Gerry Stahl's assembled texts volume #19

Theoretical Investigations:

Philosophic Foundations of Group Cognition (pre-publication version)

Gerry Stahl

Gerry Stahl's Assembled Texts

1. *Marx and Heidegger*
2. *Tacit and Explicit Understanding in Computer Support*
3. *Group Cognition: Computer Support for Building Collaborative Knowledge*
4. *Studying Virtual Math Teams*
5. *Translating Euclid: Designing a Human-Centered Mathematics.*
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Theoretical Investigations

(pre-publication version)

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2018

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Published by Gerry Stahl at Lulu.com
Printed in the USA
ISBN: 978-1-105-61697-6 (eBook)
ISBN: 978-1-387-81829-7 (paperback)

Notice

Pre-publication version

This volume is a pre-publication version of *Theoretical Investigations*, published by Springer Press in 2021. These materials were last revised November 7, 2018, from the final manuscript. This version has not been edited, laid out or paginated by Springer Press. Please do not cite page numbers from this version or quote from it. This version is only for informal use and may not be duplicated. Please refer to the Springer Press version for official usage, citation and pagination.

Errata of the published book

The following pages of the published version contain errata known to the author as of the latest revision of this pre-publication version. All have been corrected in this version. Please notify the author at Gerry@GerryStahl.net if you discover any additional errors.

Page xvii: “Appendices” should be “Appendix: Notes on the Investigations”

Pages 33 and 34: The term “cooperative” should be hyphenated as “co-operative: in the section title, i.e., Co-operative Action,” and below, where it explains that Goodman “chose the term ‘co-operative’ because people typically perform specific operations in coordination with each other.”

Page 36: The two references to issue “10 (2)” should be to issue “10 (3).”

Page 446: In the second paragraph, “inscribed squares” should be “inscribed triangles.” In the bottom full paragraph, “inscribed triangle” should be “inscribed equilateral triangle.”

Page 540: The two references to Suthers, et al (2010) should be combined.

Page 548: “[Interaction 12]” should be “[Investigation 12].”

Page 550: “(n/2) 2” should be “(n/2)^ 2.”

Page 556: “[Interaction 12]” should be “[Investigation 12]” and “[Interaction 16]” should be “[Investigation 16].”

Forward

Christopher Hoadley, Series Editor

In the current time, unlike any other, we think both as individuals and as groups, cultures and as a species, mediated by powerful tools that yoke the thinking of people around the world, if loosely. Never before in human history has most of the human species been linked by instant telecommunication, nor have we had planetary-scale access to information. While the digital divide remains, a tipping point has been crossed; the International Telecommunications Union estimates that the portion of the human race using the Internet crossed 50% sometime in 2017-2018[1]. These observations are not new. What is new is the need to come up with coherent ways to reconceptualize some very basic ideas in this new reality, including knowledge, research and learning. In this context, the field of computer-supported collaborative learning, and this book series, offer some important insights.

There are narrow and broad framings of computer-supported collaborative learning. The narrower framing, focused on the intersection of "computer-supported," "collaborative" and "learning" reflects the initial impetus behind much of the work in the field. How could we use the power of new technologies to support innovative pedagogies where learners work together? As Stahl nicely points out in his Introduction, the ideas and people gelling around this possibility were hard to concisely circumscribe or define: Is it a paradigm? A vision? In its narrower conception, CSCL is simply a sub-sub-area of applications of educational technology. However, the particular group of people working in this area and who carried the banner 'CSCL' explore much deeper issues.

Unique to the vision described by Stahl is an opportunity to define a much broader perspective, one that unpacks decades- or centuries-old assumptions about thinking, learning and knowing; and the epistemologies we use to explore those assumptions. This volume helps bring forth how studying this one narrow context—teaching kids to learn through technology-supported collaboration—can help us develop philosophical and practical approaches to new ways of understanding the relationship between information and meaning; between the psychological vs. the social and cultural sciences; and between our

[1] www.itu.int/en/ITU-D/Statistics/Documents/facts/ ICTFactsFigures2017.pdf

philosophies of science, our sciences of learning and our models for growing, sharing and perpetuating knowledge.

There is a dialectic in the field of CSCL, one which is illustrated by both this book series and the journal from which this volume draws. In CSCL there is a constant ebb and flow between what you might call on the one hand the science, or better yet, the natural philosophy, of learning and collaboration, and on the other hand the practical wisdoms encountered by inventing, designing, reforming and implementing new possibilities for knowledge and learning with the latest technologies. In many cases the dialectic produces astonishing results, not because of some outrageously successful teaching strategy, or because of some thunderous research finding on discourse or learning, but *because the dialectics help us reframe basic assumptions about what it means to know in a global, networked knowledge society*. Other volumes in the series have helped illustrate this in the past, including notably Stahl's (2006; 2009) earlier monumental work on group cognition which helped reframe the question of "How individual is knowledge anyways?" or the edited volume by Suthers, Lund, Rosé, Teplovs & Law (2013) on productive multivocality which helped reframe the question of "to what extent and how is conversation actually knowledge, and knowledge actually conversation?"

This book is in a somewhat unusual format, for important reasons. As a juxtaposition of classic articles from the journal and commentary, it is itself an example of (hopefully productive) multivocality. However, to assume it's simply a "greatest hits" volume that rehashes old ground would be to misunderstand its contribution. The difficulty in CSCL of synthesizing a common theoretical basis on which to build, a paradigm in the Kuhnian sense, an orthodoxy but also a cumulation, does relate to the ways in which CSCL is an interdisciplinary crossroads, and in which researchers are drawing eclectically on many traditions in ways that have not been solidified. But this is not simply a case of "just wait a few more decades and we'll have this sorted out into something neat and tidy and paradigmatic." Rather, it is an example of the field of CSCL struggling with, as the tech startups say, "eating your own dogfood." If knowledge is socially constructed, if we learn in the middle spaces between monologue and dialogue, if our understandings of knowledge contest both linear, accretive positivism and kaleidoscopic but subjective interpretivism, then we need to question what forms our scholarly output can take.

This book extends and builds on Wittgenstein's idea of *investigations*. The book is a learning tool that invites the reader along on a journey that invites not only apprehension of prior scientific, philosophical and design work, but also a reconstruction and co-construction of knowledge. Stahl consistently enhances the work by others in the field with his own research legacy in the VMT project, bringing his own inimitable voice to the analysis. Is it a summary of what *ijCSCL* has produced? Or is it a masterclass in building theories that take into account new models of knowledge (including new roles for the academics most likely reading these words)? I argue it is both: a summation and an invitation to think along with one of the most qualified guides to this way of studying and fostering thinking and learning that we happen to call CSCL.

