Bridging and Persistence in Sustained, Collaborative Problem Solving Online

Johann W. Sarmiento, Gerry Stahl
Drexel College of Information Science and Technology /The Math Forum
jsarmi@drexel.edu, gerry.stahl@drexel.edu

Abstract

The Virtual Math Teams (VMT) project investigates the innovative use of online collaborative environments to support effective mathematical problem-solving by small groups of learners in an online community. A key research issue for us is to understand how team interactions evolve over time and how to support them effectively. As designers, we expect the persistent records of the teams' interactions provided in the VMT environment to contribute to the sustainability of the teams' knowledge work. Our analysis of data collected from distributed virtual teams working on open-ended mathematical tasks across multiple sessions shows the situated usage of such persistent artifacts. In particular, we observe a series of "bridging" methods used to co-construct mathematical knowledge over time, evolve a sense of collectivity, and interlink the online environment with other interaction spaces. We propose that bridging, the purposeful crossing of interactional boundaries, is a consequential and often unsupported aspect of the collaborative user experience.

1. Introduction

It is almost eight at night and David, a high school student in suburban Phoenix is about to log into the online collaboration environment where, in the last few weeks, he and three other members of his virtual math team have been exploring a non-Euclidian geometrical world, generating their own mathematical questions, and exploring them. David missed the last team session so he is curious about how far the team got on the topics they were discussing the last time he was with them. He had received a summary of the missed session from the system and a message with comments written by a math hobbyist who is mentoring the team, but he is now eager to look at the original discussion and follow the evolution of ideas developed. After logging into the chat room and reviewing the postings he had missed, David gets an idea for a new problem: a geometrical space with holes which make the distance between certain points in space zero. Since he has some extra time before the session, he starts working on some images to present his new idea to the team and to share it in the community space where other interested teams can explore it as well.

The availability of persistent records of online interactions (e.g. via threaded discussions, persistent chat, wiki histories, etc.) provides unique opportunities for participants to orient in new ways towards the activities they carry out jointly in diverse interaction spaces. For instance, designers and analysts have pointed out potential altering effects of relaxing some of the requirements imposed by face-to-face conversation. Both, opportunities (e.g. carrying out very-large-scale conversations) and interactional difficulties (e.g. lack of coherence) have been studied (e.g. [1], [2], [3]). In the particular context of long-term, team problem-solving conducted online (as described in the scenario above), persistent records of the teams' interactions over time and the artifacts created by them, may support the establishment of continuity, aid in the sustainability of the joint knowledge work, and provide flexibility in how participants take part in the interactions. However, the effectiveness of these persistent artifacts has to be validated through the observation and analysis of the methods used by teams to conduct their interactions and the strategies used to establish their own situated sense of continuity. In this paper we present a preliminary analysis of a number of experimental virtual math teams using diverse social and technical supports.

2. Background: Virtual Math Teams @ the Math Forum

The Math Forum is an intentional, online community, active since 1992. It promotes technology-mediated interactions among teachers of mathematics, students, mathematicians, staff members and other interested parties committed to learning, teaching, and doing mathematics. As the Math Forum community continues to evolve, the development of new interaction supports becomes increasingly essential for sustaining and enriching the mechanisms of community participation available [4]. As an example of these endeavors, the Virtual Math Teams (VMT) project at the Math Forum investigates the innovative use of online collaborative environments to support effective secondary mathematics learning in small groups [5]. The VMT project is an NSF-funded research program designed to investigate sustained collaborative problem-solving in computer-supported environments and to characterize how members of the Math Forum's community of learners constitute their interactions over time to foster their development as learners of mathematics. VMT implements a multidisciplinary approach to research and development that integrates the quantitative modeling of students' online interactions, ethnographic and conversation analytical studies of collaborative problem solving, and an iterative process of software design.

Central to the VMT research program are the investigation of the nature and dynamics of group cognition [6] as well as the design of effective technological supports for quasi-synchronous small-group interactions, and its linkages with distributed, asynchronous interactions at the level of the online community. We are currently studying how middle school students do mathematics collaboratively in online chat environments. We are particularly interested in the *methods* that they develop to conduct their interactions in such an environment. Taken together, these methods define a culture, a shared set of ways to make sense together. The methods are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to the participants. These methods help define the nature of the collaborative experience for the small groups that develop and adopt them.

