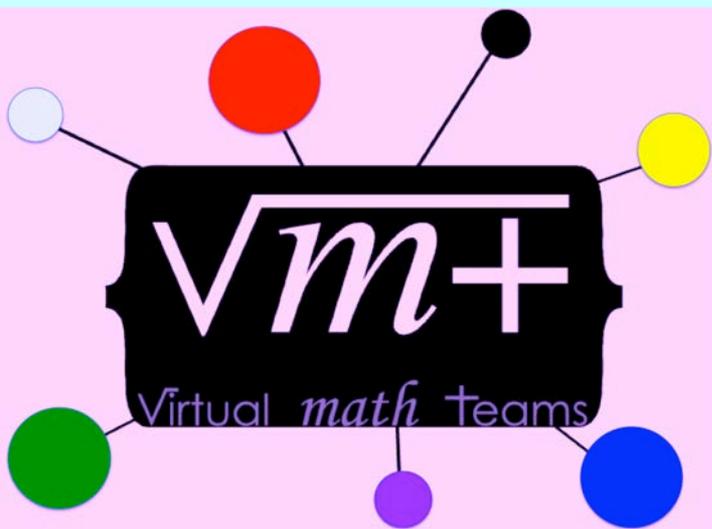


# DESIGNING A LEARNING ENVIRONMENT FOR GEOMETRY

University of Oslo  
October 16, 2013

Gerry Stahl



# Dimensions of Design

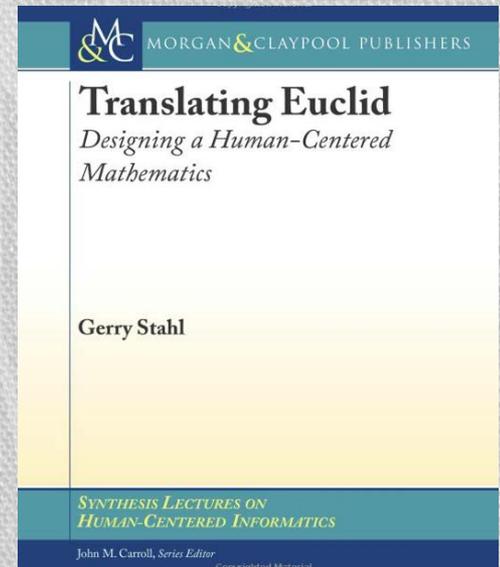
- 1. Viewing the whole design agenda**
- 2. Translating the history of geometry**
- 3. Understanding the philosophy of geometry knowledge**
- 4. Mastering the mathematics**
- 5. Building the technology**
- 6. Supporting collaboration**
- 7. Researching the learning processes**
- 8. Theorizing the approach & resources**
- 9. Defining the pedagogy**
- 10. Developing the curricular resources**
- 11. Design-based research of human-centered geometry**

# 1. Viewing the whole design agenda

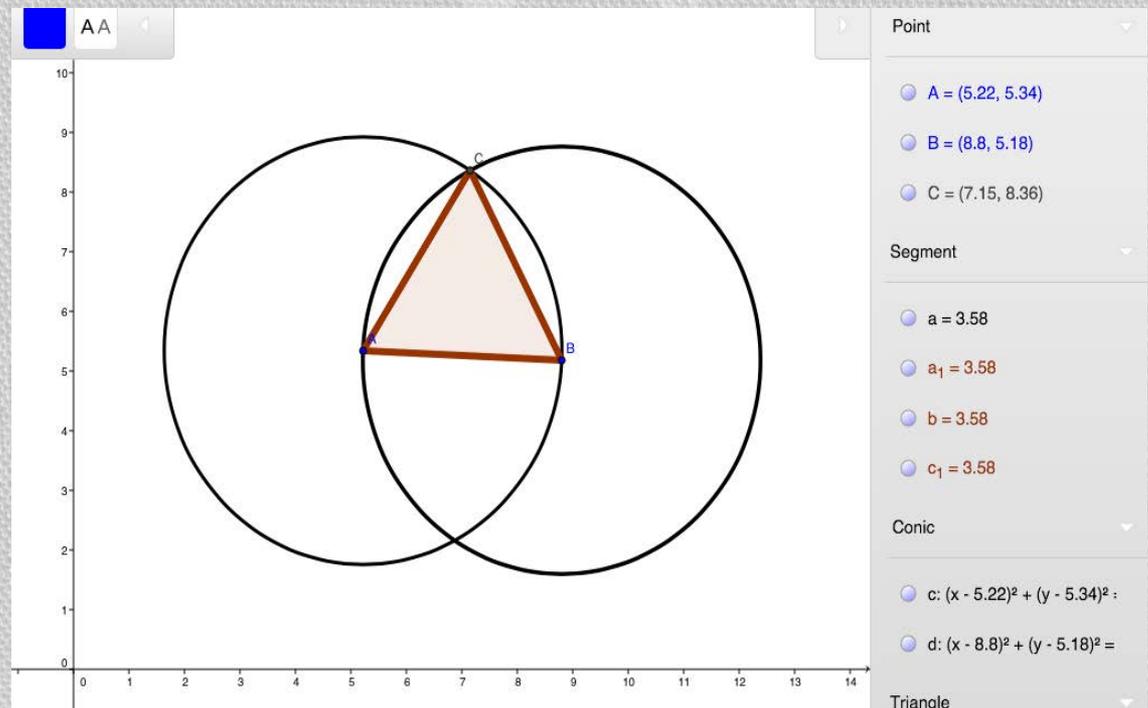
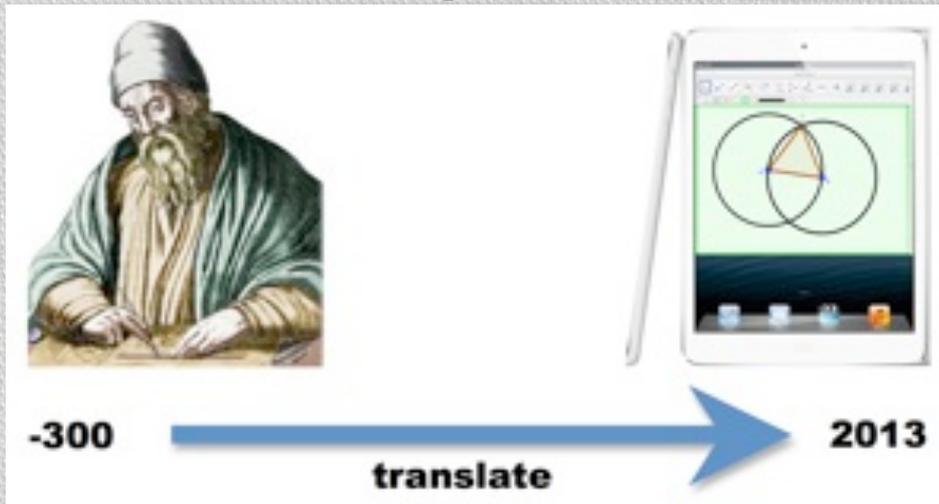
*How should one translate the classic-education approach of Euclid's geometry into the contemporary vernacular of social networking, computer visualization, and discourse-centered pedagogy?*