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Part I: Overview

Introducing Theoretical Investigations

By compiling his aphoristic *Philosophical Investigations*, Wittgenstein (1953) provided a provocative image of how philosophy could apprehend the world—in particular how it should understand language about the world. Just as Wittgenstein believed that a certain form of conceptual analysis was needed in philosophy, I am convinced of the relevance of certain kinds of theoretical reflection to the burgeoning field of computer-supported collaborative learning (CSCL). In founding and editing the *International Journal of Computer-Supported Collaborative Learning* (*ijCSCL*)—as a collective effort with many leading researchers from around the world—I intended to craft a venue for CSCL researchers to publish theoretical reflections on their work and on the nature of computer-supported collaborative learning. In addition, I always tried to derive theoretical insights from my own research, with its analysis of recorded student discourse.

In the spirit of Wittgenstein's collection of deliberations, I now assemble highlights of the journal and of my writings that I believe can contribute to an understanding of concepts and themes central to the field of CSCL. Wittgenstein's presentation was self-consciously anti-systematic. His paragraphs (like Einstein's transformative papers in physics) primarily pose "thought experiments," which tend to problematize established ways of thinking that have become second nature. Similarly, the Investigations of the present volume do not intend to lay out a detailed roadway for educational transformation or a logical edifice of theory. Rather, they hope to question outmoded assumptions and stimulate creative exploration in CSCL theory, methodology and practice—like Wittgenstein providing examples of a different kind of theory. However, in distinction from the thought experiments of Wittgenstein's philosophy, the *ijCSCL* papers and my own research reports are firmly grounded in analysis of empirical interaction data.

Introducing Part I

Looking over the collection of papers collected for this volume of *Theoretical Investigations*, I perceive an emergent vision of CSCL, quite distinct from traditional educational research. I have therefore written two new introductory essays to provide an overview that suggests this vision and that connects it to theoretical concepts. The Introduction and these two essays constitute Part I, a synopsis and foreword to this volume.

- The first of these essays [Investigation 1] argues for a particular vision of CSCL, centered on a specific paradigm of collaborative learning, which is expanded by the sequence of *ijCSCL* papers [Investigations 3-14] that constitute Part II of this volume.
- The second essay [Investigation 2] reviews contributions to a theory of group cognition as foundational for CSCL research and practice. The papers covered by this essay [Investigations 15-25] are gathered from reports of the Virtual Math Teams (VMT) research project I directed. These papers constitute Part III of this volume.

The impetus driving the research field of CSCL has been evolving over several decades. However, the multifaceted knowledge required to implement CSCL pedagogy widely in schools was not available until now. Widespread assumptions (e.g., Schwarz and Wise, 2017) to the contrary, elements of such knowledge now largely exist—albeit in a preliminary, fractured, distributed and uncoordinated manner. For instance, much of the knowledge needed for educational transformation is described, pointed to or illustrated in past volumes of *ijCSCL*. Unfortunately, however, some of the most innovative or penetrating analyses published there have not been further pursued or integrated with each other and with the accepted wisdom of the CSCL field. The present compendium of selected papers from *ijCSCL* is an attempt to substantiate this claim that the necessary components are available and to indicate a possible path forward to implementing a CSCL vision.

The view that students should be active learning agents is as old as our culture, enunciated by Socrates and Buddha, for instance. Over a century ago, Dewey argued for a progressive-inquiry approach in modern public schools—although, despite widespread recognition, his approach had limited impact on schooling. CSCL was founded to pursue a potential to transform learning from the memorization of facts instructed by authorities to inquiries of student groups assembled and supported by networked computer technologies. Still today, many people conduct research or introduce classroom interventions that they call CSCL, but that lacks the elements that we have discovered to be central to effective collaborative learning. It is not sufficient to place groups of students together with arbitrary computer communication apps; one must design, identify and support the required processes and practices—such as intersubjective meaning making and mediated knowledge building—for establishing a culture of group inquiry and collective knowledge building.

Introducing Part II

The publications selected for this volume from *ijCSCL* build upon historical sources and early CSCL investigations. They suggest: how to simultaneously focus CSCL theory and broaden the field's scope; how to analyze the processes of collaborative learning and mediation of group cognition by computer artifacts or supports; and how to develop innovative technological tools and educational infrastructures to facilitate collaborative knowledge building. Accordingly, they transform and potentially integrate elements of CSCL theory, methodology and practice that can contribute to an ambitious effort to realize the CSCL vision on an international scale.

The papers included here from *ijCSCL* all emerged out of CSCL labs around the world. Significant CSCL investigations generally require teams of researchers, pooling different expertise and perspectives on cognitive theory, analytic methodology and educational practice. They often involve consortia of labs. However, the effort to go beyond the scattered research efforts of CSCL to date and to implement the long-range vision in schools would require an even greater collaboration of researchers and educators—one on a global scale. The present volume aims to motivate the claim that this is possible through a review of the central points of selected investigations published in *ijCSCL* and reproduced here. The overview in Investigation 1, written for this volume, indicates how a synthesis of these proposals for CSCL theory, methodology and practice could allow us to reach toward implementation of a CSCL vision. The effort required for achieving this CSCL vision would involve a global collaboration, supported by computer technologies and funded by progressive political will.

As founding editor (with Friedrich Hesse and a distinguished Board) of *ijCSCL* from 2006 through 2015, I selected favorite articles for this volume and commented on them from the perspective of influences on my own evolving understanding of CSCL. I include some articles related to the VMT project, which is the CSCL research I know firsthand. Several of the other publications represent the work of leaders in the field of CSCL research. Many of these articles were among the most cited and downloaded publications in *ijCSCL*. I selected those that have a strong theory focus and are suggestive for implementing the CSCL vision. There are, of course, many other insightful theoretical papers available in *ijCSCL*; it was not possible to include them all in this volume. I hope this compilation will stimulate readers to return to early *ijCSCL* issues to unearth other gems.

Throughout the history of CSCL, there has been a tension between various paradigms of research, colloquially referred to as "quantitative" versus "qualitative." The thrust of this collection of papers is that the defining characteristic of CSCL methodology should not be the genre of techniques applied in data analysis, but a focus on small-group interactions. The focus on the group level is definitive of collaborative learning or knowledge construction in CSCL.

