Extending the Joint Problem Space: Time and Sequence as Essential Features of Knowledge Building

Johann W. Sarmiento & Gerry Stahl
Drexel University, Philadelphia, USA
jsarmi@drexel.edu, gerry.stahl@drexel.edu

Abstract: Our attempts at describing the processes involved in learning and knowledge-building activities depend on our ways of conceptualizing the context in which such activities take place. Here we trace the development of the concept of "problem space" from its inception within the information-processing perspective as a characterization of individual problem-solving activity. We review reformulations and extensions made to the concept within the Learning Sciences, and explore them as attempts to better describe small-group interactions in complex knowledge-building contexts. Using a detailed analysis of sustained, online collaborative problem-solving activity, we propose that a new aspect of the problem space needs to be carefully considered in order to fully account for these kinds of experiences: temporal and sequential orientation to inter-subjective meaning making.

Introduction

The challenge of identifying, describing and assessing the activities that typify the contexts in which learning and knowledge building take place lies at the core of all inquiry in the learning sciences. As Sfard (1998) has argued, even the metaphors that we use to characterize what learning is, work as lenses that focus our attention on particular aspects of learning interactions, while obscuring or ignoring others. Descriptions of features, resources and activities particular to each learning context serve as the building blocks for structuring inquiry about them and offering descriptions of their dynamics. In this paper we use the construct of the "joint problem space" and trace its development within the Learning Sciences as a way to present an expanding view of what is, or needs to be, considered relevant and significant in descriptions of learning and knowledge building activity.

In order to anchor our review of the evolution of the concept of "problem space," we will use data originating from groups participating in the Virtual Math Teams (VMT) project. The VMT project at the Math Forum (http://mathforum.org/vmt/) investigates the innovative use of online collaborative environments to support effective secondary mathematics learning in small groups. Central to the VMT research program are the investigation of the nature and dynamics of group cognition (Stahl, 2006a) as well as the design of effective technological supports for quasi-synchronous small-group interaction. In addition, we investigate the linkages between synchronous interactions (e.g., collaborative chat episodes) and distributed asynchronous interactions at the level of the online community. VMT is currently studying how upper middle school and high school students do mathematics collaboratively in an online environment that integrates electronic chat with a shared whiteboard and a series of support tools for referencing and annotating objects. Particular attention is given to the methods that students deploy to conduct their interactions in such an environment. Taken together, these methods define a culture, a shared set of ways to "make sense together". The methods are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to the participants. These methods help define the nature of the collaborative experience for the small groups.

Joint Problem Spaces

Joint activity, the kind of activity that takes place when multiple participants engage with each other, offers a unique context for the investigation of human reasoning. Not only are the reasoning processes that characterize joint activity visibly distributed across multiple participants (e.g., Hutchins, 1995; Salomon, 1993), but they are also highly shaped by the way that material and conceptual artifacts are integrated into activity (e.g., Perkins, 1993; Schwartz, 1995) and the way that activity evolves over time (e.g., Brown & Campione, 1994; Lave & Wenger, 1991; Scardamalia & Bereiter, 1991). For instance, in Roschelle (1992) and Teasley & Roschelle's (1993), the authors analyze dyads using a physics software simulation to explore concepts such as velocity and acceleration, and propose the notion of a *joint problem space* (JPS) to explain how collaborative activity gets structured in this context. This "knowledge structure" was presented as integrating: *goals, descriptions of the current problem state* and *awareness of available problem-solving actions*. The space was characterized as being "shared" in the sense that both members of the dyad oriented to its construction and maintenance.

At first glance, the concept of a "*joint problem space*" may appear strongly related to the original concept of "*problem space*" advanced within the information-processing perspective on human problem solving which originated in the collaborative work of Allan Newell and Herbert Simon (Newell & Simon, 1972).

