Re-inventing Maths Problem Design: Using Guided Collaborative Critique (GCC) in Chat Environments

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Abstract: Our research focused on the analysis of face-to-face (FTF) social interaction of groups working on maths problems using the *GCC* framework in traditional classroom setting. The *GCC* framework requires students to analyze maths problems with "solutions" that contained conceptual or oversight errors, critique these solutions with mathematical arguments, and finally fix the "solutions" collaboratively. In this study, the *GCC* framework is extended to the VMT chat medium which consists of a shared whiteboard, chat message box and tools for students to construct mathematical representations. The analyses of these sessions were based on the Collaboration Interaction Model (*CIM*), a model designed to study the knowledge construction process of complex chat transcripts. This paper will discuss how participants mediate shared understanding of mathematical representations and form mathematical arguments to construct new knowledge in the chat medium, using the *CIM* as the key instrument of analysis.

Keywords: mathematical discourse, collaborative environment, uptakes, computer-mediated communication

Introduction

The objective of education is to prepare students beyond school, as well as to teach them to apply concepts and skills learned in school to problems faced in their everyday lives (Verzoni, 1997). It is not uncommon to see engineers coming together to analyze an engineering problem, a group of specialist doctors collaborating as a team to diagnose a complicated symptom or a team technicians attempting to locate a fault in the pipeline. Typically under such circumstances, there is often a situation with a “solution” presented to the team. The team is to fix this “solution” by locating and correcting faults embedded in the “solution” itself. There is an increasing need for our students to be equipped with not only problem solving skills but with fault detection skills in a knowledge-based economy. The solution of a problem does not always require a fully constructed solution, rather the "solutions" presented at times are often distorted and the analysis of such problems with "solutions" requires adequate metacognitive skills to detect the fault, monitor the progress of the fault and perform the appropriate adjustment (Clark & Mayer, 2003). Thus one challenge for mathematics educators today will be to utilize existing mathematics curriculum in school to design problems for accommodating such learning needs.

Technology can serve as a tool, offering opportunities to integrate peer interaction with the construction of artifacts for facilitating collaborative knowledge construction. Peer interaction plays a significant role in collaborative learning (Vygotsky, 1978; Wertsch, 1985; Harasim, 1990; Baker, Jensen & Kolb, 2002) where students appropriate understanding of situations by constructing their knowledge, by acting upon it and by making their own interpretations. Conversation is a tool used in such exchanges between peers, enabling ideas to being further built and developed (Duffy & Cunningham, 1996; Christiansen & Dirckinck-Holmfeld, 1995). Appropriate pedagogies need to be devised to enable the effective use of computer-mediated resources. Indeed, educators have been urged to revamp their pedagogy in order to take advantage of the new opportunities offered by computer technologies in contrast to traditional classroom assignments, laboratory work and other face to face (F2F) methods (Laurillard, 2002; Godwin, Thorpe & Richardson, 2008). One such technology is the quasi-synchronous text chat, used by several studies (Garcia & Jacobs, 1999; Romiszowski & Mason, 1997) as a medium to facilitate computer-mediated discourse. Learners access the chat software through a network of computers connected through a server to communicate and co-construct knowledge geographically apart. A quasi-synchronous environment allows participants to read, interpret and construct messages in textual/ diagrammatic representations (rather than hearing them). They lack the audio and visual cues that are found in FTF interaction. The responses are not visible to participants until the "enter" key is pressed, not permitting any reaction from other participants until the message is externalized on the chat (Garcia & Jacobs, 1998).

This research proposes the use of the *Guided Collaborative Critique (GCC) Framework* (Wee, 2007) and computer technology such as the chat environment to help students reinforce and develop mathematical reasoning through the *GCC* online activity. This activity is designed for students to identify and rationalize their identification of errors detected within the proposed mathematical solution, and to collaboratively explore approaches to remedy the errors. Subsequent sections will elaborate further on the implementation of the *GCC* in a quasi-synchronous chat environment (the VMT chat system) and the use of the *CIM* (Wee & Looi, 2007) to analyze knowledge construction patterns in chat interaction.