3. Persistent Resources for Virtual Math Teams

In our iterative design-based research approach [7], we started by conducting chats in a variety of commercially available environments: AOL Instant Messenger, Babylon, Blackboard, WebCT. Based on these early investigations, we concluded that we needed to add a shared whiteboard for drawing geometric figures and for persistently displaying notes. We also found a need to minimize "chat confusion" by supporting explicit referencing of conversation threads [8, 9]. We decided to try ConcertChat, a research collaboration environment combining persistent chat with a shared whiteboard and a set of referencing tools [10]. By collaborating with the software developers, our educational researchers have been able to successively try out versions of the environment with groups of students and to gradually modify the environment in response to our research. Some of ConcertChat's interactional supports include:

- Chat conversations are persistent during and after each session. Latecomers automatically receive the last ten messages when joining a session and can load all previous messages at will.
- A chat user can post a new message with an explicit graphical link pointing to one or more previous messages. The graphical link between the two messages is displayed until a new message gets posted in the chat but can be shown again by a user by clicking on the linking message.
- The shared whiteboard allows chat participants to create drawings and shared graphic information with each other. Every whiteboard action is recorded as part of the evolving history of the whiteboard. Users can manipulate a slide bar to navigate through all changes made to whiteboard since the creation of the chat room.
- When someone types a new chat message, they can also select and point to a rectangular area in the whiteboard. When that message appears in the chat as the last posting, a bold line appears connecting the text to the area of the drawing (see Figure 1).

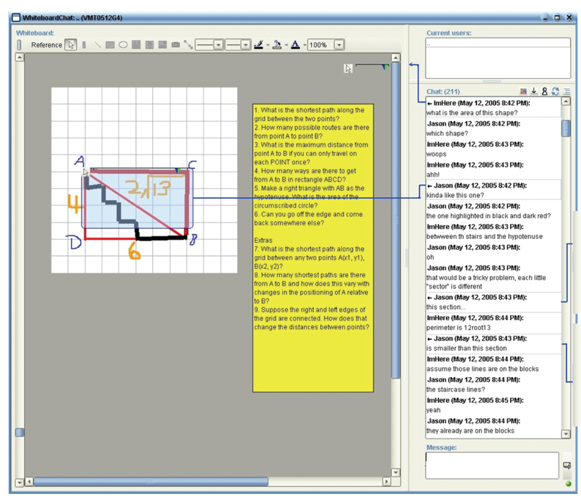

Figure1. ConcertChat collaboration environment with references from chat to whiteboard

4. Pilot Experiments

During the spring of 2005 and 2006, we conducted two pilot experiments to explore the sustained use of ConcertChat for virtual math teams.

Each team contained about four non-collocated upper middle-school and high-school students selected by volunteer teachers at different schools across the USA. The teams engaged in online math discussions for four hour-long sessions over a two-week period. They were given a brief description of a non-traditional geometry environment: a grid-world where one could only move along the lines of a grid [11]. The students were encouraged to generate and pursue their own questions about the grid-world, such as questions about shortest paths between two points in this world. The chats were facilitated by a member of our research project team. In each session, the facilitator welcomed students to the chat, introduced the task, and provided technical assistance regarding the special features of the collaboration environment. The facilitator did not actively participate in the team's mathematical collaboration. The teams used a different room for each one of the four sessions during the sessions conducted in 2005 and the same persistent room for all four sessions in those conducted in 2006.

The analysis presented below uses the approach of ethnomethodology [12] to examine recordings and artifacts from the team sessions in order to draw design implications for a full-scale online math discussion service. Ethnomethodology is a phenomenological approach to qualitative sociology which attempts to describe the methods that members of a culture use to accomplish what they do, such as carrying on conversations [13], using information systems [14, 15, 16] or doing mathematics [17]. As part of the phenomenological perspective, ethnomethodology is based on naturalistic inquiry to inductively and holistically understand human experience in context-specific settings [18].

To guide the presentation of our preliminary findings, we will use the trajectory of participation of team 5 in the 2005 pilot experiment and compare some of the features of their interactions with related aspects of the interactions derived from our analysis of the sessions held in 2006. The problem-solving trajectory of team 5 is particularly interesting since its subtle changes in membership across the four sessions offered a series of opportunities for the team's past and projected future to be an interactional focus for the participants. Figure 2 illustrates the structure of the sessions and the changes in membership of team 5 as well as the relationship of its members to two other teams participating in the 2005 pilot experiment.