**A multi-dimensional, iteratively evolving design-based research approach to designing a human-centered, 21<sup>st</sup> century geometry education using computer-supported collaborative learning**

**11 chapters of *Translating Euclid* book**



# Translating from clay tablets to iPad tablets



## 2. Translating the history of geometry

- **Early Greeks explored using persistent diagrams and specialized text in small, distributed community**
- **Euclid systematized a set of postulates that built on each other to construct and prove properties of figures. Established deductive argumentation**
- **Successive further bureaucratization of proofs as objective truth**
- **Axiomatization and formal logic. Euler extensions. Non-Euclidean geometries**
- **Dynamic geometry using computer interface**

### **3. Understanding the philosophy of geometry knowledge**

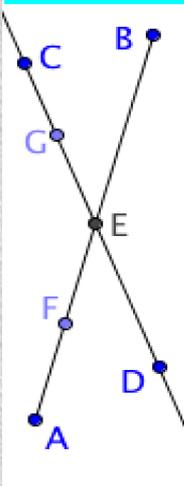
- **Alienation of geometric objects from human creators – starting with Plato’s world of ideas**
- **Abstraction from the construction process enabled progress, but reified the constructs into other-worldly, mental objects**
- **The social construction of geometry was obscured by a focus on individual thought and knowledge; a widespread ideology of individualism**

## 4. Mastering the mathematics

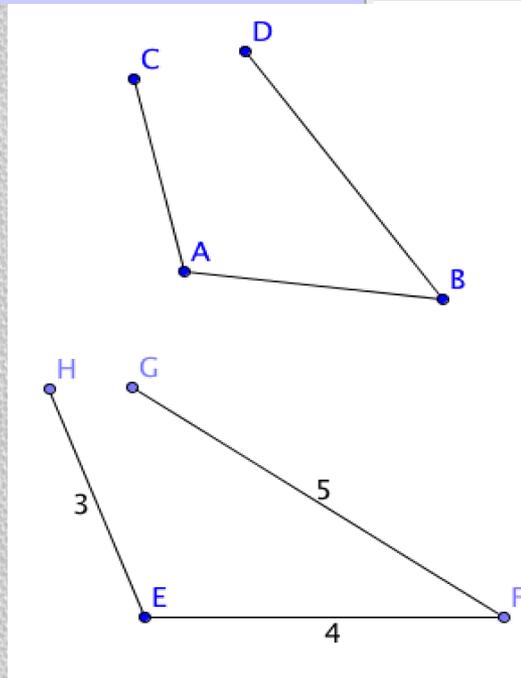
- **Dynamic geometry offers a potential to retrieve the human-centered nature of geometry as a product of creative-discovery**
- **Collaborative dynamic geometry offers a potential to retrieve the social nature of geometry as a product of community discourse**
- **At the core of dynamic geometry is the concept of *dependency*. Proof can be understood as a consequence of constructed dependencies, based on a dialectic of discovery and creation**



An example of what you will construct:



- \*\*\*Construct dependent objects\*\*\*  
 Take turns controlling the construction.
1. Select the Segment Tool and click on two points to construct a segment like AB.
  2. Select the Line Tool and click on two points to construct a line like CD that crosses the segment.
  3. Select the Intersection Tool and construct a point where the line cuts the segment.
  4. Construct another point on the segment and another point on the line, like F and G.
  5. Drag each point, the line and the segment.
  6. Discuss in the chat how each object is free, constrained or dependent on other objects.



If one triangle is congruent to another, then all its angles and all its sides are dependent on the corresponding angles and sides of the other triangle.

Given three segments -- AB, AC, BD -- for constructing a triangle, how many angles or sides do you have to constrain to fully constrain the triangle?

The three segments EF, DH, FG have been constructed with the Segment-with-Given-Length-from-Point tool to constrain their lengths. How many triangles can you construct with these segments?

What do you conclude?

## 5. Building the technology

- **Geometer's Sketchpad, Cabri, etc. pioneered the creation of dynamic geometry**
- **GeoGebra makes it freely available and integrated with other math (3-d, algebra, statistics, spreadsheet, trig, calculus, etc.)**
- **VMT provides a collaboration environment and integrates the first multi-user dynamic geometry**
- **The next dozen slides describe the design of the Virtual Math Teams (VMT) online environment**

# Virtual Math Teams (VMT)

- **VMT is an online environment for students to work on and discuss math problems synchronously.**
- **VMT combines support for dynamic geometry with media for collaborative learning.**

# VMT with multiple GeoGebra tabs

The image shows a VMT (Virtual Meeting Tool) interface with multiple GeoGebra tabs. The main window is titled "Demo\_1: student1 (CID:1368226612637)". The interface includes a menu bar (File, Edit, Chat, GeoGebra), a toolbar with various icons, and a central workspace. The workspace contains a geometric diagram with points A through I, a circle, and a triangle. A pink text box on the right side of the workspace contains the following text:

Welcome to the WARM-UP space for Dynamic Geometry!  
This is a space for you to explore the most important tools of this mathematical software.  
You can try out things on your own or collaboratively with the other members of your team.  
Try to create and move around the basic OBJECTS of Dynamic Geometry: points, lines, circles, triangles, etc.  
To get started, press the 'Take Command' button below. Use the chat to communicate with group members.