Newell and Simon concentrated on building a "process theory" describing the performance of individual "intelligent adults in our own culture," working on short and "moderately difficult problems of a symbolic nature," (p. 3) where "motivation is not a question and emotion is not aroused" (p. 53). To achieve this, they explicitly excluded group activity as well as "long-term integrated activities" involving multiple episodes of action over longer periods of time (p. 4). Central to their theory is the idea that to solve a task or problem, one must "adapt" to the environment presented by the problem (the "*task environment*") by constructing an internal representation of the problem's relevant elements (a "*problem space*"). The concept of *problem space* was then introduced as a "neutral and objective way of talking about the responses of the subject, including his internal thinking responses, as he goes about dealing with the stimulus situation" (p.59). This space, mostly viewed as internal or mental but sometimes related to external resources as well (e.g., Kotovsky & Simon, 1990), is commonly presented as a graph with nodes and links. A person is assumed to understand a task correctly when she has successfully constructed a problem space representation containing or "encoding": a set of *states of knowledge* including the *initial state* of the problem, the *goal state* and the necessary intermediate states, as well as *operators* for changing from one state into another, *constraints* determining allowable states and moves, and any other encodings of knowledge such as problem-solving heuristics and the like (pp. 59 & 810). Problem solving proceeds as the subject works from the initial state in her mental space, purposefully creating and exploring possible solution paths, testing and evaluating the results obtained. This process is commonly characterized as "search" on the problem space and search, as an activity, becomes the central phenomena theorized. The level of detail offered about candidate search processes is, undoubtedly, one aspect in which this theory rivals other less specified proposals. For instance, search methods such as *breadth first, depth first*, *branch and bound*, *bidirectional*, *heuristic best first*, *hill climbing*, etc. have been offered as descriptions of the processes followed by human problem solvers in different contexts (Newell, 1980).

The characterization of the *joint problem space* advanced by Teasley and Roschelle (1993), despite superficial similarities, goes beyond simply being a collective reformulation of the information processing concept of *problem space*.[1] From their perspective, social interaction in the context of problem-solving activity occurs *in relation to* a shared conception of the problem which is in itself *constituted through* the collaborative process of coordinating communication, action and representation in a particular context of activity; not restricted to or primarily driven by individual mental states. This perspective as well as the authors' method of analysis are closely related with the ethnomethodological position regarding the nature of shared agreements as *"various social methods for accomplishing the member's recognition that something was said-according-to-a-rule, and not the demonstrable matching of substantive matters."* From this perspective, a common understanding becomes a feature of an interaction (an operation, in Garfinkel's terms) *"rather than a common intersection of overlapping sets*" (Garfinkel, 1967, p.30), as "shared mental models" (Salas & Fiore, 2004) or "common ground" (Clark & ???, 19??) sometimes seem to portray. A "shared agreement" or a "mutual conception of the problem" is then the emergent and situated result of the participants' interactions tied to their context of activity. In the words of Roschelle and Teasley, it is "*the coordinated production of talk and action by two participants (that) enabled this construction and maintenance (of the joint problem space) to succeed*."

Beyond the sole identification of relevant resources, an effective account of the problem solving process requires a description of the fundamental activities involved. Roschelle (1992) presents the most compelling description of such activities associated with the joint problem space when he states that the process of the students' incremental achievement of convergent meaning through interaction can be characterized by the four primary features of activity synthesized in Figure 1.

(a) the production of a deep-featured situation, in relation to
(b) the interplay of physical metaphors, through the constructive use of
(c) interactive cycles of conversational turn-taking, constrained by
(d) the application of progressively higher standards of evidence for convergence.

Figure 1. Primary features of the process of achieving convergent conceptual change. From (Roschelle, 1992)

Testing and expanding the proposed construct of the *joint problem space* requires, then, the ability to recognize these features in interaction. In order to do this and to support the next steps in our exploration of the construct of *problem space*, we would like to introduce here one particular problem situation used as part of the Virtual Math Teams (VMT) project mentioned earlier:

[1] This seminal paper straddled the cognitive and interactional perspectives, causing ideological barriers to publication (personal communication Roschelle to Koschmann at CSCL 02 and Teasley to Stahl at GROUP 07).

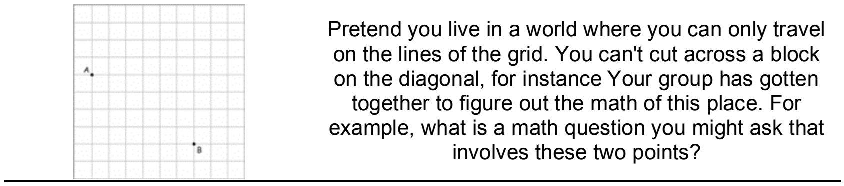

Figure 2. Grid-world task.