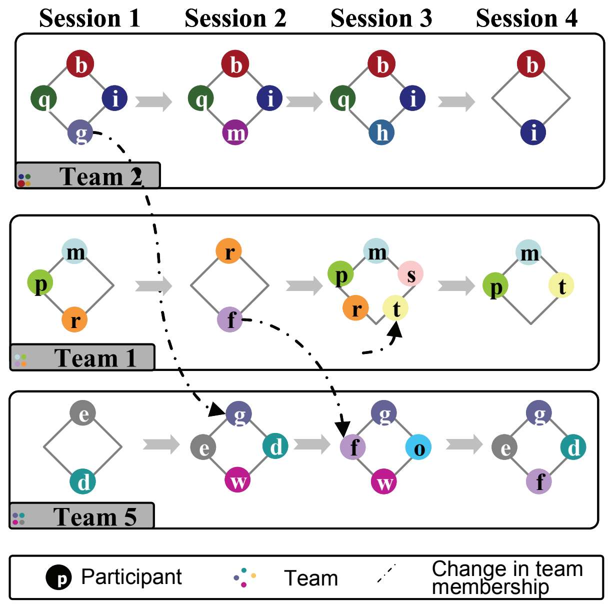

Figure 2. Teams, membership and session structure from 2005 experiments

4.1 The local interactional persistence of chat

As the group of participants in a chat conversation increases, the number of messages posted usually increases as well and, as a result, the task of reading them becomes a challenge. It has been argued before that it is precisely thanks to the persistent aspect of some electronic chat systems that a participant can attempt to pace the reading of messages and manage the coherence of the conversation (e.g. [3], [8], and [20]). As we have mentioned before, ConcertChat provides a series of special features that assist in this task. On the one hand, latecomers are able to load all previous messages posted to a chat session (or all sessions conducted in the same room). In addition, when a user posts a message she can create an explicit graphical reference to any other previous message in order to, for instance, clarify what she is responding to. When another user sees the message displayed in the chat he can follow an arrow that points to the original message and also scroll back in the persistent history of the chat to reflect on the relationship between the two messages. For the purpose of our current analysis, we are interested in exploring the ways in which these basic system supports were used during the teams' problem-solving interactions.

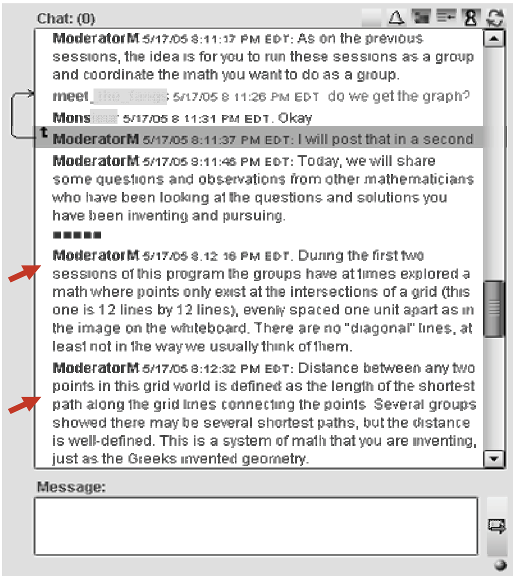

Figure 3. Chat excerpt from Team 5, session 3 (2005)

In Figure 3 we can see how the long messages posted by the facilitator rapidly occupy the chat window. Since these messages contain important task information it is unfortunate that they will soon be out of sight when new messages get posted. The problem-solving trajectory of this team reveals that especially for new team members it was difficult to adopt the perspective of the problem situation (i.e. a non-Euclidian world where you can only move along the lines of the grid). In Figure 4 we can see how, *meet* tries to resolve this problem of understanding by creating a graphical reference to the original problem description posted by the facilitator.

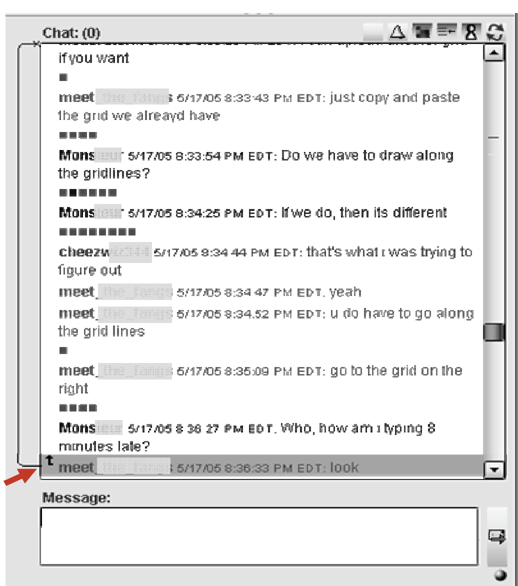

Figure 4. Chat Excerpt from Team 5, Session 3 (2005)

The end result of this action —re-framing critical information posted previously—achieves a crucial and common interactional goal for a team and one apparently supported effectively by the affordances of graphical references between persistent chat postings. In our complete dataset we see a diverse set of usages of this system feature which confirm the value of these types of system supports as already documented in the literature. (A more complete analysis of the role of shared referencing in problem-solving chats can be found in [20]).

