Collaborative Exploration of Geometric Dependencies in Dynamic Geometry

Gerry Stahl, Stephen Weimar, Annie Fetter, Anthony Mantoan
{Gerry, Steve, Annie, Tony}@MathForum.org
The Math Forum at Drexel University, Philadelphia, PA, USA

Abstract. The Virtual Math Teams Project (2002-2014) at the Math Forum developed a collaborative-learning environment for mathematics, combining text chat and a multi-user version of GeoGebra. It created curricular activities aligned with Common Core and provides teacher professional development. It has deployed the technology and curriculum with groups of students year after year and analyzed some of the student interactions in micro-detail. Study of how collaborative learning takes place in this GeoGebra-based environment has been used to refine the environment and curriculum. Student teams learn how to collaborate, work online, use GeoGebra, analyze and construct dynamic-geometry figures, think about dependencies among geometric objects and talk about mathematics. This presentation demos the approach and shows how learning about dynamic-geometric dependencies is displayed in a data excerpt.

Themes. Research, CSCL Environments, Curriculum, Practices, Technologies.

Keywords. Multi-user GeoGebra, VMT collaboration environment, geometric dependencies, interaction analysis, CSCL, collaborative learning

Research Context and Problem: Guiding Students to Math Cognition

The contemporary fields of science, technology, engineering and mathematics (STEM), in particular, require a mindset that emerged historically within the community of ancient Greek geometers (Heath, 1921). For many people, learning basic geometry still represents a watershed event that determines if an individual will or will not be comfortable with the cultures of mathematical cognition. GeoGebra provides a promising tool for supporting transformational mathematical thinking.

At the Math Forum (www.MathForum.org), we have embedded GeoGebra in an online collaboration environment (Stahl, 2009) and converted it to a multi-user version (Stahl, 2013), so that groups of students can construct and drag figures together, while chatting about what they are doing. To guide student exploration, we have developed a cohesive curriculum focused on the construction of figures with geometric dependencies (Stahl, 2015b)—for use by student teams as well as in teacher professional development. Collaborative GeoGebra is now available for iPad, tablets and laptops (vmtdev.mathforum.org). The curriculum is in a GeoGebraBook (http://ggbtu.be/b154045), which is not yet multi-user.

In this paper, we demo collaborative GeoGebra and illustrate how we analyze case studies of students engaged over multiple sessions with our online collaboration-learning environment: multi-user GeoGebra, challenging topics and inquiry pedagogy. For the past decade, such analysis of student usage has been driving the iterative design of our approach. We want to indicate how student teams under these conditions display that they are learning fundamental insights about dynamic geometry.

Theory of Collaborative Dynamic Geometry: Group Cognition and Dependencies

Learning is often conceived as a change in propositional knowledge possessed by an individual student. Opening up an alternative to this view, Vygotsky argued that students could accomplish knowledge-building or learning tasks in small groups before they could accomplish the same tasks individually—and that much individual learning actually resulted from the earlier group interactions (Vygotsky, 1930/1978), rather than the group being reducible to its members as already formed individual minds. Vygotsky conceived the group interactions as mediated by artifacts, such as representational images and communication media. More recently, educational theorists have argued that student processes of becoming mathematicians or scientists, for instance, are largely a matter of mastering the linguistic practices of the field (Lemke, 1993; Sfard, 2008).

Our pedagogical approach emphasizes collaborative learning through discourse in small groups (Stahl, 2015c). Carefully designed topics guide student exploration and bring in historically developed concepts from the mathematics community. Teachers prepare students before sessions and discuss findings and conjectures in whole-class discussions after the collaborative sessions. The group cognition (Stahl, 2006) that takes place in the group work can lead to learning by individual students in their zones of proximal learning, based on their joint meaning making and task accomplishments.

Our curriculum focuses on learning to construct *geometric dependencies* (Stahl, 2013) in GeoGebra, a challenging but important skill. While much classroom use of dynamic geometry today merely uses it as a visualization tool, to allow students to drag existing diagrams around, the technology has a greater potential: to empower students to construct their own diagrams, to build their own dependencies into the objects and even to fashion their own custom construction tools. Then they can view Euclid's propositions as guides to designing and constructing their own interesting mathematical objects, rather than as impersonal eternal truths to be memorized.

Research Method: Sequential Interaction Analysis

The data we collect from hundreds of students using our system each year includes a complete record of their interactions, which we can replay just as it appeared to the students. We also have detailed logs generated automatically.