One could argue that the task presented in Figure 2 does not properly specify a problem yet. The "problem" at hand is, in fact, to create a problem. Within the information-processing perspective, the foundational activities which contribute to the creation of a problem are, in fact, poorly understood. As a recent review of psychological research on problem solving stated, "problem-solving research has not revealed a great deal about the processes involved in problem recognition, problem definition, and problem representation (Pretz, Naples, & Sternberg, 2003, p. 9). It is only after a *problem space* has been constructed internally in the mind of a subject, at least partially, that one can start to trace the solution process as a *search process*. However, observing these early phases of problem solving can, indeed, inform us about how problem spaces are constituted in interaction and how some of the features of collaborative activity described by Roschelle contribute to this important phase. For instance, in our study of the ways that small online groups in VMT engaged with this task, we observed a number of activities that could help characterize certain aspects of these early phases of the creation of a problem space. The groups often identified and appropriated specific elements of the task, and purposefully and iteratively structured them into a problematic situation. Resources such as graphical manipulations (e.g., grid annotations), related mathematical concepts (e.g., straight distance), constraints (e.g., you can only travel on the lines of the grid) or analogous problems were used to construct and evolve a set of possible inquiries about this world. We can characterize these constructions as creating a "*deep-featured situation"* in the sense that they embody the sustained exploratory activities of the participants. As an example, many groups promptly oriented to finding the shortest distance between points A and B in the grid world, a familiar problem to school-aged students. Some purposefully attended to the constraints of the grid world while others simply ignored them and proceeded to explore diagonal distances. Building on this initial problem, many groups embarked on the problem of finding the number of shortest paths between any two points on the grid. Figure 3 contains some snapshots of the artifacts the different groups created to help constitute a problem from the original situation.

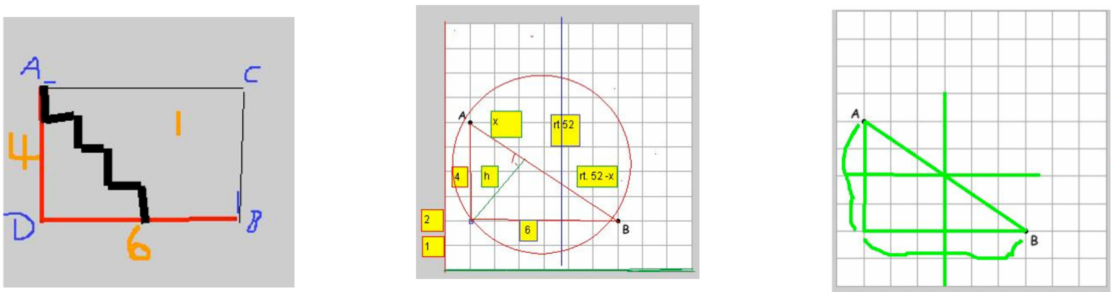

Figure 3. Snapshots of grid-world problem resources created by VMT groups.

In this particular situation, potential problems were constantly defined as sets of artifacts with specific properties (e.g., constraints) sometimes constituted as "discoverables." Multiple trajectories of reasoning were explored, sometimes in concerted fashion, others in parallel. A central aspect of the group's activity seemed to be concerned with "adding structure" to the resources used to think with. From an interactional perspective it certainly does not seem appropriate to characterize such activities as search, although, on the other hand, one could certainly agree that a "space" or network of problem objects and relations was being constructed and that specific features of the resources available were being attended to. Metaphors played a role in some instances but perspectives, or points of view, seemed more interactionally relevant. In this context, the groups did not necessarily orient to the application of "progressively higher standards of evidence for convergence" but, within those teams that seemed more intensively engaged with the grid world as an expansive situation to think with, they seemed to orient strongly to the continuity and sustainability of their inquiry. Overall, these collective problem-solving activities appear to be much more interactive than what the original concept of search in a mental problem space may have suggested (as Kirsh (forthcoming) has eloquently argued for individual problem solving as well). Next, we continue to trace the evolution of the concept of problem space within the Learning Sciences and explore its role in defining the relevant elements that characterize engagement with problem-solving and knowledge-building activity in different contexts.