In addition, “socio-cognitive” and “socio-cultural” approaches have often been contrasted. The vision arrived at in this volume moves beyond viewing individual cognition (thinking) as peripherally affected by its social context to considering human cognition as itself an inter-personal, social or small-group phenomenon, evolving in a biological and cultural background. Today, cognition incorporates a tightly entangled complex of external memories, mediating artifacts, communication partners and networked interactions. So conceptualized, collaborative learning is no longer a niche educational activity subservient to the needs of individual minds, but a foundational mode of being-in-the-world-with-others, from which individual cognition is itself a derived narrative.

Various efforts are underway to harness the opportunities of global networking of information to make course materials from advanced educational centers more broadly available around the world. These include wiring schools for the Internet, distributing networked tablets, offering massive-open-online courses (MOOCs) as well as offering open-educational resources (OER), although these initial attempts rarely adopt pedagogies of collaborative learning. A CSCL approach would add support for engaging students in joint inquiry of the available resources, involving intersubjective meaning making and collaborative knowledge building. This volume stresses the importance of supporting the collaboration in order to make technological innovations truly transformative. Part II concludes with two reports of tentative, but systematic attempts to deploy CSCL initiatives at the level of national school systems. They document efforts to develop cultures of collaborative learning in school districts. They are suggestive of an international effort that could prove transformative. As technology transforms and interconnects working, learning and thinking around the world, it calls for recognition of the importance of collaboration, which currently lags behind. Within the vision of human cognition as increasingly global, the goal of promoting worldwide collaborative learning seems inevitable, if currently challenging.

The selected papers from *ijCSCL* in Part II raise issues of CSCL theory, such as the nature of intersubjectivity, joint attention, shared experience, meaning making, artifact usage, reference, temporal sequentiality, discourse structure, multiple levels of description, primary unit of analysis, external memory, group practices and group cognition.

Introducing Part III

These issues are further explored in the VMT research papers in Part III. The Virtual Math Teams (VMT) Project has already been extensively documented in four previously published volumes:

- *Group Cognition: Computer Support for Building Collaborative Knowledge* (Stahl, 2006a). This collection of research reports motivates the design of the VMT Project. It begins with several attempts to design support for collaborative learning and cooperative work. Challenges that arose in these efforts showed a need for deeper theoretic foundations, raising questions concerning the preconditions for productive collaboration. The concept of "group cognition" emerged during the compilation of this book as a label for the shift of focus in research on learning to the small group as the primary unit of analysis for investigation. It seemed important to begin to collect data systematically documenting student interaction within a paradigmatic CSCL setting. Final chapters report on initial findings from students chatting about mathematics problems. At this stage, the proposals that mathematics could be learned collaboratively, that successful CSCL outcomes could be generated, recorded and analyzed or that interaction in such data could be understood in theoretical and practical terms were all hypothesized as questions to be investigated.

- *Studying Virtual Math Teams* (Stahl, 2009) documents the VMT Project as it began to explore technology for supporting student mathematical discourse. Core issues of pedagogy, analysis and theory are considered in relation to technological features. The VMT system integrated a shared whiteboard with text chat. Sessions were automatically recorded so that student interaction could be replayed and analyzed in detail by researchers. First examples of successful CSCL sessions are presented here, along with analysis of many aspects of the technology, pedagogy and methodology brought to bear. Presented case studies show that collaborative learning could provide a powerful approach to mathematics instruction.

- *Translating Euclid: Designing a Human-Centered Mathematics* (Stahl, 2013) reflects on the final version of the VMT Project from a dozen perspectives. The co-evolution of theory, methodology, pedagogy and technology through iterative cycles of design and testing illustrate a design-based research approach. At this point, a multi-user version of GeoGebra was integrated into the shared whiteboard, to allow teams of students to construct and explore strategically selected geometric figures and gradually learn to think/discuss geometrically and solve problems collaboratively. This book confirmed the hypotheses about the possibility of generating, recording and analyzing successful CSCL sessions of collaborative online learning in the illustrative domain of dynamic geometry.

- *Constructing Dynamic Triangles Together: The Development of Mathematical Group Cognition* (Stahl, 2016a) provides a book-length longitudinal study of how a specific group of three young girls began to learn dynamic geometry together. The detailed analysis shows how the group successively adopted a productive set of group practices for collaboration, geometric construction, problem solving and mathematical discourse. This provides a paradigmatic example of a CSCL approach to teaching a challenging school subject. It illustrates a method for analyzing longer sequences of interaction that build group competencies—showing how interaction in such data can be understood in theoretical and practical terms.

Part III of the current volume elaborates theoretical issues that were raised in these books. It thereby supplements them and completes the documentation of the VMT research effort.

During several years the VMT Project conducted Fests, in which students were invited through their teachers to participate in online small-group sessions of mathematical problem solving. This generated most of the data analyzed by researchers. Some sessions were particularly well suited for analysis due to continuity of participants. Key examples of interaction data from these VMT sessions appear in multiple Investigations. The most intensively analyzed sessions were those of the following student teams:

- Teams in the VMT Spring Fest 2005, including the students ImH and Jas as well as Sup, Pin and Avr [Investigations 22, 23] (see also Çakir, 2009).
- Team B in the VMT Spring Fest 2006, including the students Quicksilver, Bwang and Aznx [Investigations 5, 8, 16, 19, 25] (see also Medina, 2013).
- Team C in the VMT Spring Fest 2006, including the students Qwertyuiop, 137 and Jason [Investigations 12, 16, 17, 19, 21, 24] (see also Sarmiento-Klapper, 2009; Zhou, 2009).
- The Cereal Team in the VMT Winter Fest 2013, including the students Cheerios, Fruitloops and Cornflakes [Investigations 9, 16, 18, 22] (see also Stahl, 2013).

The Investigations of Part III draw theoretical consequences from the analysis of interaction in these case studies.

It is not necessary to read the Investigations in this volume in order. The twenty-five presentations are structured so that they can be skimmed, read, studied or skipped in any order. Each is self-contained, incorporating its own problematic, argument, literary style and reference section. Most of the Investigations are reprints or adaptations of earlier publications (see Notes on the Investigations), originally focused on a special point for a particular audience. They retain some of the emphasis deriving from their origin during a particular point in the development of the theory of group cognition.

To aid in integrating the whole presentation, connections and references among the Investigations abound—both implicitly and explicitly. It is hoped that the different presentations support and enhance each other, gradually building a sense of the depth, evolution and power of group-cognition theory, as well as of the potential of the CSCL field to empower students to tackle the daunting challenges of the future collaboratively.