At the bottom of the workspace, there is a "Take Control" button and a "Move Graphi" button. The "Take Control" button is currently disabled, and the text "[nobody has control]" is displayed next to it. The "Move Graphi" button is currently selected, and the text "Move ..." is displayed next to it.

On the right side of the interface, there is a "Current users" section showing "student1". Below that is a "Chat (2)" section with a list of messages:

- student1 7:04:11 PM EDT: Here is our triangle
- student1 7:05:22 PM EDT:
- student1 7:05:23 PM EDT:
- student1 7:05:24 PM EDT:
- student1 7:08:52 PM EDT: And here is our circle
- student1 7:09:51 PM EDT: Note the reference to the whiteboard

Below the chat is a "Message" section with the text "Here is a GeoGebra circle!".

Labels on the left side of the image point to various elements:

- VMT menu
- GeoGebra menu
- Tool Bar
- Views Bar
- History Slider
- Throttle
- Referencing Tool

Labels at the bottom of the image point to various elements:

- Take/ Release Control Button
- Current Tool Indicator
- GeoGebra Reference
- Chat Reference

# Integration with GeoGebra

- **Remote students can synchronously work on a shared construction together.**
- **Users can take turns manipulating the construction.**
  - Adding, deleting, modifying and moving objects
- **The construction will stay in sync on each user's screen.**
- **Users can chat about the problem as they work.**

# Multi-user GeoGebra

The image displays two side-by-side screenshots of the GeoGebra multi-user interface. Both windows show the same content, representing the user's perspective. The left window is for user 'demo4: tony' (CID:1347034769142) and the right window is for user 'demo4: amantoan' (CID:1347034769142). Both windows have a menu bar with 'File', 'Edit', and 'Chat'. Below the menu bar, there are tabs for 'GeoGebra', 'Task', and 'GeoGebra2'. The main content area is titled 'Goal of the activity' and contains the following text:

In this activity, you will use the equivalent of straightedge-and-compass tools to construct parallel lines, and a midpoint. Then you will construct a right triangle. These are basic con relationships, which are used over and over in geometry. To make it easier to do these fre you can program your own custom tools in GeoGebra. In this activity, you will program a n constructing a dynamic-geometry perpendicular.

**Warning:** This activity has many steps. Give yourself plenty of time to work on this before y

### Construction of a perpendicular at a point

We want to construct a line GH perpendicular to line AB and passing through point C to inte C.

1. Clear anything on the drawing area with the menu "File" | "New" | "Don't Save".
2. Construct line AB with the Line tool. Construct an arbitrary point C with the Point too AB. Now you want to construct a perpendicular to line AB, which intersects line AB at
3. Construct a circle with center at C using the Circle tool D not on AB). (passing throu
4. Use the intersect tool to construct points E and F at the two intersections of the cir Notice that points E and F are equidistant from point C.
5. Construct a second circle with center at E passing through F.
6. Construct a third circle with center at F passing through E (and therefore having the previous circle).
7. Use the intersect tool to construct points G and H at the two intersections of the cir E and F) with each other.
8. Construct line GH.

Use the angle tool for angle ACH to see if line GH is perpendicular ( $90^\circ$ ) to line AB at Point C

Use the drag test to see if line GH stays perpendicular to line AB at point C.

Think about why GH is perpendicular to AB at point C. Was every step necessary? Can you s construction?

Retrieved from "[http://vmttest.mathforum.org/vmtwiki/index.php?title=Demo1\\_-\\_demo&Categories:](http://vmttest.mathforum.org/vmtwiki/index.php?title=Demo1_-_demo&Categories:)

- Demo1
- Demo
- Geometry

The chat window on the right of each window shows a conversation between the two users:

amantoan  
tony

Chat (11)

tony leaves the room 12:33:19 PM EDT

tony joins the room 12:34:56 PM EDT

tony 12:36:11 PM EDT: Hello.

amantoan 12:36:15 PM EDT: HI again.

tony 12:36:25 PM EDT: What is our assignment today?

amantoan 12:36:34 PM EDT: Let's look at the Task tab to see.

tony 12:36:59 PM EDT: We are going to create a perpendicular line based on Euclid's method

amantoan 12:37:13 PM EDT: Right.

amantoan 12:37:28 PM EDT: We will only use straight edge and compass like tools.

tony 12:37:39 PM EDT: Sounds fun.

amantoan 12:37:45 PM EDT: Let's get started.

tony 12:37:53 PM EDT: OK, I'll start

Messages

# History Tracker

- **Built-in history tracker allows users to scroll back and forth in time to see how the construction developed.**
- **Shows everything that happened including style changes and object movements.**
- **Each GeoGebra workspace is tracked separately.**

# History Tracker In Action

The image displays two side-by-side screenshots of the GeoGebra web application interface, illustrating the 'History Tracker' feature. Both windows show the same geometry construction: a horizontal line  $a$  with points  $A$  and  $B$ ; a vertical line  $b$  intersecting  $a$  at point  $C$ ; a circle centered at  $C$  with radius  $CE$  (point  $E$  on  $a$ ); a circle centered at  $A$  with radius  $AC$  (point  $D$  on  $a$ ); a circle centered at  $B$  with radius  $BC$  (point  $F$  on  $a$ ); and a circle centered at  $C$  with radius  $CH$  (point  $H$  on  $b$ ). A green box highlights the angle  $\alpha = 90^\circ$  at point  $C$ .