One could argue that the facilitator of the session should have posted the content of the task description in the whiteboard instead of using the chat; since the objects shown in that interactional space are potentially much more persistent. Participants also used the chat interface often to present problem solving information and creating interactional troubles as can be seen in Figure 5. In this exchange we can observe an instance in which the fact that a problem-solving proposal (i.e. *Mons*' "theory") is no longer visible generates confusion in the team.

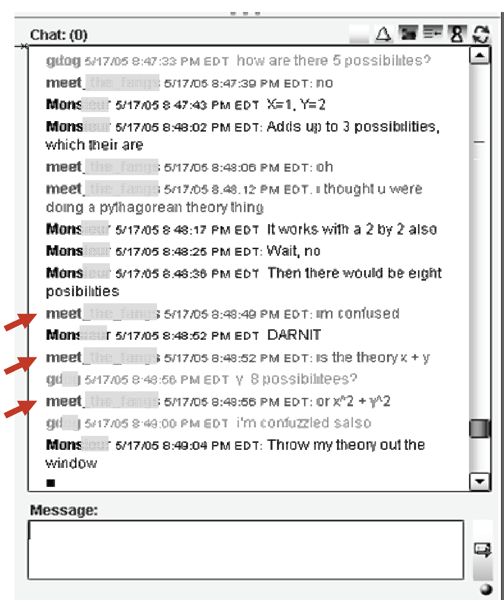

Figure 5. Chat excerpt from Team 5, session 3 (2005)

Over time, however, we can see in the ways that the different teams organized the use of the chat and the shared whiteboard how some of them developed an awareness of this interactional difference and attempted to use the whiteboard space as a place to manage mathematical proposals, intermediate findings, and results. We will analyze the uses of the persistent aspect of the whiteboard space for sustained problem solving in the next section.

4.2 Managing persistent resources through the shared whiteboard

As often documented in the literature on computer-supported cooperative work (CSCW) and collaborative learning (CSCL), the interactional persistence offered by the shared whiteboard is different than that provided by a chat interface (e.g. [21], [22]). As expected, the teams in our experiment demonstrated a variety of ways of producing and managing relevant information resources for each session using the shared whiteboard. These included creating visualizations of strategies and ideas already expressed conversationally through the chat, producing several contrasting representations of a problem situation, recording math proposals and their results, and coordinating different problem-solving paths among different team members. Our interest here centers on the ways that such resources were used in sustaining collaborative work across sessions.

Following the trajectory of Team 5, we can observe the effect of the fact that in our first pilot experiment records of prior work were not available to the teams. In the last session of Team 5 the participants attempted to continue the work some of them had started previously in order to discover the number of shortest paths between two points. This recommencement activity demanded a significant amount of work, much of it repetitious when compared with prior sessions. Despite this apparent inefficiency, the joint work done on the shared whiteboard seems to suggest great potential in capturing the reasoning procedures of the team and, as such, could serve as a resource for future work by this same team or by other participants.

Despite the fact that Figure 6 is a restrictively static representation of the team's use of the whiteboard, it allows us to trace how in the last session *drago* and *meet* organized their joint exploration of the number of paths between two points in the grid world. Initially, *meet* who was recounting what the team was doing in the last session reported having trouble "seeing" the paths in a 2x2 grid. *Drago*, who was not present in the previous session, shows him the different paths by labeling vertices in the grid and naming the paths using the labels (e.g. "from B to D there is BAD, BCD …"). The strategy is then re-used later when the two of them work in parallel, labeling and counting paths in two new grids. This use of the whiteboard represents an interesting way of making visible the procedural reasoning behind a concept (e.g. shortest path). The fact that a newcomer can use the persistent history of the whiteboard to re-trace the team's reasoning seems to suggest a possible strategy towards preserving complex results of problem-solving activities. However, the actual meaning of these artifacts is highly situated in the doings of the co-participants, a fact that challenges the ease of their reuse despite the availability of detailed records such as those provided by the whiteboard history.