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Investigation 1. Advancing a CSCL Vision

Gerry Stahl

Abstract. The field of computer-supported collaborative learning (CSCL)—as a unity of educational practice and academic research—is characterized in this Investigation by a specific vision of learning, illustrated by a prototypical research effort. A number of recent publications are reviewed to extend the scope of CSCL in response to contemporary theory and current social issues. This leads to advancing theoretical concepts and frameworks for conceptualizing CSCL research and practice, which contrast with traditional educational approaches. Although these ideas were originally proposed in disparate contexts, they provide the conceptual skeleton of a unified theory for CSCL, which would be distinguished from popular theories of individual learning and would integrate technological support with collaborative cognition. These insights concerning theory have methodological implications for analyzing CSCL interventions in terms of group knowledge-building practices mediated by interactionally appropriated artifacts. Revised forms of analysis can help innovators evaluate CSCL trials during iterations of design-based research, leading to revisions of the collaborative-learning theory and research methods. Bridging from academic research to educational practice, two examples of efforts to bring the CSCL vision to scale within national school systems are then reviewed. Finally, a global collaboration among CSCL researchers is recommended for effective implementation of the CSCL vision in education worldwide, based on the presented conceptualizations of a unified theory of collaborative learning and their implications for evaluation of CSCL technical and pedagogical designs. This could advance the field of CSCL in its theory and practice, toward its underlying vision of cognition at the group level.

Keywords. CSCL theory, group practice, design-based research, scaling up, cognitive evolution, group cognition, sequential

analysis, knowledge objects, referential resources, temporal analysis, instrumental genesis, intersubjective meaning making.

Defining a CSCL Vision

Previous attempts to circumscribe the field of CSCL have faltered; the target is so nebulous, controversial, disjointed, multi-dimensional and agonistic. Most of these endeavors have tried to specify operational criteria for inclusion of papers in the CSCL corpus (Akkerman et al., 2007; Jeong & Hmelo-Silver, 2016; Jeong, Hmelo-Silver & Yu, 2014; Kienle & Wessner, 2006; Lonchamp, 2012; Schwarz & Wise, 2017; Tang, Tsai & Lin, 2014). However, such attempts to apply "objective" standards generally fail to include some of the most important contributions, especially those that are more theoretically oriented. As a multi-disciplinary field, CSCL papers bear more of a "family resemblance" (Wittgenstein, 1953) to each other, sharing diverse constellations of characteristics and relationships, rather than fitting a definition with clear and distinct necessary and sufficient conditions (Descartes, 1633/1999).

Perhaps that is why the first definition of CSCL (Koschmann, 1996) presented it as a "paradigm," contrasting it with earlier educational-technology research paradigms like computer-assisted instruction, intelligent tutoring systems and constructionist exploratory environments—which all focused on learning by individuals, conceived in terms of behaviorist, cognitivist or constructivist psychology, respectively. However, Koschmann (2001) soon realized that actual CSCL research did not form a neat paradigm, contrasting with earlier, incommensurate research approaches, but included an eclectic mixture of mutually conflicting theories, methods, pedagogies and settings.

A frequently cited introduction to CSCL (Stahl, Koschmann & Suthers, 2006) characterizes its approach as: "studying how people can learn together with the help of computers." This generic characterization is immediately followed with the warning that CSCL "has a complex relationship to established disciplines, evolves in ways that are hard to pinpoint and includes important contributions that seem incompatible." It suggests that one should "view CSCL as a vision of what may be possible with computers and of what kinds of research should be conducted, rather than as an established body of broadly accepted laboratory and classroom practices."

It seems that what we need is neither a definition of past work nor a paradigm of an ideal science, but a focused yet open vision for the future—along with a concrete "prototype" example to serve as a cognitive reference point (Lakoff, 1987). A prototype example is a typical instance that often comes to mind, like a robin is a prototypical bird, although it has various similarities and differences to other birds, like turkeys or penguins. Therefore, I will here sketch a vision of CSCL based on my own efforts to develop a prototypical CSCL design. In addition, I will consider a selection of papers published in *ijCSCL* that I feel have until now been undervalued in setting future directions for CSCL. These papers suggest how to extend existing examples of CSCL research to a growing family of related efforts.

The vision of CSCL advanced here is that students working in small groups can productively incorporate collaborative learning centrally in their schooling and in their intellectual development, taking advantage of appropriate forms of computer support. As CSCL is adopted as a foundational form of learning in educational systems around the world, students will acquire collaborative group practices, individual cognitive skills and technology-enhanced abilities to enable them to address the challenges of contemporary social issues.

Collaborative learning is a primary form of human learning, and facility in collaborating can enhance student participation in other learning. Meanings and practices developed by small groups can result in understandings and skills of the individual group participants—although the correspondence between learning at the different levels is by no means direct or necessary. Increasingly today, with the Internet, students and others can form spontaneous, opportunistic or long-term networks to discuss, debate and explore topics of interest—including issues of global importance; students can learn to build knowledge together and refine understanding by sharing perspectives. Formal education in schools can involve mutually supportive mixes of individual, small-group, classroom and networked activities. Collaborative learning can be extended outside the classroom as well.

Although knowledge has always been a social product in many senses, the ubiquity of computers and networking tremendously expands the potential to collaborate in building knowledge, to take advantage of computational support for knowledge creation, or to share and preserve knowledge. On the other hand, the proliferation of technology has also contributed to enormous societal problems: climate change, income inequality, over-population, fake news, nuclear proliferation and political schisms. The skills acquired during CSCL sessions in working, problem solving, conceptualizing and reflecting together in small groups may be critical for addressing such pressing social issues of our times, as this Investigation will suggest.

Two major sources for CSCL theory are Vygotsky (1930/1978) and Lave and Wenger (1991); they proposed influential perspectives on mediated cognition and social practices—i.e., shifting the traditional focus from methodological individualism (including positivism, behaviorism and cognitivism) to the mind-in-society mediated by artifacts, and the community-of-practice as the primary level of analysis. Two early investigations following these perspectives and also definitive of the CSCL vision were those of Scardamalia and Bereiter (1996) and Teasley and Roschelle (1993); they extended the unit of analysis to the group or classroom and to the joint-problem-space as represented by knowledge artifacts and as observable in shared discourse. These initiatives have been conceptually elaborated in subsequent CSCL theoretical papers, as we will see in the following.