The left window, titled 'demo4: tony (CID:1347034769142)', shows the chat history window with the following messages:

- amantoan 12:37:28 PM EDT: we will only use straight edge and compass like tools.
- tony 12:37:39 PM EDT: Sounds fun.
- amantoan 12:37:45 PM EDT: Let's get started.
- tony 12:37:53 PM EDT: Ok, I'll start
- amantoan 12:39:38 PM EDT: Let me try!
- tony 12:39:40 PM EDT: ok
- amantoan 12:40:23 PM EDT: I think that's it.
- tony 12:40:33 PM EDT: Let's check.
- tony 12:40:46 PM EDT: Looks good.
- amantoan 12:41:05 PM EDT: Yep
- tony 12:41:24 PM EDT: ok

The right window, titled 'demo4: amantoan (CID:1347034769142)', shows the chat history window with the following messages:

- amantoan 12:37:28 PM EDT: we will only use straight edge and compass like tools.
- tony 12:37:39 PM EDT: Sounds fun.
- amantoan 12:37:45 PM EDT: Let's get started.
- tony 12:37:53 PM EDT: Ok, I'll start
- amantoan 12:39:38 PM EDT: Let me try!
- tony 12:39:40 PM EDT: ok
- amantoan 12:40:23 PM EDT: I think that's it.
- tony 12:40:33 PM EDT: Let's check.
- tony 12:40:46 PM EDT: Looks good.
- amantoan 12:41:05 PM EDT: Yep
- tony 12:41:24 PM EDT: ok

At the bottom of the right window, the 'Message' input field contains the text 'Let's review how we did this.' and the status 'nobody has control' is displayed. The status 'tony is typing' is visible at the bottom right of the right window.

# Other Shared Tools

- **Shared Whiteboard**
  - Users can doodle on the shared whiteboard
  - Draw simple shapes
  - Summarize work, draft shared statements, note observations or hypotheses in text boxes
  - History of the whiteboard is also tracked.
- **Web browser**
  - Simple web browser
  - Can be used to show instructions for the student's assignments or other related information on the web.

# **VMT has built in tools for session analysis**

- **VMT records every chat, and action in the session.**
- **Provides a session replayer to go back and forth through the session step by step.**
- **Chat, whiteboard, and GeoGebra events all playback in the order they occurred in the original session.**

# The VMT Session Replayer

Chat player

CID:1346942242703

GeoGebra Task

File Edit View Perspectives Options Tools Window Help

Move Graphics View: Drag graphics view or one axis (Shift + Drag)

Refresh View Take Control nobody has control

Speed: 1

Time to previous: -0:00 (Awareness info)  
Current action at: 11:57:54 AM (Message by: tony)  
Time to next:

Current users:

CHAT (0)

amantoan 9/6/12 11:27:00 AM EDT: That's it.  
tony 9/6/12 11:27:04 AM EDT: Lets check.  
tony 9/6/12 11:27:21 AM EDT: Looks good.  
amantoan 9/6/12 11:28:01 AM EDT: It stays that way!  
tony 9/6/12 11:28:40 AM EDT: Lets show the teacher how we did this.  
amantoan leaves the room 9/6/12 11:57:44 AM EDT  
tony leaves the room 9/6/12

Message:

# **VMT has built in tools for session analysis**

- **Spreadsheet log files can be downloaded for each VMT room – by anyone: students, students from other teams, parents, teachers, researchers.**
- **Log files and the VMT Replayer provide unique insights for teachers and researchers.**
- **Allows teachers to go back and see anything that teams did in the chat rooms.**

# Session Log Files

	A	B	C	D	E	F	G	H
1	Line	Date	Start Time	Post Time	Duration	EventType	tony	<u>amantoan</u>
2	1	09/06/2012		10:38:42	00:00:00	chat	joins the room	
3	2	09/06/2012		10:39:54	0:1:12	chat		joins the room
4	3	09/06/2012		11:24:49	0:44:55	chat	Hello	
5	4	09/06/2012		11:25:07	0:0:18	chat		Hi. Lets start by looking at the task description.
6	5	09/06/2012		11:25:11	0:0:4	chat	Ok.	
7	6	09/06/2012		11:25:31	0:0:20	chat	Alright, I'll start.	
8		09/06/2012		11:25:42	0:0:11	<u>Geogebra: GeoGebra</u>	added point:Point "A"	
9		09/06/2012		11:25:44	0:0:2	<u>Geogebra: GeoGebra</u>	added point:Point "B"	
10		09/06/2012		11:25:44	0:0:0	<u>Geogebra: GeoGebra</u>	added line:Line "a"	
11		09/06/2012		11:25:50	0:0:6	<u>Geogebra: GeoGebra</u>	added point:Point "C"	
12		09/06/2012		11:25:52	0:0:2	<u>Geogebra: GeoGebra</u>	added point:Point "D"	
13		09/06/2012		11:25:52	0:0:0	<u>Geogebra: GeoGebra</u>	added conic:Circle "c"	
14		09/06/2012		11:26:01	0:0:9	<u>Geogebra: GeoGebra</u>	added point:Point "E"	
15		09/06/2012		11:26:02	0:0:1	<u>Geogebra: GeoGebra</u>	added point:Point "F"	
16	7	09/06/2012		11:26:18	0:0:16	chat		Great, I'll take it from here.
17		09/06/2012		11:26:29	0:0:11	<u>Geogebra: GeoGebra</u>		added conic:Circle "d"
18		09/06/2012		11:26:33	0:0:4	<u>Geogebra: GeoGebra</u>		added conic:Circle "e"
19		09/06/2012		11:26:40	0:0:7	<u>Geogebra: GeoGebra</u>		added point:Point "G"
20		09/06/2012		11:26:43	0:0:3	<u>Geogebra: GeoGebra</u>		added point:Point "H"
21		09/06/2012		11:26:52	0:0:9	<u>Geogebra: GeoGebra</u>		added line:Line "b"
22	8	09/06/2012		11:27:00	0:0:8	chat		That;s it.
23	9	09/06/2012		11:27:04	0:0:4	chat	Lets check.	
24		09/06/2012		11:27:15	0:0:11	<u>Geogebra: GeoGebra</u>	added angle:Angle "?"	
25	10	09/06/2012		11:27:21	0:0:6	chat	Looks good.	
26								
27								

# VMT Is Publicly Available

- **VMT is open source.**
- **Our Math Forum VMT server is available for all to use – [vmt.mathforum.org](http://vmt.mathforum.org)**
- **Anyone may set up their own VMT server.**