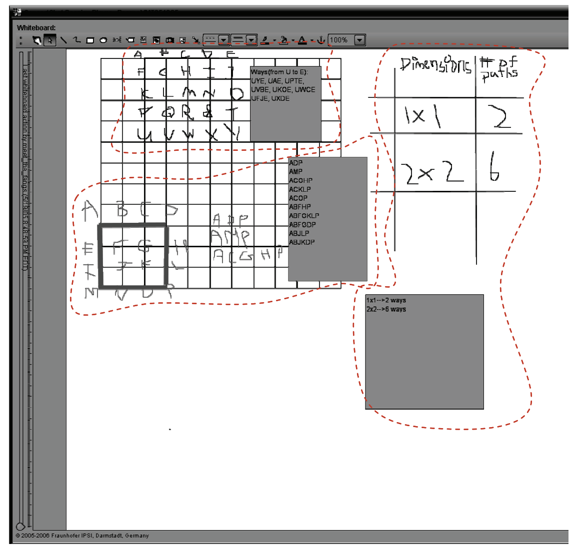

Figure 6. Shared whiteboard: Team 5, session 4.

On the other hand, we could view the artifacts created by Team 5 in their last session as particular kinds of "boundary objects" [23]. More precisely, we could see them as "bridging" objects which, in addition to being a representation of the teams' moment-to-moment joint reasoning, could also serve for their own future work and for other members of the VMT online community. These particular objects are constructed *in situ* as a complex mix of resources that "bridge" different points in their own problem-solving and, potentially, those of others. As highlighted in the different areas of Figure 6, the two team members combined the depiction of the cases being considered, the labeling and procedural reasoning involved in identifying each path, a summary of results for each case (i.e. the list of paths expressed with letter sequences) and a general summary table of the combined results of both cases. The structure of these artifacts resembles Star's original view of boundary objects as being "plastic enough to adapt to local needs and constraints of the several parties employing them" ([23] p. 46) but at the same time are transient and highly situated in the local interactions of this team requiring extra work for them to become useful.

Despite the fact that the problem-solving artifacts and conversations are the result of the moment-by-moment interactions of a set of participants and, as such, require a significant effort for others to reconstruct their situated meaning, they can serve as one of the resources used to “bridge” problem-solving episodes, collectivities or even conceptual perspectives. Here, we use the term "bridging" to characterize interactional phenomena that cross over the boundaries of time, activities, collectivities, or perspectives as relevant to the participants themselves. Bridging thereby might tie events at the local small-group unit of analysis to interactions at larger units of analysis (e.g. the community). Bridging may reveal linkages among group meaning-making efforts, across collectivities or events in time. Next, we will explore our preliminary analysis of this interactional phenomena that is closely related to the persistent aspect of the teams’ resources.

4.3 “Bridging” problem-solving episodes, trajectories and teams.

As we consider scenarios such as the one described in the introduction of this paper, in which teams with evolving membership collaborate in knowledge work over time and perform tasks that inform and are affected by other outsiders, it becomes critical to analyze how different aspects of the collaboration environment (computational and social) play a role in the crossing of interactional boundaries.

In order to investigate the dynamics of bridging we designed our experiments so that a number of teams worked on the same task for a series of four sequential sessions. In our 2005 experiments, teams used a different ConcertChat room for each session and had no direct access to archives of their previous interactions. Despite this apparent limitation, they demonstrated several strategies to reconstruct their sense of history and to establish the continuity of their interactions. The following excerpt represents and example of such attempts recorded during the second session of Team 5, where two new team members, “*gdo*” and “*mathwiz*”, join the dyad that had collaborated in the first session (“*drago*” and “*estrickm*”).

302	gdo:	now lets work on our prob [Points to Whiteboard]
303	drago:	last time, me and estrickm came up
304	drago:	that
305	gdo:	
306	drago:	you always have to move a certain amount to the left/right and a certain amount to the up/down
307	gdo:	what?
308	drago:	for the shortest path
309	drago:	see
310	drago:	since the problem last time
311	drago:	stated that you couldn't move diagonally or through squares
312	drago:	and that you had to stay on the grid
313	*gdo leaves the room*	
314	mathwiz:	would you want to keep as close to the hypotenuse as possible? or does it actually work against you in this case?
315	drago:	any way you go from point a to b
316	*gdo joins the room*	
317	drago:	is the same length as long as you take short routes
318	gdo:	opps
319	gdo:	internet problem
320	gdo:	internet problem
321	drago:	you always have to go the same ammount right, and the same ammount down