My prototypical example of computer-supported collaborative learning involves a team of three 13-year-old girls interacting in the Virtual Math Teams (VMT) online environment to investigate dynamic geometry. The software allows a team of students to explore mathematical tasks in a shared dynamic-geometry workspace, which responds interactively to their actions constructing and dragging points, lines, triangles, and so on. The student discourse takes place through textual chat in the same software environment. Tasks from the teacher and curriculum displayed in the workspace include example constructions, technical terminology and prompts for collaboration and discussion. The analysis of the team's eight hours of interaction (Stahl, 2016) is carried out at the small-group unit, documenting how the team adopted over 60 "group practices" [Investigation 16] of collaborative interaction, geometry construction, problem solving and mathematical discourse. Without speculating about what took place in the individual students' minds, the analysis shows how the team achieved impressive geometry accomplishments as a group and documents that each individual significantly increased her geometry skills through participation in the collaborative learning.

This example prototype is specific in many ways that are typical of some CSCL projects but not others: The team is a small group of students meeting online in an after-school club. It interacts through chat and actions in a multi-user application (see Figure 1). Pedagogical guidance is supplied by a carefully crafted sequence of tasks. Interaction in the group takes place as mediated by reference to the task descriptions, previous chat postings, construction actions and graphical figures. Analysis tracks the sequentiality of chat and math events as they develop within a network of artifacts, meanings, questions, technical terminology (e.g., "dependency") and practices (e.g., dragging points to test for geometric dependencies). The subject domain has broad implications for learning: Studying Euclidean geometry has served since Plato as the classic gateway to logical thinking and deductive argumentation (Stahl, 2013); collaborative, computer-supported dynamic geometry could similarly serve as a training ground for the group cognition required for democratic responses to contemporary social issues through deeper understanding of interconnections among actors and factors.

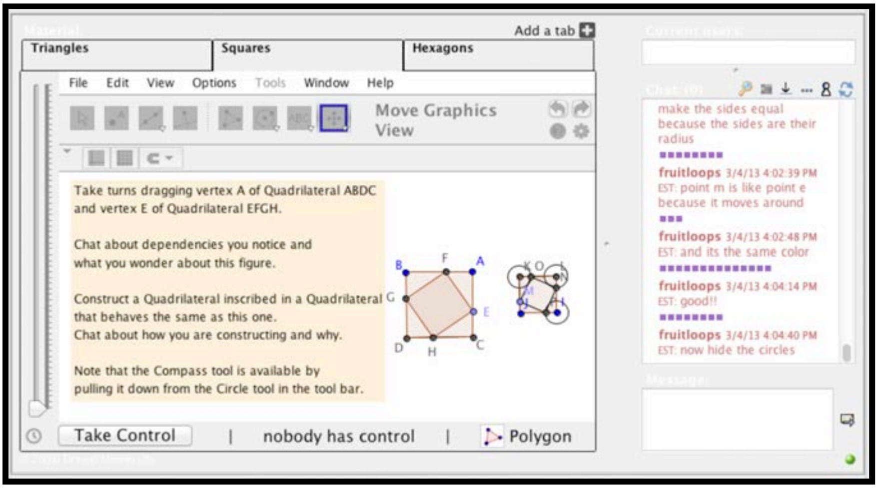

Figure 1. The VMT interface. The team has constructed square IJKL and inscribed another square inside it, based on exploration of the given example of square ABCD and past group experience constructing inscribed triangles.

Each of this prototype's specifics could be expanded by other CSCL efforts with family resemblances to it. The VMT Project illustrates one typical approach to CSCL, but it has differences from other current or future instances. To extend from this example, synchronous text chat can be replaced by asynchronous discussions, perhaps increasing reflection but lessening the flow of thinking together. Other knowledge domains can be supported with appropriate tools and curriculum. The role of computers in collaborative knowledge building can switch from communication medium to face-to-face workspace or embodied virtual reality. The after-school math club can grow to international networking, bringing different cultures together. CSCL environments can include scientific models, simulations or artistic media. They can be supported with feedback and analytics of the interaction for student awareness, teacher overview and researcher analysis.

The following consideration of several evocative papers in *ijCSCL* suggests possible dimensions for fruitful advances in the scope of CSCL from a focus on the micro-level interaction within small groups of students. This could lead to a growing family of theories, research projects and institutional interventions resembling each other in various ways and all pursuing the underlying CSCL vision.

We will now review a number of *ijCSCL* papers that are suggestive of directions for progress in CSCL. This will provide an overview and contextualization of the Investigations published in Part II of this volume. The following comments on these papers are only meant to highlight some themes addressed by the papers and to motivate the careful reading of the Investigations themselves.

Extending the CSCL Vision

In the first year of *ijCSCL* publication, **Jones, Dirckinck-Holmfeld and Lindstrom (2006)** [Investigation 3] proposed dramatically broadening the concerns of CSCL to include the larger socio-technical context and infrastructure. These authors argue for a relational, indirect, meso-level approach to CSCL design, which would go substantially beyond the traditional paradigm of educational studies. In this approach, the phenomena at the micro level are understood as outcomes of processes of development within their larger contexts.

Most educational research aims at objective results based on a view of the world as having fixed characteristics: it is assumed that technologies have inherent affordances, individual utterances have definite intended meanings, subjects have rational thoughts (logically connected mental representations), and analysis can be carried out algorithmically. Investigation in this tradition is conducted at the individual unit of analysis, classifying student utterances as expressions of imputed intentions of individual speakers.

The paper by Jones, Dirckinck-Holmfeld & Lindstrom takes a very different tack. It proposes that affordances of CSCL technologies should be understood in terms of how they are taken up by users in the interactions that the technologies mediate. Meaning is here seen as an intersubjective product of the interaction among multiple people within their conversational context, including its technological artifacts and infrastructure. The concern is with the unfolding process of (group) meaning making within these settings, rather than in traditionally conceived (individual) learning outcomes.

Analysis in this approach is complex, viewing each aspect of task, technology, personality, role, utterance, response or knowledge as inter-related or relational. Data is not directly determinant, but negotiated by participants and necessarily interpreted by researchers who understand colloquial language and human interaction. Furthermore, analysis of CSCL interactions is understood on many interpenetrating levels: the micro level of individual utterances and brief interactions, the small-group level of interacting teams of learners, the classroom level of teacher-led instruction, the local-culture level of schooling, the global level of geo-political and historical influences. Such multi-faceted analysis requires computer-supported collaboration among the multidisciplinary researchers themselves; it is notable that Investigation 3 was written by authors from three different countries.