# Create your own topic rooms

← → ↻ 144.118.94.160:8080/VMTLobby/commons/index.jsp

 **The Math Forum @ Drexel**  
PEOPLE LEARNING MATH TOGETHER

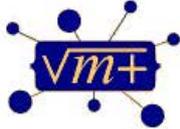
**Collaboration and Dynamic Mathematics in Middle and High School**  
Online professional development course offered Fall 2012 by the Math Forum, Drexel University, and Rutgers University (Graduate credit and stipends available)

Home Math Help Problems & Puzzles Math Talk Resources & Tools About The Math Forum

Welcome What's New Students Educators Parents & Citizens Researchers

## Virtual Math Teams 3.0-Alpha-1

Welcome tony



- ☒ New to VMT?
- ☒ List of All Rooms
- ☒ My Profile
- ☒ My Teammates
- ☒ My Rooms
- ☒ Messages
- ☒ Manage Activities

[VMT Help](#) Pages  
[VMT Sandbox](#) Room  
[VMT Lounge](#) Room  
[VMT Wiki](#) Pages  
[VMT Replayer 3 Alpha-1](#)  
[Logout](#)

### View Chat Rooms as

**Math Subject Tree** Tabular List

Filter Chat Rooms By...

Project: IGI 2012 Last Activity: Show All

Apply filters Use default filters

► **Geometry** (1 Topic)

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## **6. Supporting collaboration**

**Virtual math teams are guided by carefully designed and tested curricular resources**

**They drag points to discover, discuss and reflect on dependencies in figures**

**They discuss how to construct figures with the needed dependencies and take turns constructing and testing the figures**

# Teachers collaborate on inscribed triangles

Chat player  
CID:1353122104578

Material: Add a tab +

Summary Original Figure Our Team's Construction

File Edit View Options Tools Window Help

Drag vertex A or vertex D to explore the dependencies in this figure.

Explain in the chat what the dependencies are.  
Can you prove why these dependencies are needed to make this figure?

Then construct a similar figure with the same dependencies.  
Explain in the chat how you constructed the dependencies.

Take Control | nobody has control | Move Graphics View

Current users:

Chat (10)

michele\_colon 11/26/12 8:07:23 PM EST: ending early, woo hoo

JL123 11/26/12 8:07:24 PM EST: that seems to work

sholland 11/26/12 8:07:32 PM EST: sounds good to me

sholland 11/26/12 8:08:24 PM EST: So did we use the compass tool for all 3 points or just 2?

emilyL 11/26/12 8:08:45 PM EST: only 2

JL123 11/26/12 8:08:49 PM EST: yes lets just discuss dependencies further and I think our work is done here

emilyL 11/26/12 8:08:52 PM EST: in ours G and U

JL123 11/26/12 8:09:04 PM EST: compass for 2

sholland 11/26/12 8:09:30 PM EST: I laugh thinking how hard it was to create an equilateral triangle the first time. We are much better with geogebra now

emilyL 11/26/12 8:09:31 PM EST: CG and BU are dependent of AD because we used the compass tool to keep the same radii

JL123 11/26/12 8:09:31 PM EST: points were made be intersecting for E and poin tool for U

emilyL 11/26/12 8:09:42 PM EST: lol me too!!

emilyL 11/26/12 8:09:48 PM EST: well the ind also helped

JL123 11/26/12 8:09:55 PM EST: I know its crazy n always beneficial working with this group

sholland 11/26/12 8:10:11 PM EST: yes

michele\_colon 11/26/12 8:10:14 PM EST: so true!

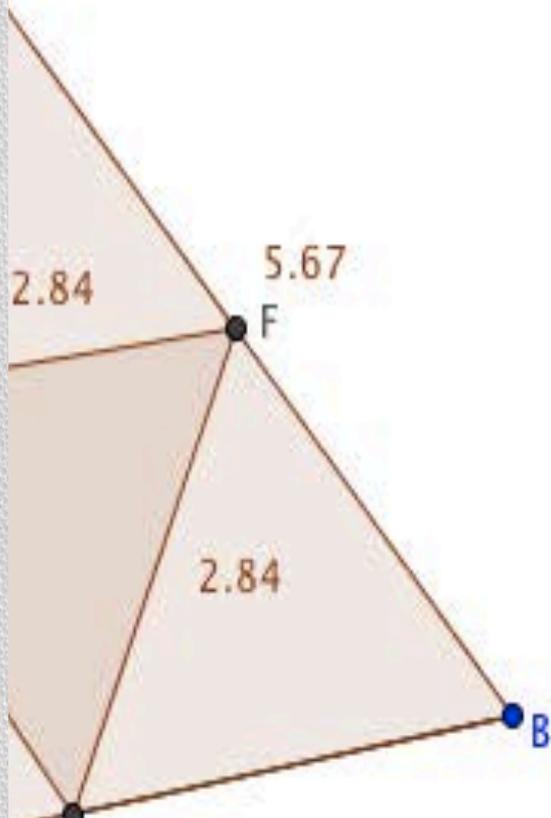
sholland 11/26/12 8:10:40 PM EST: So the first point D was just a point on the object or what?

Message:

emilyL is typing

Speed: 1

Time to previous: -0:00 (Message by: sholland)  
Current action at: 8:10:40 PM (Awareness info)  
Time to next: 0:06 (Awareness info)



**sholland** 11/26/12 8:09:30 PM EST: I laugh thinking how hard it was to create an equilateral triangle the first time. We are much better with geogebra now

**emilyL** 11/26/12 8:09:31 PM EST: CG and BU are dependent of AD because we used the compass tool to keep the same radii

**JL123** 11/26/12 8:09:31 PM EST: points were made be intersecting for E and poin tool for U

↑ **emilyL** 11/26/12 8:09:42 PM EST: lol me too!!

**emilyL** 11/26/12 8:09:48 PM EST: well the ind also helped

**JL123** 11/26/12 8:09:55 PM EST: I know its crazy n always beneficial working with this group

↑ **sholland** 11/26/12 8:10:11 PM EST: yes

↑ **michele\_colon** 11/26/12 8:10:14 PM EST: so true!