This excerpt illustrates how the participants of this interaction chose to start a current collaborative task. Understandably, when teams sustain their collaborative work over multiple individual sessions, this task of recommencing knowledge-building activity becomes an issue that participants have to address. We can see that *drago*’s posting in line 303 (“*last time, me and estrickm came up*”) stands as an uptake of the proposal for collective action put forward by *gdo* in line 302 (*“now lets work on our prob*”). By contrasting “*last time*” with *gdo’s* “*now*”, *drago* attempts to establish a particular kind of episodic continuity or “relevant history” of the team (unavailable elsewhere in the collaboration environment), while at the same time categorizing *gdo* and *mathwiz* as newcomers and opening up the possibility of orienting to them as such.

Drago’s posting in line 306 (“*you always have to move a certain amount to the left/right and a certain amount to the up/down*”) completes the initiation of his bridging move not only in a temporal sense but also as far as the problem-solving trajectory, since a prior discovery (“*you always have to go…*”) is presented as relevant to re-start the problem-solving task of the team. Naturally, it is not a simple task for the new members of the team to fully understand the meaning of *drago*'s summary but they engage in doing the situated work of making sense of it and using it. In fact, the reply posted in line 307 by *gdo* ("*what?*") and the subsequent elaboration attempted by *drago* suggest that the posting in 306 was taken as a problematical response to the proposal to initiate the problem solving work. Perhaps additional work was necessary for line 306 to be fully sensible for the team— in other words, for *drago* to successfully bridge prior work into the present. In the subsequent lines we can see the beginnings of an instance of the kind of interactional work that seems to be necessary for the team to engage with the reported past that *drago* is presenting.

Even without a thorough understanding of the mathematical task at stake, one can see that *drago* elaborates on his initial posting by providing additional problem information (308, "for the shortest path") and adding further references to elements of the past problem-solving activity (310-312, "since the problem last time stated that you couldn't…"). Furthermore, *mathwiz*'s posting in line 314 (*"would you want to keep as close to the hypotenuse as possible? or does it actually work against you in this case?*) engages with the bridging activity opened up by *drago* in a particular way. *Mathwiz* seems to suggest a specific way of clarifying *drago*'s presentation of how the grid-world works while at the same time doing the interesting work of positioning *drago* as the one who is to assess this suggestion (i.e. testing whether this case "*works against you*"). This short sequence signals only the beginnings of the type of interactional work necessary to fully bridge prior knowledge-work into present joint- activity and yet it is sufficient to provide us with significant evidence of the nuanced aspects of this type of activity.

This scenario of change in membership and continuity of task work is a clear example of the need for persistent supports in collaboration environments. However, simply providing *drago*, *mathwiz, gdo,* and the rest of the teams direct access to raw recordings of the team's prior sessions would probably be an inefficient solution in this case. Even if the team was reusing the same persistent room for each session, the interactional ground that is so essential to the meaning of the chat and whiteboard records is not easily recovered and certainly not easily transferred or summarized. On the other hand, the successful bridging achieved by this team can be partially linked to the sophistication of their problem-solving work in the last session introduced in section 4.2, especially when compared with other teams which did not establish such strong sense of continuity of their problem-solving trajectory.

As a result of our initial findings, we designed in our 2006 experiments a setting in which "bridging" could be investigated more conspicuously. We arranged for the teams to reuse the same persistent chat rooms so that they had direct access to the entire history of their conversations and their manipulations on the whiteboard across the four sessions. In addition, mentors provided explicit feedback by leaving a note on the whiteboard of each team's room in between sessions. Finally, we also provided a wiki space to help the teams share their explorations (e.g. formulas found, new problems suggested by their work, etc.). Below we present our preliminary impressions and contrast them to our observations about the first set of experiments.

The reuse of the same room by teams that were much more stable in their membership over time proved effective in stimulating the constructive establishment of continuity in the problem-solving task of the teams. The feedback provided by the external mentors, however, was in several cases problematic since it re-framed past experiences in ways that seemed unfamiliar or problematic to the participants themselves. Furthermore, the use of the wiki space provided us with a set of interesting examples of new "bridging" activity being conducted by the teams.