The meso level of the community points to the realm of social practice as the locus within which interactional processes are situated; the social practices are taken up in small-group activity. This focus corresponds to the "practice turn" in contemporary social theory (Schatzki, Knorr Cetina & Savigny, 2001). In a practice-oriented analysis, structures are emergent; they grow out of recursive interactions among people, technologies and social action. In this post-cognitivist view [Investigation 15], it is not mental representations in individual minds or designed properties of technology that directly structure the practice. Rather, it is through a recurrent and situated practice over time—a process of enactment of a relevant practice by a group—that people constitute and reconstitute a structure of technology use.

CSCL designers have only limited direct control over how their designs are actually used by students. How learners respond to, understand and enact artifacts in relation to any educational design is a complex structuration process that has to be studied in practice. Investigation 3's authors contend that the CSCL tradition has pursued a relatively narrow focus that places in the background issues concerning the politics, policies, institutions and infrastructures in which the processes of CSCL take place. They argue for a greater inclusion of what they call the meso level of collaborative learning, as opposed to the trend towards networked individualism—the conception of collaborative groups in terms of their individual members. They asked—already back in 2006—whether CSCL, and education more generally perhaps, should act as a critical opponent to some of the trends identified in the networked society and stand up against networked individualism.

Several books published in recent months highlight the acute and growing importance for the survival of modern society of issues at the technological meso level or the knowledge infrastructure. Collaborative learning could prepare students to address such issues in the future, if CSCL develops effective appropriate interventions. The social issues have arisen in part as a result of the prevalence of individualism: understanding things from the epistemological perspective of a rational individual mind seeking its own personal benefit, rather than seeing how things are increasingly interrelated and interdependent. By bringing multiple personal perspectives together to analyze dependencies in studied phenomena, collaborative learning provides both an approach and a model that transcends the individualistic in favor of the collective or collaborative.

With agent-network theory (ANT), Latour has expanded the group to include artifacts as well as humans as interacting agents of change. Climate change and ecological corrosion are widespread concerns, which Latour views as results of complex networks of dependency and interaction. In his last major book, **Latour**

(2017) argues that the unforeseen consequences of industrialization have gone so far as to transform our relation to the natural world in a threatening way. What is needed is not a set of technological fixes, but a re-conceptualization of the distinction between nature and society. Not only are the new-age strictures about living in harmony with Mother Earth inadequate, but even the metaphors of ecological science need to be rethought. The complexity of climactic trends involves networks of interactions among countless human and non-human actors. The analysis of these interactions requires collaborative knowledge building on a global level, as does the designing of effective responses.

CSCL curricula can acculturate student teams to such knowledge building on a novice scale. CSCL software like Knowledge Forum, VMT and argumentation-support apps provide illustrative forms of computer support. For instance, many lessons in classrooms around the world using Knowledge Forum (Figure 2) already focus on group theorizing about environmental phenomena and historical conflicts; the geometric dependencies explored in VMT provide a metaphor for team thinking about interdependencies affecting the climate; argumentation-support systems model the forms of discourse needed for meaningful and democratic discussion of climate policy.

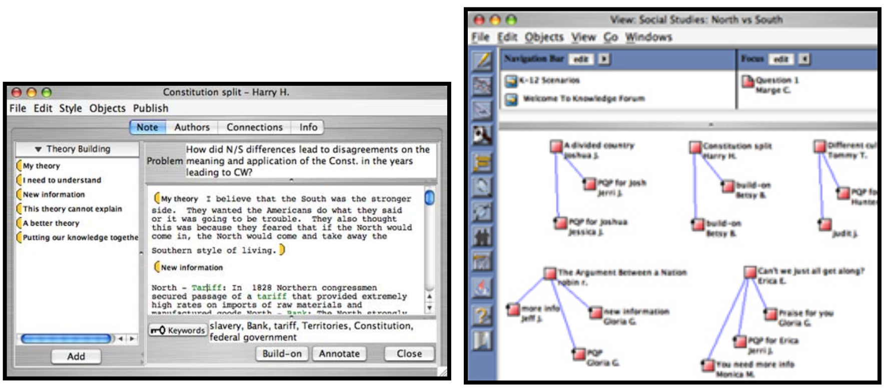

Figure 2. Knowledge Forum interface. Students enter theory-building notes (left). A view of interrelated notes is displayed graphically (right).

Computer technology—such as social media—provide a powerful infrastructure role in our society, including influencing the economy and politics. Technological tools, social institutions and human roles are not independent fixed entities. **Ekbia and Nardi (2017)** suggest that the very nature of capitalism is being transformed as people turn to online sources of information generated by unpaid participants. Companies can produce new products without having to supply manuals and training, as these are provided by the public through YouTube videos and product reviews. Other corporations provide information services through apps like Siri, Google, Alexa or FaceBook, which rely on volunteer-generated information like Wikipedia and the WorldWideWeb. This shifts labor costs from corporate wages to the unpaid public—from the producer to the consumer. Economically, this can be seen as a new strategy of capital to reduce its production costs. Consumer inputs are monetized by software giants like FaceBook, Amazon, Apple, Microsoft and Google for use by corporate and political targeting.

The pervasive technological infrastructure of social media also plays a central role in the production and dissemination of "fake news," leading to the chaotic and simplistic character of public comprehension of the political world. **Rushdie (2017)** provides a sense of some of how this emerged with the Trump campaign. The modern-world ideals of rational thought, reasoned discourse and graspable truth seem to have dissolved in a flash. Training in thoughtful group cognition and deliberative argumentation may be the best antidote to the destructive "group-think" of emotionally charged political bubbles.

CSCL research has explored argumentation-support environments (as illustrated in Figure 3) to accustom students to logical debate, to teach them to view social issues from multiple perspectives and to discuss controversial topics through discourse platforms that support rational argumentation. These systems are often designed based on Toulmin's popular theory of argumentation structure. However, as **Schwarz and Baker (2017)** make clear, the Toulmin (1958) model is most appropriate for legal briefs; it does not apply to deductive mathematical proofs or to scientific hypotheses, let alone to informal debates, which feature emotion, prejudice, identity politics and power relationships. The book by Schwarz & Baker reviews in detail traditions of multiple world cultures that led to the potential of deliberative discourse as a basis for informed democracy. Such deliberation in small groups of students can prepare them to make sense of the world and to negotiate equitable shared understandings. Skill in conducting reasoned discourse and collaborative knowledge building is the only antidote to the spin of fake news and the blinders of emotionally charged political bubbles. Students need to internalize critical debate practices in order to evaluate online information analytically.