Guided Collaborative Critique Framework

Several studies have supported the use of Collaborative Critiques (e.g., Lewis, Perry & Murata, 2003; Swan, 2006; Cathey, 2007) as an effective learning process for participants during collaboration. For example, Cathey (2006) reported the effectiveness online collaborative peer critique on an assignment. The concept of *Guided Collaborative Critique* (*GCC*) is derived from above mentioned studies and complements traditional methods of mathematical problem solving where students construct knowledge via reading the problem and solving it from scratch. "*Guided*" refers a sequence of structured steps to aid students in the analysis of the problem. "*Collaborative*" emphasizes use of dialogue in the group solving process to construct knowledge. "*Critique*" is associated with the group's negotiation ability to locate errors embedded in the proposed "solution", to build an argument collaboratively to substantiate their identification of the error(s) and to defend the validity of the proposed argument. In the context of this research, an error is defined as a representation identified as mathematically inappropriate in the "proposed solution".

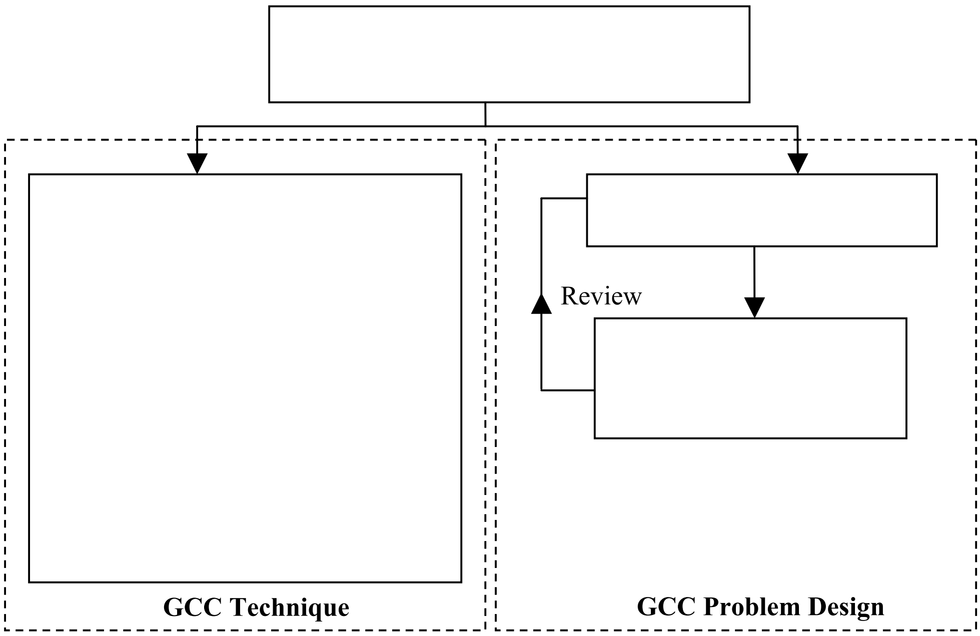

Figure 1. Guided Collaborative Critique Framework.

The Guided Collaborative Critique Framework (see figure 1) consists of two phases: *GCC Problem Design* and *GCC Technique.* The latter is a proposed problem solving method employed by students while the former emphasizes the teacher's design of the *GCC* problem. The implementation of the *GCC Technique* will be explained in the next section. The *GCC Problem Design* is formulated through the construction of the *Error Table* (see table 1 for typical student errors in junior college mathematics in Singapore), and is designed to assist teachers in rationalizing frequent conceptual issues (errors) encountered by students and incorporating these issues in the problem design.

Table 1: Error Table.

	Error Description	**Rationale**
1.	Identify the common term as 3 and not 3^{-1} when factorizing the term $(1+\frac{x}{3})^{-2}$	Students often miss out the power term when factorizing terms like $(1+\frac{x}{3})^{-2}$
2.	Identify $\left\|\frac{x}{3}\right\|>1$ as $\|x\|<3$	Students have to understand the validity (the value of x) of binomial expansion series.
3.	Take into account the $(-1)^r$ when simplifying the term	Students will learn to use the $(-1)^r$ to replace the negative signs of the numerator of

	$\frac{(-2)(-3)(-4)...(-2-r+1)}{r!}3^{-r-2}x^r$ into $(r+1)(-1)^r 3^{-r-2}x^r$	$\frac{(-2)(-3)(-4)...(-2-r+1)}{r!}3^{-r-2}x^r$ When r is even, $(-1)^r$ will be positive and when r is odd $(-1)^r$ will be negative.

