety in how groups constitute themselves, define their work, socialize, problem solve, persuade, guide, decide, conclude, etc. Nevertheless, a number of principles can already be suggested. It is important to start thinking about groupware design because ideas for innovative functionality and prototypes of new components will have to be tried out with online groups and the resultant logs analyzed. One cannot know how new technologies will lead to new member methods without such investigation.

Here are some very preliminary suggestions for groupware design principles:

**Persistence and visibility.** Make the group work visible and persistent so that everyone in the group can easily see what has been accomplished by all members. Ideally, important contributions should stand out so that people do not have to search for them, but are made aware of them with little or no effort. This is a non-trivial requirement, since the work of a group quickly becomes too extensive for everyone to read and keep track of. The software must somehow help with this.

**Deictic referencing.** As discussed above, the references from one message to another or to objects in the problem context are essential to the meaning making. Software could make these references visible under certain conditions. Patterns of references
among proposals, adjacency pairs and responses between different group members
could also be displayed in order to give participants indicators about how their group
interaction is going.

**Virtual workspaces.** Ideally, the groupware would encourage noticing, recognizing
and reflecting on related contributions. There should certainly be group workspaces
for different kinds of work to be done together, creating shared artifacts. For instance,
there could be group workspaces for taking notes and annotating them, for jointly
navigating the Internet, for constructing shared drawings, for building formal
arguments together, for collecting annotated bibliographies and other lists or
collections. Issues of turn-taking, ownership and control become important here.

**Shared and personal places.** It may be useful to distinguish and sometimes to
separate individual and group work. However, it may be important to make even
the individual work visible to everyone. Group accomplishments build on the
individual contributions. Even contributions that the proposer does not consider
significant may, as we have seen above, provide a key to progress of the group. In
addition, group members often want to know what people are doing when they are
not active in the group. Content should move fluidly from place to place.

**Computational support.** Of course, a major advantage of having groupware systems
running on computers is that they can provide computational support to the work of
their users. They can filter or tailor different views or computational perspectives
chapter 6 of materials in the chat or workspaces, as well as providing search, browsing
and annotating facilities. They can play various moderator roles.

**Access to tools and resources.** Another advantage of the networked computer
infrastructure is that groupware can provide structured shared access to information,
tools and other resources available on the Internet, for instance in relevant digital
libraries and software repositories. Group collections can be supported.

**Opening new worlds and (sub-)communities.** Finally, Internet connectivity allows
for groups and their members to participate in larger online communities and to
interact with other groups—either similar or complementary. Groupware could
facilitate the building of open-ended networks of individual, group and community
connections, or the definition of new sub-communities.

**Allowing natural language subtleties.** While computer support brings many
potential advantages, it also brings the danger of destroying the extreme flexibility
and adaptability of the natural language used in conversation and group interactions.
Groupware designs should be careful not to impose rigid ontologies and sets of
allowable speech acts for the sake of enabling automated analyses. It should permit
the use of overloaded, multiple functioning, subtle linguistic expression that is not
reified, stereotyped, coded or packaged, but that opens space for interpretation,
engagement, creativity, problem solving. As we saw in the chat, even a simple laugh
can perform multiple complex roles simultaneously. Chat is a vibrant form of human
interaction in which people exercise their creativity to invent linguistic novelties such
as abbreviations, contractions, emoticons and new ways of interacting textually.
Groupware should support this, not cramp it.
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