A Dual Space Model of Collaboration: Content and Relational Spaces

Barron (2000; 2003) investigated triads of 6th grade students engaged in collaborative mathematical problem solving. Her analysis proposed that it was necessary to differentiate between the *social* and *cognitive* aspects of the interactions observed and investigate the ways in which both are interwoven in the establishment of a joint problem-solving space, especially, when attempting to characterize successful and unsuccessful collaborations. Both cognitive and social aspects are, in a sense, integrated in the features of collaborative activity described by Roschelle (1992) and reproduced in Figure 1. However, Barron's analysis illuminates a new set of specific activities that the participants engaged in when explicitly orienting to this duality, attending to social and cognitive factors in the development and maintenance of a "between-person state of engagement" (p. 349) which resembles the joint conception of the problem proposed by Teasely and Roschelle. Interestingly, patterns of interaction related to a group's inability to attend to common aspects of the problem or to coordinate their reciprocal participation while solving the problem were particularly salient in groups that failed to achieve and maintain "mutual engagement" and, as a result, were unable to capitalize on the ideas and proposals of the group members (p. 311). As a result, Barron proposes a *dual-space model* of collaboration integrating a *content space* pertaining to the problem being solved and a *relational space* pertaining to the ways that participants relate to each other. Naturally, these two spaces are not separate entities but essentially mutually constitutive of each other. Participants simultaneously "attend to and develop" such spaces.

Similar proposals have been made, for instance, in the field of Small Group Research since Robert Bales (1953) first proposed his principle of "equilibrium," which states that a group continuously divides its attention between *instrumental* (task-related) needs and *expressive* (socio-emotional) concerns. More recently, McGrath (1991) suggested in his "Time, Interaction, and Performance" theory that work groups orient towards three "inseparably intertwined" functions: working on the common task together (*production function*), maintaining the communication and interaction among group members (*group well-being* function), and helping the individual member when necessary (*member support function*, p. 151). Poole, also suggested that group decision-making discussions can be characterized by three intertwining "tracks" of activity and interaction*: task progress*, *relational track*, and *topical focus*. The *task track* concerns the process by which the group accomplishes its goals, such as doing problem analysis, designing solutions, etc. The *relation track* deals with the interpersonal relationships between the group members (e.g., sharing personal information or engaging in social joking). The *topic* track includes a series of issues or concerns the group has over time. Interspersed within these tracks are *breakpoints*, marking changes in the development of strands of work.

The power of these proposals to advance our understanding of group activity lies, however, not in their ability to appropriately name dimensions of interaction or group functions but in their ability to characterize and describe the activities that groups engage in. Consequently, the value of Barron's proposal, in our opinion, lies on her careful way of calling our attention to the interactional methods employed by the students to orient to and constitute the "responsivity" and "connectedness" (p. 353) of their content and relational spaces. In her descriptions, we see participants' degrees of competence in attending and relating to their own "epistemic process" while "tracking and evaluating others' epistemic processes" (p. 310). Similar descriptions have been provided by Engle and Conant as "positioning" (Engle, 2006; Engle & Conant, 2002). In order to expand these concepts, next we extend the type of group phenomena studied from collaborative interactions to longitudinal sequences of joint activity and attempt to inquire about ways in which the concepts of "joint problem space" and "dual problem space" are sufficient to understand them.

Continuity of Joint Problem Spaces in Virtual Math Teams

Undoubtedly, the difficulty of constructing and maintaining a "cognitive" and "social" joint problem space—the intersubjective space of interaction emerging from the active engagement of collectivities in problem solving— represents the central challenge of effective collaborative knowledge building and learning. In fact, several studies have shown that what determines the success of the collaborative learning experience is the interactional manner in which this intersubjective problem space is created and used (Barron, 2003; Dillenbourg *et al.*, 1995; Hausmann, Chi, & Roy, 2004; Koschmann *et al.*, 2005; Wegerif, 2006). Furthermore, the complexity of the challenge of maintaining a joint problem space rises when, as in many naturalistic settings, joint activity is dispersed over time (e.g. multiple episodes of joint activity, long-term projects, etc.) and distributed across multiple collectivities (e.g. multiple teams, task forces, communities, etc.). As a result of these gaps, sustained collaborative learning in small virtual groups and online communities of learners might require that co-participants "bridge" multiple elements of their interactions continuously as they interact over time. Motivated by the need to understand such activities, we set out to investigate the challenges associated with such discontinuities of interaction over time.