The Error Table is compiled from the consensus and perspectives of a group of teachers. The first column in the Error Table describes the errors to be implemented in the problem design. The second column explains the rationale behind the error. Following the construction of the Error Table, the problem and the proposed "solution" are designed, and then iteratively reviewed to ensure the quality of test. The next section elaborates further on the GCC mathematics problem in the chat environment.

GCC Activity and the GCC Technique

The participants collaboratively work on a *GCC* activity under the topic of binomial series. All participants logged onto their terminals at a designated time allocated by the instructor and were geographically apart. The overview of the activity is as follows:

> Expand $(3+x)^{-2}$ as a series of ascending powers of x, up to and including the term in x^3, expressing the coefficients in their simplest form. State the range of values of x for which the expansion is valid. Find also the coefficient of x^{25} in the form $\frac{k}{3^m}$
>
> where k and m are constants to be found.
>
> **Proposed solution:**
>
> $$(3+x)^{-2} = 3\left(1+\frac{x}{3}\right)^{-2}$$
>
> $$= 3\left[1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{3}\right)^3+\text{L}\right]$$
>
> $$= 3-2x+x^2-\frac{4}{9}x^3+\text{L}$$
>
> Expansion is valid for $\left|\frac{x}{3}\right|>1 \Rightarrow |x|>3$.
>
> The general term is $(3)\frac{(-2)(-3)(-4)\text{L }(-2-r+1)}{r!}\left(\frac{x}{3}\right)^r$
>
> $$= \frac{(2)(3)(4)\text{L }(r+1)}{r!}3^{-r+1}x^r$$
>
> $$= \frac{r!(r+1)}{r!}3^{-r+1}x^r$$
>
> $$= (r+1)3^{-r+1}x^r$$

Embedded in the "proposed solution" are three common errors found in assignments. The students were instructed to work collaboratively in their group to identify any mistakes in the proposed solution and to follow four steps known as the *GCC Technique* to solve the problem.

Step 1: First read the question and the proposed solution. Use the VMT drawing tool to circle any error/fault identified in the proposed solution on the whiteboard.

Step 2: Explain using mathematical evidence (formula, concept, and proof) how your group identified the circle term as an error/fault. Working is to be shown in the VMT chat room.

Step 3: If any error/fault is identified, work out the solution to amend the error/fault found in the proposed solution.

Step 4: Reflect on the steps your group has taken to identify the error/fault under the Summary whiteboard.

GCC Technique is adapted from Polya's four-stage model of problem solving (Polya, 1952). The steps are designed to guide students to work collaboratively to externalize mathematical representations appropriately during discourse, to scaffold students in analyzing and solving the problem, and to conclude the session with a post-problem solving reflection.

Participants and Affordances of Chat Environment

Our target groups are 16-17 year old junior college students. They have a basic foundation in mathematics and are among the top 20% of the cohort in terms of academic ability. A total of 7 groups consisting of three college students collaboratively solved the maths problem (see figure 2) in VMT-Chat.