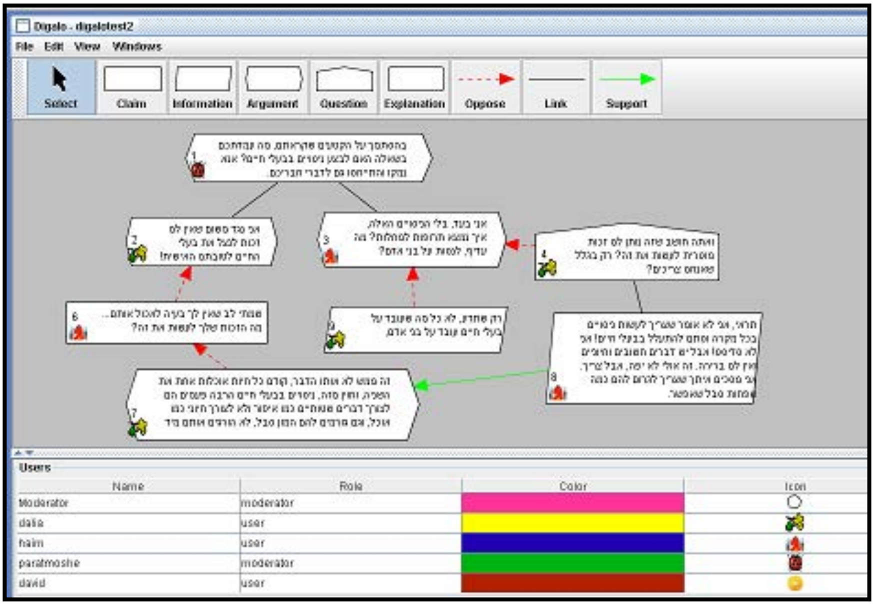

Figure 3. The Digalo interface. Students contribute elements of an argument and represent their role in the overall structure of the argument.

Investigation 3 has opened up the CSCL vision, suggesting a post-cognitive epistemology centered on group interaction. The application of this vision to various domains of understanding could have positive implications for addressing current social issues. The following Investigations zoom in and detail some of the theoretical and methodological issues involved in this expanded view of CSCL, such as intersubjective meaning making, discourse reference, artifact affordance, instrumental genesis and knowledge practice.

Conceptualizing the CSCL Vision

One of the first *ijCSCL* papers to make an important contribution to a conceptual framework for the extended CSCL vision suggested above was another paper from *ijCSCL*'s first year. It proposed an analytic focus on intersubjective meaning making. **Suthers (2006)** [Investigation 4] claims that to study the accomplishment of collaborative learning we must necessarily study the practices of intersubjective meaning making. In contrast to individualist epistemologies, where the individual is the learning agent, in intersubjective epistemologies the group is the learning agent. Collaborative knowledge construction locates the meaning making in its group context; the process of meaning making is itself constituted of social interactions. In CSCL, even if we sometimes ultimately want to track learning by individuals, we need to understand the processes of learning highlighted by intersubjective epistemologies, at both the interpersonal (group) and social (community) levels.

Meaning making in CSCL environments generally involves reference to representations, such as icons, words and drawings. Jointly constructed representations become imbued with meaning for the participants by virtue of having been produced through an interactional process of negotiation (discourse aimed at a consensual conclusion). These representational constituents then enable reference to prior interpretations with deictic pointing (through gesture or language), or by direct manipulation in a digital environment. In this manner, collaboratively constructed external representations facilitate subsequent negotiations—increasing the conceptual complexity that can be handled in group interactions and facilitating elaboration of previous conceptions. The expressive and indexical affordances of a technological medium affect its value as a referential resource.

The notion of referential resource is further elaborated in terms of practices and usage by **Zemel and Koschmann (2013)** [Investigation 5]. They analyze how a group of students in a VMT session specify how they understand the mathematical problem they are given on the computer screen and what for them might count as a solution to that problem. The authors focus on referential practices, understood as the ways that actors refer to and represent problems and solutions. References are indexical, that is, dependent on their situation or circumstances of their occurrence for their local sense or meaning; they point or index into their context of production.

Math problems, for instance, are indexical phenomena that can be indexed in various ways. Students constitute the problem on which they are working by indexing it, pointing to it, referring to its constituent properties, elements and features in particular ways. (E.g., a line "looks perpendicular," it forms a right angle, it was constructed to be perpendicular.) The more refined their referential work, the more developed their understanding of the problem.

If some object or matter is something students communicate about and work with, they must have a set of shared interactional resources that allow them to refer to that object or matter in mutually intelligible ways. Thus, collaborative learning necessarily and centrally involves the interactional, shared construction of intersubjective meaning using referential resources. (See also Garfinkel (1967); Stahl (2015)).

Investigation 5 details the work of problem solving as involving referential practices. Zemel & Koschmann show how when students in the VMT session build a representation of a problem in a particular manner using some combination of text and graphics, the key to meaning making is not the representation per se. It is the process of building the representation and working with it in a way that allows for the selection and identification of its relevant indexical properties. (The building of representations and the identification of indexical properties—which take time or effort and distract from immediate accomplishments—may explain the intriguing paradox of "productive failure" in CSCL groups (Kapur & Kinzer, 2009)). The specific indexical or referential properties of a math problem emerge through the way in which whiteboard objects and text postings are sequentially produced in relation to each other.

The idea that the meaning embodied in representations and other artifacts is interactionally constructed as a group repeatedly uses them is further explored by **Overdijk, van Diggelen, Andriessen and Kirschner (2014)** [Investigation 6]. They refine the concept of affordance (Dohn, 2009) by arguing that a technical artifact's potential for action only becomes available when learners and artifact connect, and that the availability and realization of this potential is relative to the students who interact with the artifact and to the socio-cultural context in which this takes place. When a group uses an artifact, the meaning of the artifact for the group undergoes a process of "instrumental genesis" (Rabardel & Bourmaud, 2003), in which the artifact is taken up in a specific way by the group, determining its possible significance for the group. To evaluate an innovative CSCL technological artifact, one must observe how it is used in practice. This implies a methodology of design-based research (DBR) and the identification of adoptions of group practices, as discussed below.

How a CSCL artifact is brought into use, or appropriated by students, involves a tension between the artifact as it is used by students and the intentions invested in the artifact by the designers or teachers. This tension may develop within a brief period in the context of joint activity, and be eventually resolved through a complex set of group negotiations. The effective affordances of CSCL technology result from the interaction of the implicit intentions invested in the artifact by instructional designers and the active intentionality of the learners who perform actions upon the artifact. In this way, utilization of a technical artifact can be seen as a process of social construction that is generated through a dialectic of resistance and accommodation between human agency of the student group and material agency of the designed artifact.