Through the wiki postings, teams working on the same or similar task were made aware of the parallel work being conducted by their counterparts. In several cases, the wiki acted as a successful third space from which materials generated by one team could be used, validated, and advanced by other teams. The authors of the postings also used them to sustain their own problem-solving across the four sessions. Postings and trajectories of use of the wiki also showed a structure that was very different than the conversational and interactional style of the chat room artifacts. Some postings were purposively vague and others resembled highly elaborate summaries of the teams' findings. In a few cases, postings included a narrative structure abstracted from the chat sessions (e.g. "*So in session 3, our team tried to understand Team C's formula …*"). In one instance, the wiki presented evidence of cross-team asynchronous interactions: Team B found a new problem generated by another team in addition to a possible solution. Team B proceeded to work on the problem, found a mistake in the solution formula originally reported, and proceeded to re-work the original solution and post the corrected result back to the wiki.

These preliminary findings seem to suggest both the potential of explicit bridging spaces to promote continuity and sustain problem-solving work, especially in the context of an online community formed of multiple virtual teams with overlapping interests and activities. Naturally, the availability of bridging resources like the wiki does not by itself shape the ways participants interact over time. In addition, the fact that certain social practices were promoted (e.g. reporting to others, imitating, reflecting, etc), also influenced the way these resources were used.

5. Conclusions and Implications for Design

Our analysis of the ways that virtual math teams used a wide array of methods to sustain their joint activity suggests that in the particular domain of team problem-solving, access to special supports for long-term work might be crucial to the success of these interaction environments. On the other hand, direct access to "raw" records of interactions (e.g. chat transcripts) might be only of limited value to the teams' knowledge work. As a result, it remains as a challenge to researchers and designers to explore effective supports for sustaining the complexity of multiple, long-term problem-solving teams. In particular, support for "bridging" —the purposeful crossing of diverse interactional boundaries—seems to be a critical design challenge.

Because continuity in itself is important to the success of virtual teams, we have observed how participants develop a series of "bridging" methods used to co-construct mathematical knowledge over time, evolve a sense of collectivity, and interlink the online environment with other interaction spaces. Our particular interest in "bridging" work resembles other researcher's analyses of the structure of synchronous collaboration such as Linebarger et al's [24] proposal for combining "episodic", "sub-group" and "agenda" supports for collaboration.

Based on these observations we will continue to explore three main aspects of bridging work:

Episode bridging: Supports for interlinking sequences of knowledge-building interactions (e.g. quasi-synchronous and asynchronous) while empowering users to make sense of knowledge from previous collaborative activities and use it for present or envisioned group interactions.

Perspective bridging: Artifacts and activities oriented towards allowing participants to discover and negotiate multiple conceptual perspectives within and across team interactions or held by other interested parties (e.g. mentors, supervisors, etc.).

Identity bridging: Facilitating the awareness of teams' knowledge trajectories over time and the opportunistic discovery of potential collaborators. Exploiting the interdependence between small-group problem-solving interactions and community participation over time.

6. References

[1] T. Erickson and S. Herring, "Persistent Conversation: A Dialog between Research and Design," presented at The Persistent Conversation minitrack and workshop at the Hawai'i International Conference on Systems Science (HICSS), 2005.

[2] H. H. Clark and S. E. Brennan, "Grounding in Communication," in *Perspectives on Socially Shared Cognition*, R. M. L. L.B. Resnick, and S.D. Teasley, Ed. Washington, DC: American Psychological Association, 1991, pp. 127-149.

[3] S. Herring, "Interactional Coherence in CMC," in *Journal of Computer-Mediated Communication.* vol. 4, 1999.

[4] A. Renninger and W. Shumar, "Community building with and for teachers: The Math Forum as a resource for teacher professional development," in *Building virtual communities: Learning and change in cyberspace*, A. Renninger and W. Shumar, Eds. New York: Cambridge University Press, 2002, pp. 60-95.

[5] G. Stahl, "Group cognition: The collaborative locus of agency in CSCL," presented at international conference on Computer Support for Collaborative Learning (CSCL '05), Taipei, Taiwan, 2005.

[6] G. Stahl, *Group cognition: Computer support for collaborative knowledge building.* Cambridge, MA: MIT Press, 2006.

[7] G. Stahl, "Group cognition in chat: Methods of interaction / Methodologies of analysis," presented at Kaleidoscope CSCL SIG Workshop on Analysis of Interaction and Learning (NAIL 2005), Gothenburg, Sweden, 2005.

[8] M. Cakir, F. Xhafa, N. Zhou, and G. Stahl, "Thread-based analysis of patterns of collaborative interaction in chat," presented at international conference on AI in Education (AI-Ed 2005), Amsterdam, Netherlands, 2005.