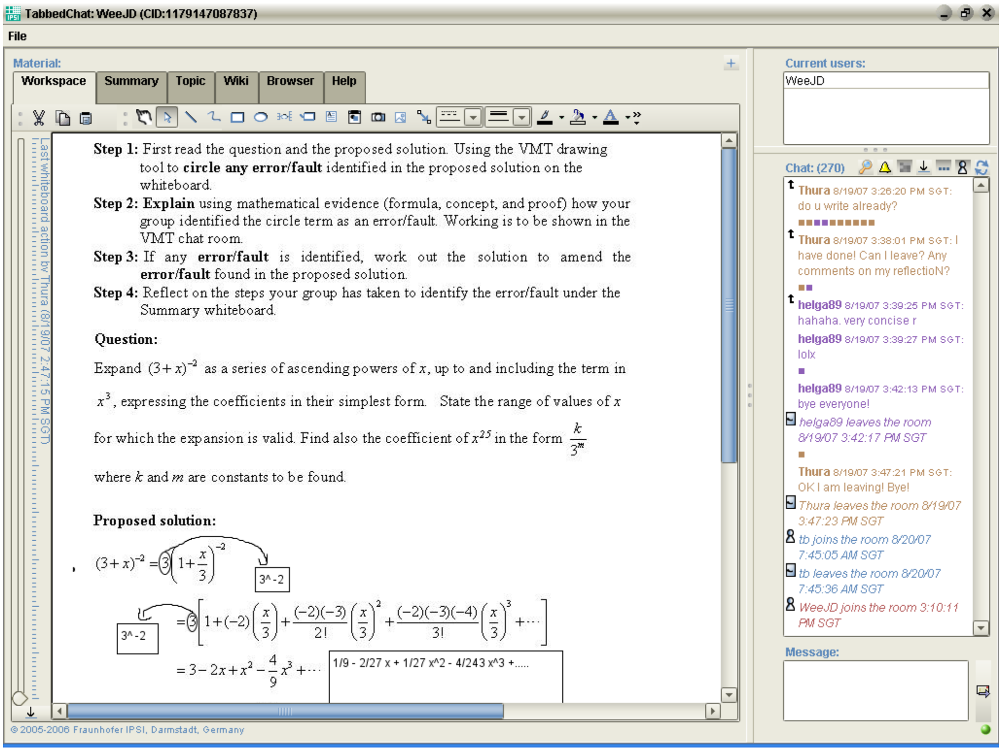

Figure 2. Interface of VMT chat room: *Workspace* Tab.

Figure 2 shows the interface of the VMT chat room under the *Workspace* Tab accessible by members of the group. The maths problem is placed on the shared whiteboard by the teacher. The chat line is used as a platform for students to converse synchronously. The students used the toolbar to construct artifacts on the shared whiteboard. For example, one student drew a circle over the maths symbol "3" (see figure 2), linking with a line connected with an arrow to the text symbol "3^-2", illustrating the amendment to the error identified.

Methods and Data Collection

In this study, a segment of the chat transcript between 3 college students solving the *Guided Collaborative Critique (GCC)* problem (see table 4) is analyzed. The chat transcript is created using the VMTplayer, a recorder that reproduces the session as it was experienced by the students. The VMTplayer plays back the entire session, capturing the moment to moment interaction between the students as they posted messages in the chat line and manipulated artifacts on the shared whiteboard. The messages are then coded into contribution units denoting a logical unit of meaning. A contribution number is analogous to the speaker taking the turn in conversation analysis (Sacks, Schegloff & Jefferson, 1974). Messages can be interrupted by any participants since visual or auditory cues are not visible to the students in such environments. When such interruptions occur, the logical unit will be separated into two contribution numbers. The Collaboration Interaction Model (CIM) is constructed based on the coded messages in the chat segment (table 2) to increase the reliability of relationships between the contribution nos. There is a probability that messages arrive out of sequence or responses may not have an adjacency relationship. An additional tool known as the *Individual Uptake Descriptor Table (IUDT)* is designed for researchers to understand the rationale of each message constructed from the student's perspectives towards triangulation of the interpretations of the relationships between the messages in the CIM. Given the tight school curriculum, the students were required to submit within 24 hours after the completion of VMT session. The methodology of the *CIM* and concepts of the *IUDTs* will not be elaborated in this paper (see Wee & Looi. 2007 for more information on the *CIM* and *IUDTs*). For the purpose of this work, the *IUDTs* are applied in construction of the *CIM* (see figure 3).

Table 2: Chat Segment of VMT (Students: Wane, Yvonne and William).