Within the Virtual Math Team online community, participating teams might engage in multiple, collaborative sessions over time, they might work on several related tasks over time and learn about the work of other teams. To explore whether VMT teams employ specific methods oriented towards overcoming the discontinuities of time, tasks and participation, during the Spring of 2005 we conducted a pilot case study of five

Virtual Math Teams. These virtual teams were each formed with about four non-collocated upper middle-school and high-school students selected by volunteer teachers at different schools across the United States. The teams engaged in synchronous online math interactions for four hour-long sessions over a two-week period. They used the ConcertChat virtual room environment (Wessner *et al.*, 2006) which integrates a chat interface with a shared whiteboard. A new virtual room was provided for each of the sessions, so that participants did not have direct access to the records of their prior interactions. In the first session, the teams were given a brief description of the grid-world presented in Figure 2, where one could only move along the lines of a grid. The students were asked to generate and pursue their own questions about this mathematical world. In subsequent sessions, the teams were given feedback on their work as well as on the work of other teams, and were encouraged to continue their collaboration. Because of the sequential framing of the tasks provided and the continuous relevance of the properties of the grid world, we considered this a propitious setting for the investigation of members' methods related to continuity of knowledge building. We examined each of the 18 sessions recorded, paying special attention to the sequential unfolding of the four problem-solving episodes in which each team participated, to the ways that prior activities were used as resources for later team work, and also to the ways that changes in team membership triggered issues of continuity.

As a result of our analysis, we identified a number of instances where the teams were engaged in several types of "*bridging activity*" aimed at overcoming discontinuities emerging over the multiple episodes of interaction. All teams, although in different levels of intensity, engaged in this type of activity over time. In summary, the instances of bridging identified involved methods related to narrating or reporting past doings as resources for constructing a new task, remembering collectively, and managing the history of the team, among others. Constant comparison through different instances of bridging activity in the entire dataset led to our initial characterization of the structural elements that define these activities and their interactional relevance. Our analysis of the dynamics of bridging activity echoes the construction and maintenance of a "*joint problem space*" (Teasley & Roschelle, 1993) and also agrees with the proposal that such a space integrates "*content*" and "*relational*" dimensions (Barron, 2003). However, throughout our analysis of all instances of bridging activity, we noticed that a third element of interaction reoccurred as a resource and a relevant concern of the participants: The temporal and sequential unfolding of activity (see Figure 4). To illustrate this, let's turn to an actual instance of bridging activity. The conversation reproduced in Figure 5 illustrates how a team oriented to past team activity as resources for framing a current problem-solving task.

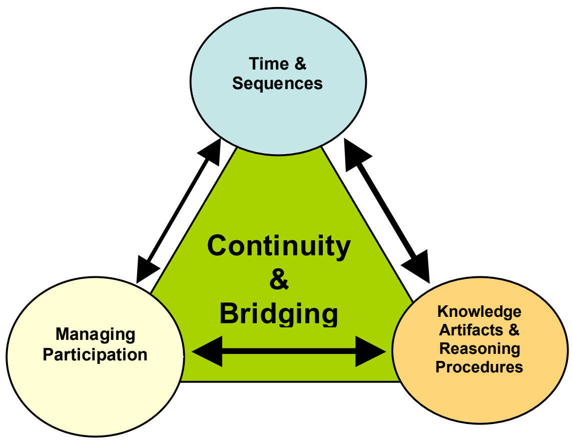

Figure 4. Three dimensions of interaction and their relationship in bridging work

144 mathis: letz start working on number 8
145 bob1: we already did that yesterday
146 qw: we did?
147 mathis: but we did it so that there was only right and down
148 bob1: i mean tuesday
149 mathis: i guess we will do it with left and up?
150 qw: It would be almost the same.
151 bob1: it's (|x2-x1|+|y2-y1|-2) choose (|x2-x1|-1)
152 bob1: try it if you like
153 mathis: nah
154 mathis: if you are so sure...
155 bob1: i'm not
156 bob1: actually
157 bob1: take out the -2 and the -1
158 mathis: then letz check it
...