## **7. Researching the learning processes**

**Students are asked to identify chat log segments that show effective collaboration and to reflect on what they noticed and wondered about**

**Teachers are asked to select and reflect on student chat log segments – and to discuss how to improve the resources, approach and experience**

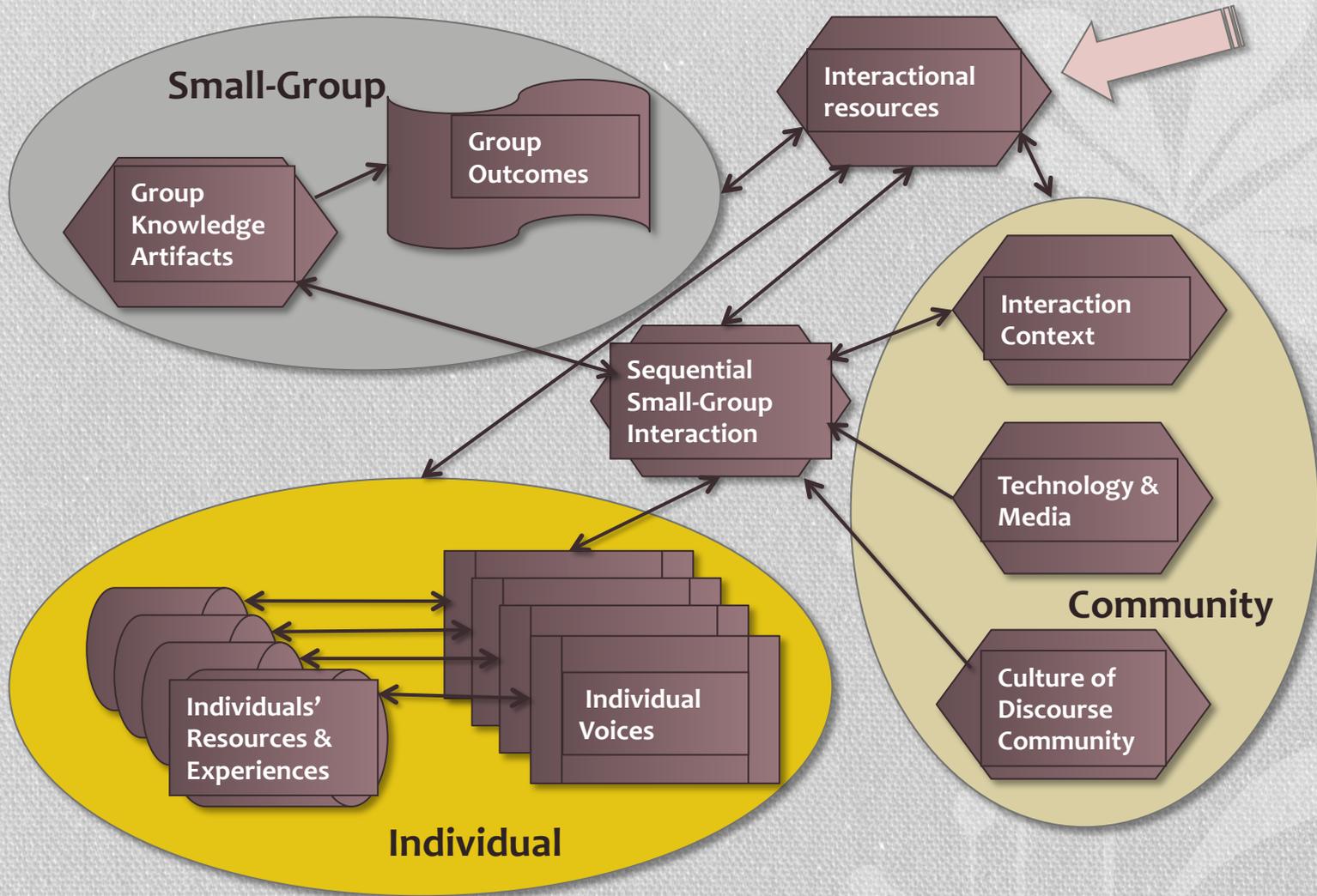
**Researchers analyze logs and replayer to understand strengths and weaknesses of resources and to investigate how student teams collaboratively master dynamic geometry practices, skills and understanding**

## **8. Theorizing the approach & resources**

**The ideology of individualism views geometry as a realm of mental ideas that individuals must master using logic.**

**But geometry is a product of the community of mathematicians, math educators and math students during the past 2,600 years; it can be learned collaboratively by teams discovering and creating under the guidance of teachers/educators**

**There is validity both to the importance of individual mental effort and socio-cultural practices. There should be support at individual, group and community levels**



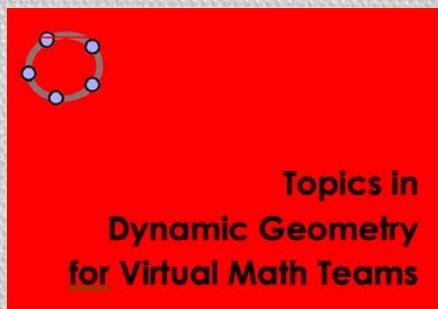
**Levels of analysis connected by interactional resources**

# My recent theoretical work

- **Group cognition as focus of analysis on the inter-personal inter-action**
- **Theory of resources mediating learning**
- **Unity of levels of analysis/interaction:  
Individual, team, community (class, schooling, math)**
- **Theory implications of analyses of collaborative dynamic geometry learning**

## 10. Developing the curricular resources

Although the VMT Project was funded to just develop the technology and analyze its effectiveness, the real problem is to design the pedagogy: approach & resources



We developed a set of about 18 “topics” in a workbook format that included tutorial “tours” of the technology. Each topic was intended for about a one-hour online, synchronous, collaborative session. Each topic included 3 to 10 GeoGebra tabs with guiding tasks