Student	Message	Contribution No
Wane	i cant remember the method for finding the coefficient	C1
Yvonne	yea	C2
Wane	do u remember tt formula we learnt in secondary school?	C3
William	it's a binomial series	C4
Yvonne	same... but there is one mistake ler	C5
William	use the binomial formula	C6
Wane	the more than sign	C7
William	yeah step by step	C8
Yvonne	i not sure cause it's power to -2	C9
Wane	the first part is correct	C10
Yvonne	can enlighten me?	C11
Yvonne	no.... that first part is wrong ler	C12
Wane	then the modules x more than 3 is wrong	C13
Wane	it should be less than	
William	first take out the 3	C14
Yvonne	when he take out constant, it will not be 3	C15
Yvonne	cos it's 3^-2	
William	yeah. coz there is a a power to -2	C16
Yvonne	but the rest of the steps i'm not very sure	C17
William	so the second line is not correct	C18
William	$3\left(1+\frac{x}{3}\right)^{-2}$	C19
Yvonne	$3\left[1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^{2}+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{3}\right)^{3}+L\right]$	C20
William	see I circle it .right?	C21
Wane	that 3 is correct	C22
Yvonne	y?	C23
William	no . that3 should also to the power to 3	C24
Yvonne	-2	C25
William	sorry.	C26
Wane	the formula is (a+ bx) power n	C27
Yvonne	the person took out the common factor	C28
Wane	William is correct	C29
Wane	i over look it	
William	but should take out the a.	C30
Yvonne	ok.... for the next part... did your spot any error?	C31
William	coz -2 is a negative value	C32
Wane	the formula is a power n (1+ bx/a) ower n	C33
William	so it cant use wingkin's formula	C34
Wane	what?????	C35
Yvonne	i don't get it	C36
Wane	hold on	C37
Wane	wait	
William	i mean if the power is a negative value. it should use(1+ax)to power of n	C38
Yvonne	ya... that's why they took out the common factor	C39
William	see?	C40
William	yes	C41
Wane	tts what i was saying	C42
Yvonne	ok...	C43
William	so move to the next line	C44

Analysis of Interaction

The following sections illustrate using the *CIM* to analyze the chat interaction, drawing on the concepts of *Conscious Effect of Situations*, *Conceptual Reinforcement, Mathematical Reasoning and Dual Feedback* to help students in the learning of mathematics.