Figure 5. Chat excerpt of a bridging episode. Spring 2005, Team B, Session two

The first of the three basic interactional dimensions that seem to be at play in bridging activity corresponds to the creation, referencing, manipulation, assessment and re-use of a set of *knowledge artifacts*. This involves constituting the problem-at-hand, identifying which resources are relevant to it, creating tasks, constituting aspects of the problem situation and its resources as known or unknown, among other activities Despite the brevity of the interaction captured in Figure 5, we can recognize some of these artifacts (e.g., problem number 8, "*only right and down*", "*left and up*", (|x2-x1|+|y2-y1|-2) choose (|x2-x1|-1), etc.). We can get a glimpse of ways in which they are attended to and manipulated (e.g., "*only right and down*" is debated as being almost the same as "*left and up*", the formula provided is offered for assessment, etc.). Interwoven with the development and use of knowledge artifacts, we also identified the active management of participation as a second relevant dimension at play in this case of bridging activity. From this perspective, teams were actively oriented towards, for instance, who was and was not involved in an activity, who could or should speak about a particular matter and how, which activities (e.g. assessing and responding to assessments) were allocated to participants, etc. In essence, the participants orient to the development in interaction of specific participation frameworks (Goffman, 1981) which "position" team members in relation to each other, the resources at hand and the activities they are engaged in. This positioning activity, for example, situated participants as problem-solving peers, experts, explainers, etc. In addition, the activities they engage in over time position them with different types of access, rights and duties with respect to relevant knowledge artifacts. The excerpt in Figure 5 illustrates this, especially toward the end of this passage, when Bob1 attempts to position Mathis as someone who could do the checking of his solution formula. After Mathis declines and Bob1 states his lack of confidence in the correctness of the formula a new participation framework gets enacted, in which the group together can engage in the work necessary to check and possibly correct the solution provided for this problem.

The first two dimensions of interaction observed matched, very closely, the "content" and "relational" spaces theorized by Barron. However, a recurring third element present in episodes of bridging activity captured our attention both because of its centrality in the interactions analyzed as well as its novelty within the theoretical frameworks considered: the temporal or sequential organization of experience. Temporality and sequentiality are constructs that are often taken for granted and which have only until recently recovered their centrality in analyses of joint activity (e.g., Arrow *et al.*, 2004; Lemke, 2001; Reimann, 2007; Sawyer, 2003, Stahl, 2006b). Our analysis suggests, however, that in the types of interactions that we observed, participants orient to time and sequences as central resources for the organization of their collaborative activity. As can be seen in Figure 5, VMT participants visibly oriented to what was done in a different episode of activity or at a different time, to the relationship between what was done before and what is being now, or to what possible actions might be available at a particular moment as related to what had been achieved so far.

The excerpt reproduced in Figure 6, illustrates a case in which a team is collectively engaged in trying to reconstruct parts of their previous session while initiating their current problem solving activity. Remembering of past activity unfolds as a collective engagement in which different team members participate dynamically. Some of the current team members were not present in the previous session and yet, they are instrumental in the reconstruction of that past and in shaping its current relevance. This was the fourth session of team E. Towards the beginning of the session (8:22:09 PM) the facilitator (MFMod) suggested in the chat that during the summer the team members could work with their friends on a new problem he posted: the "circle problem." Later, he added that they could pursue the circle question in "this chat" if they wanted or "any other questions and worlds" that they thought of. Following about a minute of silence, the facilitator posted a message in which he reported how in the previous session the team had "worked on finding a formula for the number of shortest paths between any two points A and B on the grid (…) explored multiple possibilities and figured out that x+y and x^2+y^2 work (where x and y correspond to the # of units you need to travel along x and y axis to get from A to B) but only for some points, not all". Then he suggested that they could continue "exploring more cases" and see if they could find "a general formula," work on the circle problem he had posted earlier, or on any other problem from the "original questions" presented at the beginning of their VMT experience. The team then oriented towards finding a task for themselves, and the following interaction took place:

119	8:27:42	drago:	ok
120	8:30:11	gdo:	where did u guys last leave off (To 119)
121	8:31:20	MFmod:	I think that the above section I wrote is where the group last was (To 114)
122	8:31:36	MFmod:	yes?
123	8:31:42	drago:	well
124	8:31:48	gdo:	i dont remember that
125	8:31:51	drago:	actually, my internet connection broke on Tuesday
126	8:31:56	drago:	so I wasn't here
127	8:32:12	MFmod:	so maybe that is not the best place to pick up
128	8:32:14	estric:	i wasnt able to be here on tuesday either
129	8:32:50	gdo:	how bout u meets
130	8:33:01	meets:	uh...
131	8:33:11	meets:	where'd we meet off....
132	8:33:16	meets:	i remember
133	8:33:22	gdo:	i was in ur group
134	8:33:24	meets:	that we were trying to look for a pattern
135	8:33:27	gdo:	but i didn't quite understand it
136	8:33:34	gdo:	can u explain it to us again meets
137	8:33:38	meets:	with the square, the 2by 2 square, and the 3by2 rectangle
138	8:33:42	meets:	sure...
139	8:33:45	meets:	so basically...
140	8:33:45	gdo:	o yea
141	8:33:49	gdo:	i sort of remember
142	8:33:55	meets:	we want a formula for the distance between poitns A and B
143	8:34:02	drago:	yes...
144	8:34:05	meets:	ill amke the points
(meets draws two points on the existing grid on the shared whiteboard)			
145	8:34:09	MFmod:	since some folks don't remember and weren't here why don't you pick up with this idea and work on it a bit
(meets labels the two points on the grid A and B)			
146	8:34:55	meets:	okay
147	8:34:59	meets:	so there are those poitns A and B
148	8:35:08	meets:	(that's a 3by2 rectangle
149	8:35:28	meets:	we first had a unit square
(meets draws the lines of a 3 by 2 rectangle with points A and B in its opposing corners)			
150	8:35:44	meets:	and we know that there are only 2 possible paths......
…			

Figure 6. Chat excerpt of a bridging episode. Spring 2005, Session Four, Team E

This sequence involves a number of interesting interactional features. In particular, a set of temporal and sequential markers (e.g., *Tuesday*, *last*, *again*) and the mixing of different verb tenses are used to index prior events and constitute a present task. In the facilitator's feedback, the declarative assertions constructed with past-tense verbs (e.g., you worked on finding a formula, you explored multiple possibilities, you figured out that x+y and x^2+y^2 work, etc.) were followed by future-oriented suggestions: you may want to continue exploring more cases and see if you can find a general formula, you can work on the problem I posted earlier, etc. The uptake by the team of the task assessments and proposals made by the facilitator also involved similar resources. Gdo's request in line 120 for a report of where the group "last" left off seems to use a communicative marker that allows parties in conversation to segment or index specific portions of experiences and relate them in ways that allows them to form sequences of participation and activity. Gdo is orienting the group back to a specific aspect of "last Tuesday," and after Drago and Estric both positioned themselves as not having participated in last Tuesday's session, Meets is then asked directly in lines 129 and 136 to re-produce a past ("again") explanation for the rest ("us").

One of the things that is remarkable about the way this interaction unfolds is the fact that although it might appear as if it was Meets who individually remembered what they were doing last time, the activity of remembering unfolds as a collective engagement in which different team members participate. This is accomplished by marking and using time as a central resource to organize participation and to advance their current problem solving. To organize their present activity, they reproduce a sequence of previously constructed cases (the square, the 2x2 square, and the 3x2 rectangle) and link them to knowledge artifacts and the related knowledge of the group (e.g., stating in line 150 that for the unit square "we know that there are only 2 possible paths"). In fact, later in this interaction there is a point where Meets remembers the fact that they had discovered that there are six different shortest paths between the corners of a 2x2 grid but he reports that he can only "see" four at the moment. Drago, who did not participate in the original work leading to that finding, is able to see the six paths and proceeds to invent a method of labeling each point of the grid with a letter so that he can name each path and help others see it (e.g., "from B to D there is BAD, BCD …"). After this, Meets was able to see again why it is that there are six paths in that small grid and together with Drago, they proceeded to investigate, in parallel, the cases of a 3x3 and a 4x4 grid using the method just created.