[9] H. Fuks, M. Pimentel, and C. J. P. de Lucena, "R-U-Typing-2-Me? Evolving a chat tool to increase understanding in learning activities," *International Journal of Computer-Supported Collaborative Learning (ijCSCL)*, vol. 1, 2006.

[10] M. Mühlpfordt and M. Wessner, "Explicit referencing in chat supports collaborative learning," presented at international conference on Computer Support for Collaborative Learning (CSCL 2005), Taipei, Taiwan, 2005.

[11] E. Krause, *Taxicab Geometry: An Adventure in Non-Euclidean Geometry*. New York, NY: Dover, 1986.

[12] H. Garfinkel, *Studies in Ethnomethodology*. Englewood Cliffs, NJ: Prentice-Hall, 1967.

[13] H. Sacks, E. A. Schegloff, and G. Jefferson, "A Simplest Systematics for the Organization of Turn-Taking for Conversation," *Language*, vol. 50, pp. 696-735, December 1974 1974.

[14] G. Button, *Technology in Working Order: Studies of Work, Interaction, and Technology*, London & New York: Routledge, 1993.

[15] G. Button and P. Dourish, "Technomethodology: Paradoxes and possibilities," in ACM Conference on Human Factors in Computing Systems (CHI '96), Vancouver, Canada, 1996, pp. 19-26.

[16] L. Suchman, *Plans and Situated Actions: The Problem of Human-Machine Communication*. Cambridge, UK: Cambridge University Press, 1987.

[17] E. Livingston, The Ethnomethodological Foundations of Mathematics. London, UK: Routledge & Kegan Paul, 1986.

[18] M. Q. Patton, Qualitative Evaluation and Research Methods: (2nd ed.) Newbury Park:, CA: Sage Publications, Inc., 1990.

[19] M. Pimentel, H. Fuks, and C. Lucena, "Mediated Chat Development Process: Avoiding Chat Confusion on Educational Debates," presented at international conference of Computer Support for Collaborative Learning (CSCL 2005), Taipei, Taiwan, 2005.

[20] G. Stahl, A. Zemel, J. Sarmiento, M. Cakir, S. Weimar, M. Wessner, and M. Mühlpfordt, "Shared Referencing of Mathematical Objects in Online Chat," presented at International Conference of the Learning Sciences (ICLS 2006), Bloomington, IN, 2006.

[21] S. Whittaker, E. Geelhoed, and E. Robinson, "Shared workspaces: How do they work and when are they useful?" *International Journal of Man-Machines Studies*, 39, 1993.

[22] P. Dillenbourg and D. Traum, "Does a shared screen make a shared solution? ," in Proceedings of the 1999 *Conference on Computer support for collaborative learning* Palo Alto, California International Society of the Learning Sciences, 1999.

[23] S. L. Star, "The Structure of Ill-Structured Solutions: Boundary Objects and Heterogeneous Distributed Problem Solving," in *Distributed Artificial Intelligence*, vol. II, L. Gasser and M. N. Huhns, Eds.: Morgan Kaufmann Publishers, 1989, pp. 37–54.

[24] J. M. Linebarger, A. J. Scholand, and M. A. Ehlen, "Representations and Metaphors for the Structure of Synchronous Multimedia Collaboration within Task-Oriented, Time-Constrained Distributed Teams," presented at 39th Annual Hawaii International Conference on System Sciences, 2006.

Acknowledgements

The Virtual Math Teams Project is a collaborative effort at Drexel University. The Principal Investigators are Gerry Stahl, Stephen Weimar and Wesley Shumar. A number of Math Forum staff work on the project, especially Stephen Weimar, Annie Fetter and Ian Underwood. The graduate research assistants are Murat Cakir, Johann Sarmiento, Ramon Toledo and Nan Zhou. Alan Zemel is a post-doc. The following visiting researchers have spent 3 to 6 months on the project: Jan-Willem Strijbos (Netherlands), Fatos Xhafa (Spain), Stefan Trausan-Matu (Romania), Martin Wessner (Germany), Elizabeth Charles (Canada), and Martin Mühlpfordt (Germany). The ConcertChat software was developed at the Fraunhofer Institute IPSI in Darmstadt, Germany, by Martin Wessner, Martin Mühlpfordt and colleagues. The VMT project is supported by grants from the NSDL, IERI and SoL programs of the US National Science Foundation.