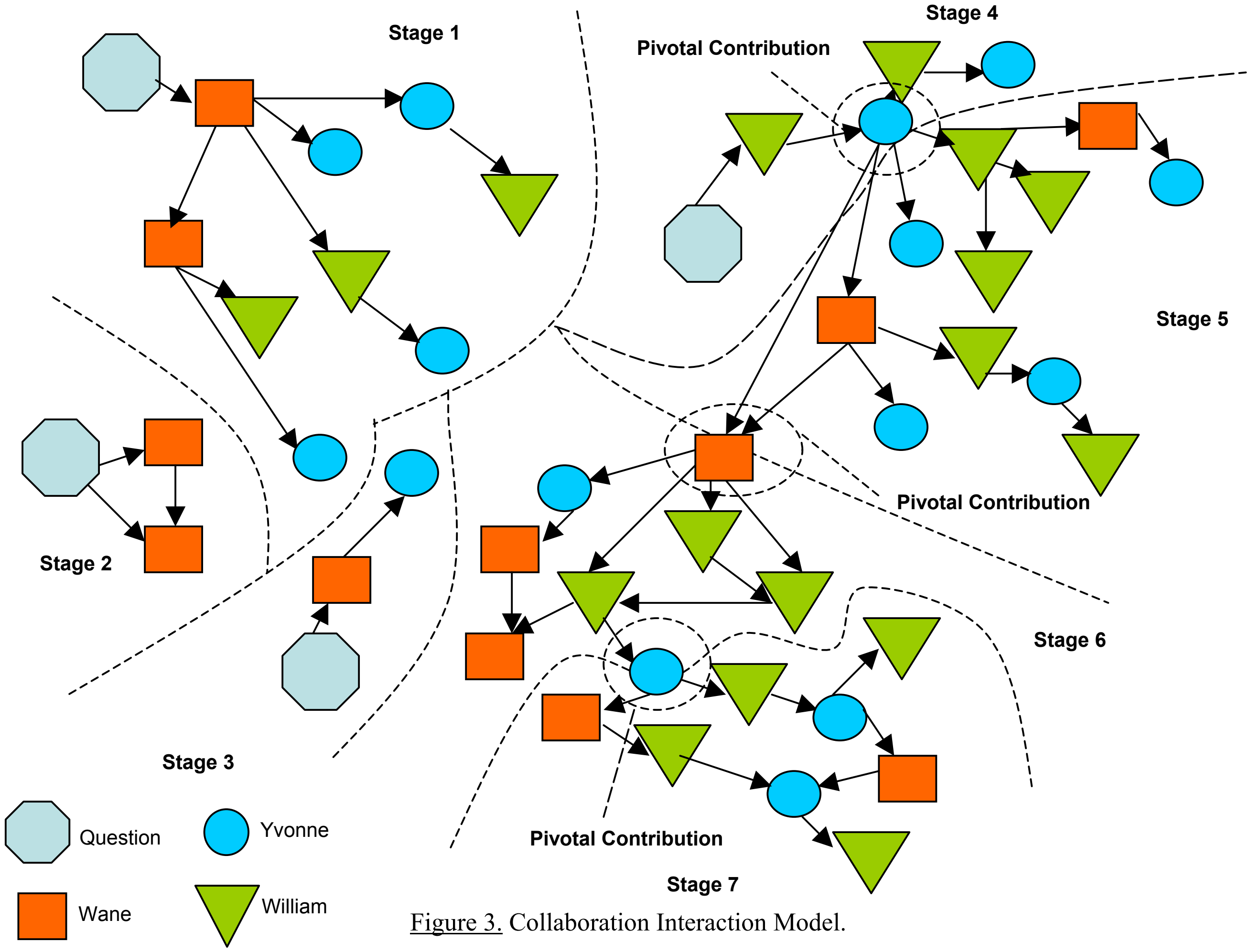

Figure 3. Collaboration Interaction Model.

Conscious Effect of Situations

The *GCC* is designed to promote awareness of embedded conceptual errors in maths problem. This is known as the *Conscious Effect of Situations.* Through negotiation (argumentation and agreement) of errors, students will become aware of possibility of encountering such errors in similar problem. Let's take the following example to illustrate how the *Conscious Effect of Situations* is realized during the discourse. The chat segment shows that Wane missed out the first error $3\left(1+\frac{x}{3}\right)^{-2}$, identified the second error $\left|\frac{x}{3}\right|>1 \Rightarrow |x|>3$ in *stage* 2 (refer to *CIM*), and proposed that the first part of the solution was free of errors [C10] (refer to table 2) leading to a rejection from Yvonne. She claimed that there was indeed an error in the first part of the proposed solution [C12]. At *stage* 4 (refer to *CIM*), Yvonne explained the need for "3" to have "the power of -2" [C16], and drew a circle over "3" of $3\left(1+\frac{x}{3}\right)^{-2}$ to reference the location of the error. William supported Yvonne's proposal [C16] for the need of the "power of -2" [C16]. At *stage* 5, Wane continued to insist that "3" was correct [C22], rejecting claims by William and Yvonne. Wane constructed the formula "a power n (1+ bx/a) power n" [C33] after several chat exchanges with Yvonne and William [C23 to C32] at *stage* 6, which is coherent with the ideas that William and Yvonne attempting to explain to Wane in *stage* 4 and *stage* 5. The negotiation of errors between the three students assisted Wane in realizing what he had missed in the first part of the proposed solution, hence creating the awareness of the need to be cautious of such mathematical conceptual errors. Although William and Yvonne have shown to understand the nature of the error, the articulation process undertaken enforced their consciousness of such mathematical situations.

Conceptual Reinforcement

The *GCC* is structured for students to investigate the "proposed" solution sequentially (the logical flow of manipulating mathematical expressions and symbols) through dialogue and external resources such as their notes and assignments in order to reinforce mathematical concepts. Conceptual reinforcement is a dialectic process between the students where representations are interpreted, co-constructed and manipulated. Let's take the following example to illustrate how this dialectic process reinforced Wane's mathematical concepts. *Stage* 1 shows William responding "it's a binomial series" [C4] to Wane's question "do u remember tt formula we learnt in secondary school?"[C3]. Wane's question was also *uptaken* by Yvonne reply "can enlighten me?" [C11].