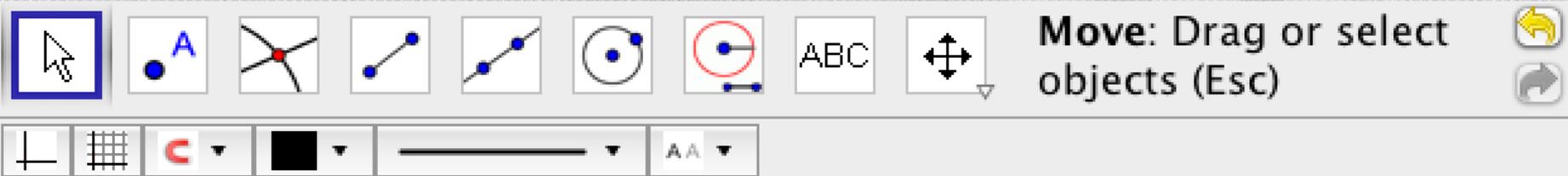
# Pedagogical focus

The approach emphasis includes:

- 1. The importance of experiencing first-hand the actual doing of mathematics: exploration, noticing, discovering, wondering, conjecturing, creating, designing, constructing, explaining, understanding, proving, teaching**
- 2. Resisting the tradition of accepting on authority the definitions and understandings of geometric objects**
- 3. Resisting the temptation to use GeoGebra just to illustrate geometric facts with pretty figures or flashy simulations**
- 4. Guiding teachers and students to design their own constructions, including the definition of custom tools**
- 5. Emphasizing the role of dependencies in dynamic geometry**

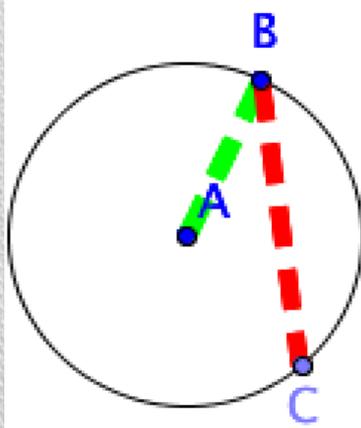
# Beginning topics

1. Warm up – login and create points
2. Messing with dynamic geometry – dragging points; copying & adding segment lengths
3. Visualizing Thales' and Pythagoras' theorems
4. Constructing equilateral triangles
5. Programming custom tools: perpendicular lines, parallel lines, equilateral triangles
6. Finding 8 centers of triangles – challenge: Euler's segment and the nine-point circle of a triangle



Move: Drag or select objects (Esc)

An example of what you will construct:



\*\*\*Construct a segment whose length = sum of two lengths\*\*\*

1. Construct a circle with center through a point, its radius and a chord.  
(A radius is a segment from a circle's center to a point on its circumference--like AB--and a chord is a segment connecting two points on its circumference--like BC.)
2. Construct a line like DE and construct a segment along it, whose length is the sum of the lengths of your radius + chord.
3. Drag each point, segment or circle to make sure that the length of the segment changes dynamically correctly.



# Intermediate topics

- 7. Rigid transformations, symmetry, proof of area of a triangle**
- 8. Exploring angles of triangles and similar triangles**
- 9. Visualizing congruent triangles**
- 10. Solving typical geometry problems**
- 11. Constructing inscribed triangles & polygons**
- 12. Building a hierarchy of kinds of triangles**

This is a tricky case.  
 Given triangle ABC, construct another triangle with an angle equal to  $\angle ABC$ , a side along the angle equal to side AB, and a side opposite the angle equal to side AC.

1. Use the compass tool to copy angle ABC to angle HGI
2. Use the compass tool to copy side AB to side GJ and
3. to copy side AC to side JK.
4. Now drag point K to meet the side extending GI.
5. Notice that for some shapes of triangle ABC, there are two points that satisfy the constraint SSA, but that only one of them constructs a triangle congruent to ABC
6. Discuss this in the chat.



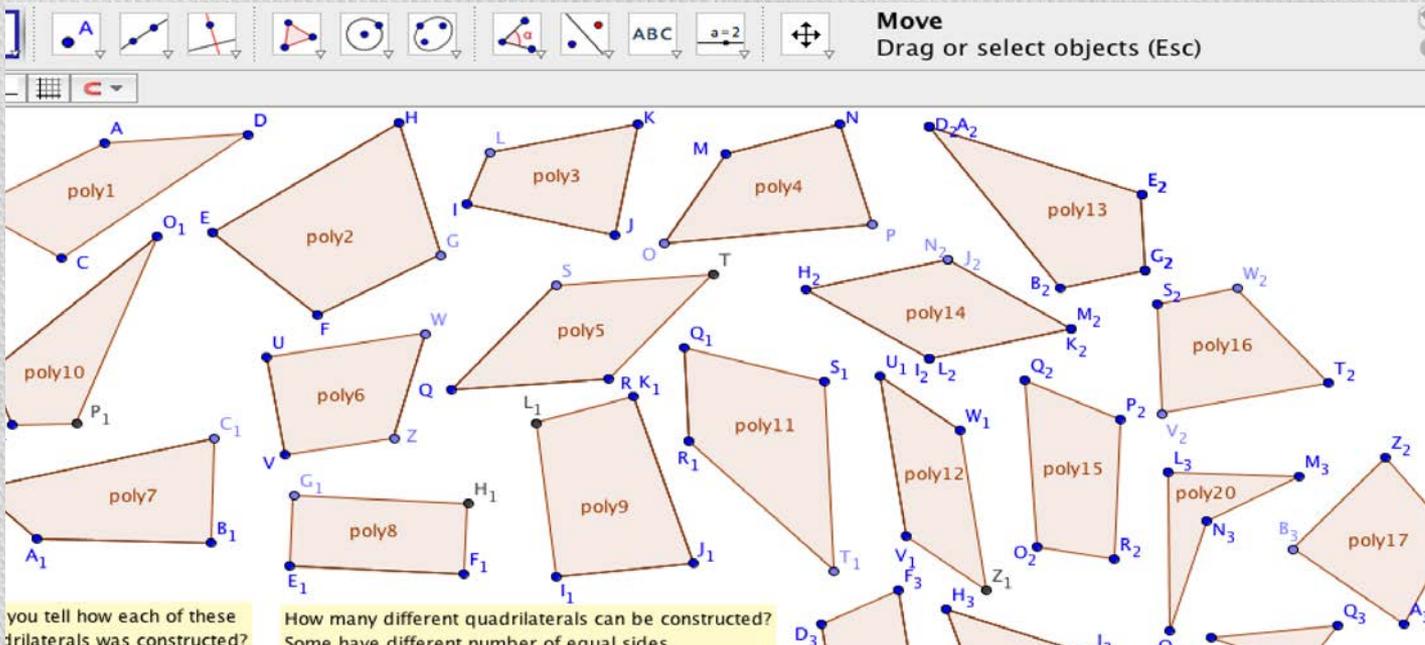
To construct the centroid of a triangle, construct the midpoints of the three sides (you can use the Midpoint tool for this). Then construct Segments from the Midpoints to the opposite vertex. Construct the Point where these Segments intersect. (Note that all three Segments intersect at the same location, so you can use the intersection of any two Segments.)