All of these resources—the knowledge artifacts used and referenced, the sequential organization of cases, and the temporal markers of prior activity—are positioned in different ways with relation to the participants in a *temporal or sequential space.* The concept of "deictic field" developed by Hanks (2005) seems especially useful to define the relationship between this new "space" and the existing *content* and *relational* spaces. Hanks describes the deictic field as composed first by *"the positions of communicative agents relative to the participant frameworks they occupy,"* for example, who occupies the positions of speaker and addressee as well as other relevant positions. Second, the deictic field integrates "*the positions occupied by objects of reference,"* and finally "*the multiple dimensions whereby the former have access to the latter"* (p. 193). From this perspective, participants constitute, through interaction, the relevant relative dimensions whereby they are to manage the positioning of agents and relevant objects of reference. In our analysis, we have confirmed that the *content* and *relational* dimensions are, in fact, relevant to collaborative problem-solving teams. However, in expanding the range of phenomena analyzed to longitudinal interactions, we have also uncovered time and the sequential unfolding of interaction as a third relevant and important dimension of activity. In the excerpt reproduced in Figure 6, the interactional field is being constituted by the participants to include problem-related objects and communicative agents associated with a prior interaction, and in doing so they position themselves and those resources within specific participation frameworks. Our central claim is that this third dimension is essential to understanding collaborative interactions of this type. This dimension is essentially interwoven with the content and relational dimensions of the joint problem space. Such interdependency can be seen as characterizing the longitudinal knowledge building of activity systems like the Virtual Math Teams.

Conclusions

The theory of group cognition (Stahl, 2006a) takes as one of its central principles the dialectical relationship between social interaction and the construction of meaning. From this perspective, the organization of action and the knowledge embedded in such action is an emergent property of moment-by-moment interactions among actors, and between actors and the activity system in which they participate collectively. The *content* space and the *relational* space, in Barron's terms, are mutually constitutive from this perspective. *Group Cognition* offers a candidate description for how the dynamic process of building knowledge might intertwine the content and relational spaces: "Small groups are the engines of knowledge building. The knowing that groups build up in manifold forms is what becomes internalized by their members as individual learning and externalized in their communities as certifiable knowledge." (Stahl, 2006a, p. 16). Time and the sequential organization of activity might be a resource and an aspect of interaction that plays a significant role in how groups and individual achieve this.

In our analysis of how groups "sustain" their group cognition while engaged in brief episodes of online mathematical problem solving, we alluded to two ways in which time might be an important element of individual episodes of problem-solving activity (Stahl, 2006b). On the one hand, the collaborative activity involved in solving a problem can be "spread across" hundreds of micro-level interactions. On the other hand, individuals might internalize the meaning co-constructed through interactions and "sustain" the group cognition by engaging in later individual or group work. In either case, groups are described as sustaining their social and intellectual work by “building longer sequences of math proposals, other adjacency pairs and a variety of interaction methods.”

The analysis presented here of interactions that bridge gaps across sessions confirms and extends these findings by suggesting that in longitudinal interactions, temporal and sequential resources are central to constituting activity as continuous by constructing and maintaining a joint problem space. Interaction is taken here in the full sense that ethnomethodologists give it, as the “ongoing, contingent co-production of a shared social/material world,” and which, as Suchman argues “cannot be stipulated in advance, but requires an autobiography, a presence, and a projected future” (Suchman, 2003). At the moment, our characterization only provides a tentative framework to organize our developing understanding of collaborative learning and knowledge building over time. We have just began the work of describing in more detail the interactional methods that allow teams to construct and manage this expanded problem “field” (e.g., Sarmiento & Stahl, 2007; Stahl, 2006b; Stahl *et al.*, 2006) by interweaving *content*, *relational* and *temporal* aspects of interaction.

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