This indicates that the group members were not sure of the binomial expansion formula used to analyse the problem. Binomial expansion was taught with the power "n" was positive in secondary school but the *GCC* problem indicated n as negative $3\left(1+\frac{x}{3}\right)^{-2}$ hence it was not appropriate to directly apply what was learned in secondary school to problem. Yvonne raised this concern "I not sure cause it's power to -2" [C9]. This issue was not only unresolved but invited strong reaction from members when Wane proposed the formula "(a+bx) power n" [C27]. William *uptook* Wane's proposal "(a+bx) power n" [C27], explaining that " 'a' needs to be taken out because -2 is a negative value" [C30, C32] and rejecting the use of the formula [C34]. Wane proposed "the formula is 'a' power n (1+ bx/a) ower n" [C33], (typo mistake of 'ower' as 'power'), and indicated that he appropriated William and Yvonne's explanations (see *stage* 5 and *stage* 6). Wane responded "what???" [C35] when William proposed that his formula could not be used [C34] indicated that he had attempted to rationalize the application of "the formula is 'a' power n (1+ bx/a) ower n" [C33], not realizing that William's proposal [C34] was actually referring to "(a+bx) power n" [C27]. Wane's mathematical concepts was reinforced when his proposal "(a+bx) power n" [C27] was re-constructed to" 'a' power n (1+ bx/a) power n", it became clear that his understanding shifted from applying the formula to cases where power n as positive to cases where power n is negative.

Mathematical Reasoning

The *GCC* aims to develop student's mathematical reasoning through the process of justification of errors. The *stages* in the *CIM* represent how students attempt to make sense of other another. Mathematical reasoning can be illustrated using three cases from *stage* 5: The first case: Yvonne is reasoning out "take out constant, it will not be 3, cos it's 3^-2" [C15] when William proposed "first take out the 3" [C14]. The second case: William attempted to reject Wane's proposal "that 3 is correct" [C22] with "no. that 3 should also to the power of 3" [C34], indicating that a power was required. The third case: Wane explained to Yvonne why he thought she was wrong to propose "take out constant, it will not be 3, cos it's 3^-2" [C15]. Wane constructed "(a+bx) power n" [C27] to substantiate his point. Although not all the cases appropriated correct mathematical reasoning (only the first two cases are correct), the *GCC* problem design assists students in making sense of mathematical situations, engage their prior knowledge on the topic and through dialogue achieve a better understanding of mathematics.

Dual Feedback

The dual feedback process informs student on the depth of their conceptual understanding of the problem, allowing them review their notes, assignment and tutorials to learn at that instance and the teacher of the group's ability in the problem. Teacher's talk in the classroom is often reflected in the student's discourse. That is to say that the way the topic is taught by the teacher will influence the student's perception and language of that topic. Using the *CIM*, teachers can understand the chat transcript (how students interpret and manipulate mathematical representations) and reflect on his/her teaching via sharing with fellow colleagues to devise an alternative approach to teach the topic. Taking for example, *stage* 1 shows that there is a possible confusion in using binomial formulas taught in secondary school and that taught in the JC. Teachers can bring across this concept to the students as differentiating between positive n and negative n power in class. Teacher may also surface this distinction of what is taught in secondary school and what is taught in JC as different to avoid conceptual confusion in preparation of future lessons.

Moment to moment Interaction

The quasi-synchronous chat environment logs the moment to moment interaction during the session. Students communicated by typing their messages in the textual representations in the chat line, construct or manipulate artifacts on the shared whiteboard basing on interpretations of representations and artifacts constructed earlier. One possible limitation of the chat environment, unlike face to face communication (e.g. classroom discussion) is that students will not be able to visually and audibly capture information (e.g. emotion, intonation of voice) during the interaction (Garcia & Jacob, 1998). However the chat environment affords them to visually capture the communicated message representation, giving the opportunity to review any representation before appropriating a response, an avenue not possible in FTF communication as information maybe lost due to selective listening.

Conclusion

Our initial explorations of the use of *GCC* designed activities show us that it can help student to be conscious of mathematical situations through the identification of errors, and it can reinforce their existing conceptual understanding of mathematics learned during normal class time. It can improve their mathematical reasoning process as they attempt to justify their choices of the errors and promote dual feedback for the educator analyzing the chat transcript. By providing an alternative approach to the learning of mathematics, the *GCC* activity implemented in chat environment provides a context for students to engage in meaningful discourse. Teachers should tailor the *GCC* problem to suit the needs of the class rather than adopting a "one size fits all" concept. They should explore the use of alternative assessment for the learning process in the chat environment rather than assessing students on producing a complete solution or answer to a mathematics problem. Subsequent research will explore running more *GCC* sessions on various mathematical topics to measure its effectiveness in providing students the opportunity to identify conceptual errors and co-construct proposed solutions in a collaborative setting.

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Acknowledgments

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