Now create a custom tool to automatically construct the centroid given the three vertices of a triangle.

Create some different triangles and their centroids. Drag the vertices of the triangle and observe how the centroid behaves. Is it always inside the triangle?

## Advanced topics

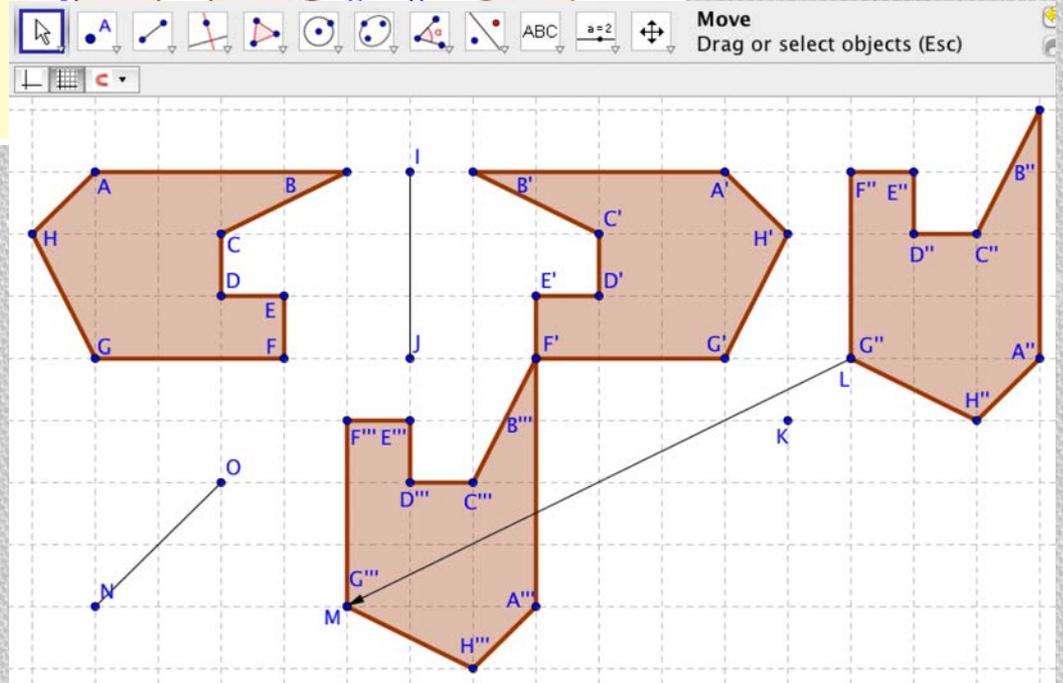
- 13. Constructing a square and other quadrilaterals**
- 14. Building the hierarchy of quadrilaterals**
- 15. Constructing a tangent geometrically and using the GeoGebra algebra interface**
- 16. Proving incenter properties and Euler segment properties with constructed dependencies**
- 17. Modeling a factory workflow with systems of rigid transformations**
- 18. Exploring taxicab transformational geometry**



How many of these quadrilaterals were constructed? What are its dependencies?

How many of these quadrilaterals are special? What is special about each one?

How many different quadrilaterals can be constructed?  
 Some have different number of equal sides.  
 Some have different number of equal angles.  
 Some have different number of right angles.  
 Some have different number of parallel sides.  
 Some have different number of lines of symmetry.  
 Some have diagonals with different characteristics.



# 11. Design-based research of human-centered geometry

**Discover:** The curricular topics guide student teams and teacher teams to discover dependencies in dynamic-geometric figures

**Create:** The curricular topics guide student teams and teacher teams to creatively design and construct dependencies in dynamic-geometric figures

**Understand, explain, prove:** Students and teachers learn to view geometric truths in terms of constructed dependencies. They begin to see the causality of the world as human/social creative-discovery involving designed dependencies

The VMT Project evolves its pedagogical approach through iterative analysis of interactions among teams of researchers or teachers or students using the technology and resources. The research process is reflected in its publications and presentations.

# The Virtual Math Teams Trilogy

## Group Cognition (2006)

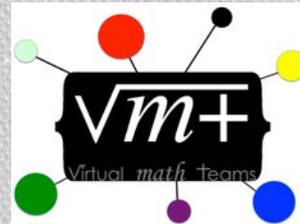


### Computer Support for Building Collaborative Knowledge

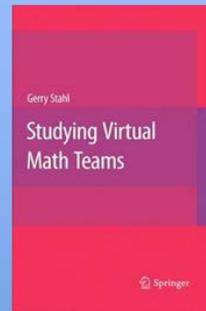
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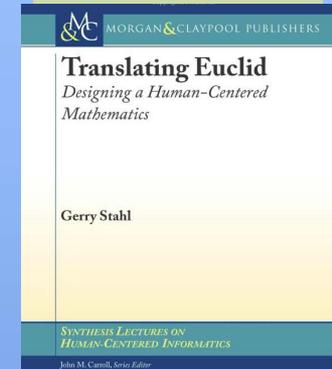


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Email:

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