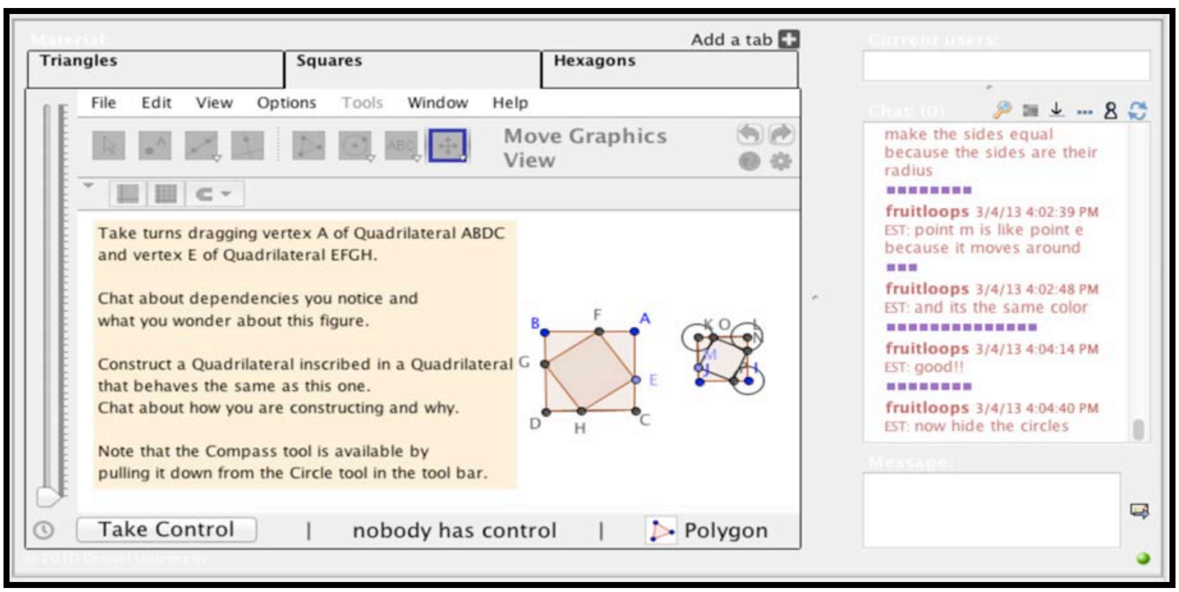

Figure 1: The interface of the collaboration environment, showing multi-user GeoGebra and text chat.

We use methods of interaction analysis or conversation analysis (Jordan & Henderson, 1995; Schegloff, 2007), adapted to our online math-education setting. This looks at how student groups engage in shared attention, joint representation and intersubjective meaning making. Although we recognize that processes at different levels are inextricably intertwined in reality, we focus methodologically on the group unit of analysis, which is where individual learning, group becoming and community practices are often most visibly displayed (Stahl, 2015c).

Findings: Collaborative Learning of Dynamic Geometry Core Principles

In our case study for this presentation, three 14-year-old girls engaged in our environment for eight hour-long sessions (Stahl, 2015a). In their sixth session, they worked on the problem shown in Figure 1, constructing inscribed squares. They had previously solved the challenge of constructing inscribed triangles, but had never constructed a square. We follow their explorations, which led to an elegant construction of a square. They were then able quickly and collaboratively to construct the inscribed squares, based on their previous experience with inscribed triangles. They displayed their group and individual learning through their GeoGebra actions, text chat and building on each other. They explicitly discussed the need to construct various geometric dependencies to accomplish this task.

Conclusions: Designing an Integration of Software, Curriculum and Practices

The Virtual Math Teams (VMT) Project (http://gerrystahl.net/vmt) at the Math Forum (www.MathForum.org) has been researching the integration of: an online collaboration environment, multi-user versions of GeoGebra, sequences of curricular units, data analysis methods and pedagogical approaches for over a decade. We now believe that a collaborative approach to dynamic geometry can support the learning of core components of mathematical cognition. Our approach integrates online software (for all browsers on computers, tablets, iPad), student-centered collaboration (with text chat), teacher orchestration of student teams and a carefully scripted sequence of curricular units (emphasizing exploration, reflection and group mathematical discourse). The curriculum is aligned with Common Core standards and focuses on the mathematical notion of dependency and techniques for constructing dependency in GeoGebra. Dependency is central to dynamic geometry, to deductive thinking and to student understanding of explanatory proofs. The dependencies constructed with GeoGebra tools—often following Euclid's procedures—result in figures with desired invariants. We have shown that even young students in groups can begin to understand, analyze, design and construct dynamic-geometric dependencies with GeoGebra.

The tablet version of multi-user GeoGebra with chat has just become available (vmtdev.mathforum.org). Teachers and groups of students can use it for free. A set of 50 GeoGebra activities (in a GeoGebraBook: http://ggbtu.be/b140867) introduces student teams to the role of geometric dependencies in exploring, articulating, creating and explaining dynamic-geometry figures and relationships—within a gaming-like context of sequenced challenges.

References

Heath, T. (1921). *A history of Greek mathematics* (Vol. I: From Thales to Euclid). Oxford, UK: Clarendon Press.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences. 4*(1), 39-103. Web: http://lrs.ed.uiuc.edu/students/c-merkel/document4.HTM.

Lemke, J. L. (1993). *Talking science: Language, learning and values*. Norwood, NJ: Ablex.

Schegloff, E. A. (2007). *Sequence organization in interaction: A primer in conversation analysis*. Cambridge, UK: Cambridge University Press.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.

Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Web: http://GerryStahl.net/elibrary/gc. Doi: http://mitpress.mit.edu/books/group-cognition.

Stahl, G. (2009). *Studying virtual math teams*. New York, NY: Springer. Web: http://GerryStahl.net/elibrary/svmt. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3.

Stahl, G. (2013). *Translating Euclid: Designing a human-centered mathematics* (paperback & ebook ed.). San Rafael, CA: Morgan & Claypool Publishers. 221 pages. Web: http://GerryStahl.net/elibrary/euclid. Doi: http://dx.doi.org/10.2200/S00492ED1V01Y201303HCI017.

Stahl, G. (2015a). *Analyzing the development of mathematical group cognition*. Web: http://GerryStahl.net/elibrary/analysis.
Stahl, G. (2015b). *The construction crew game*. Web: http://GerryStahl.net/elibrary/topics/game.pdf; http://ggbtu.be/b154045.
Stahl, G. (2015c). The group as paradigmatic unit of analysis: The contested relationship of CSCL to the learning sciences. In M. Evans, M. Packer & K. Sawyer (Eds.), *The learning sciences: Mapping the terrain.* Cambridge, UK: Cambridge University Press. Web: http://GerryStahl.net/pub/ls.pdf.
Vygotsky, L. (1930/1978). *Mind in society*. Cambridge, MA: Harvard University Press.