When groups bring an artifact into use, they call upon sets of routines and procedures that have developed around previous use of that artifact or similar artifacts. In other words, the use of artifacts is situated in group practices and motivated by routines and procedures that have become sedimented in those practices. The set of group practices incorporates resources for communication as well as classroom norms, procedures and other available technical artifacts. The group practices adopt and adapt specific social norms of the classroom that are relevant to the task at hand, and the social practices that have formed around this task. Overdijk et al. describe the appropriation of an artifact as meaningful by a group that is using that artifact as a series of enactments whereby social norms and group practices become gradually associated with the artifact. Such appropriation is framed within the constraining and enabling conditions of the local situation; through it, the group produces new conditions, affordances, meanings and understandings for future learning and action.

The idea that artifacts are brought into use and thereby granted specific meaning through the enactment of group practices is re-conceptualized at a global level of human evolution by **Ritella and Hakkarainen (2012)** [Investigation 7]. At the same time, they reflect on the difficulties of implementing appropriate educational responses implied by this new conceptualization. Key to both their theoretical and practical considerations is the concept of "knowledge practices." Knowledge (or epistemic) practices are defined as routine (recurrent and appropriated) personal, group and social activities related to working with existing knowledge and creating new knowledge. They include deliberate efforts to expand available intellectual resources by creating and building epistemic artifacts—symbols, concepts, technical terms, theories, inscriptions, visualizations, models, tools, etc.—which contribute to extending and preserving group knowledge.

Human beings do not have sufficient innate cognitive capacities to engage in the development of complex ideas within their individual brains; in order to pursue complex trains of thought, they have to, for instance, work on paper, make sketches, record information, talk things out. Inscription and visualization allow human beings to establish a theoretic culture by gradually accumulating a wide variety of external symbolic storage systems. Experts can then internalize complex reasoning and memory capabilities through sustained habits of externally embodied cognitive practices. A crucial role in the evolution of our civilization was the emergence of external memory fields (lists of numerals, art, diagrams, writing, maps, spreadsheets, wikis, networked webs) that allow us to use our powerful visual system for elaborating, sharing and building on externally represented ideas and creating exponentially growing external symbolic storage systems. In this way, human biological evolution over epochs has been extended by much more rapid cultural evolution (Donald, 1991; 2001), now amplified by technological evolution.

CSCL environments are designed to support the collaborative building of knowledge through construction of knowledge artifacts, which constitute locally created cognitive-cultural networks and mediate knowledge building. However, these goals must be brought into practice by students using them. Learning to engage in knowledge building requires the deliberate transformation of classroom-learning activities and student-participation routines, in order to capitalize on the potential epistemic mediation designed into these external artifacts. CSCL technologies allow for delegating cognitive processes to digital systems, creating mechanisms for fusing intellectual efforts in collaboration, and complementing personal epistemic resources with global networks that are accessible online. The vision of CSCL is to take advantage of forms of media in a way unthinkable in the past. Rather than assessing digital artifacts as merely isolated tools and signs, we should examine how they might radically transform human cognition and activity.

Conventional education focuses mostly on using the Internet for acquiring and consuming facts, rather than for creating new knowledge. By contrast, CSCL creates foci around which collaborative knowledge-building practices can be organized. Such environments could provide the material agency that enables even elementary-school students to participate in deliberate knowledge advancement, with adequate guidance and facilitation by teachers. The current textual practices prevailing at school, however, often guide students to use writing mostly for reporting what their textbooks say about issues being studied rather than using writing as a tool for extending thinking and deliberately generating new ideas and working theories. Adopting and cultivating a cognitive-cultural system that enables effective use of writing as a tool of thinking is difficult; it is an extended struggle to acquire embodied, largely tacit capabilities rather than direct assimilation of well-specified skills.

The CSCL vision involves educating students for future forms of cognition: technology-supported and collaborative—in groups and globally. The potential of human cognition continues to expand dramatically, and CSCL can help prepare students to appropriate the required practices and modes of learning. However, technological artifacts become instruments of human activity only through sustained and iterative efforts of using them in practice, a process through which cognitive-cultural activity gradually transforms and adapts according to evolving practices of using technologies. This evolution is reflected in deep-level changes in mental processes. Unfortunately, this must overcome considerable resistance and inertia. This is clear in the fact that it is still rare for students to appropriate the full potential of the written word after millennia of literacy (Ong, 1998). Not surprisingly, CSCL researchers have generally underestimated the in-depth challenges associated with students enacting new cognitive practices at the personal and collective levels.

Ritella & Hakkarainen argue that all successful cultures of CSCL are simultaneously also expansive-learning communities (Engeström, 1987) focused on problematizing current practices, envisioning changes and gradually, step-by-step, consolidating novel inquiry practices. Through sustained collaborative activity, ideas, artifacts, methods and practices—that do not belong to any one of the individual participants—emerge situationally and interactionally within groups from self-organized collaborative processes as meaningful and effective.

The expansion of the vision of CSCL with theoretical elaboration of concepts like intersubjective meaning making, referential resources, artifact affordance, knowledge practice, instrumental genesis and cultural evolution prepares the way for understanding how CSCL in the future could contribute to intellectual development of new generations. The problem becomes a more practical one of evaluating the potential impact of proposed innovations. How can these theories guide the CSCL design process in analyzing trial interventions of CSCL prototypes?

Analyzing the CSCL Vision

The conceptual framework discussed in the previous sections has implications for CSCL methodology. It means that it is no longer sufficient to run simple controlled studies with some student groups using an experimental CSCL tool and the other students not using it—and then concluding that if the students in the experimental condition individually tested higher, then:

a) The new CSCL tool led to more learning;

b) The tool worked as designed; and

c) Collaborative learning is effective.

Rather, the theory suggests, for instance, that:

a) Collaborative learning is a complex process that is in each case situated in specific group contexts and requires the understanding and analysis of meaning-making interactions;

b) CSCL tools must be appropriated by user groups over time to determine their affordances in specific contexts; and

c) CSCL environments ultimately aim at enhancing the power of human knowledge building by providing artifacts that extend external memory, computational ability and conceptual depth.

Analyses of interventions with new CSCL tools need to explore how teams of users take up—or fail to appropriate—the designed artifacts as knowledge-building tools. This generally involves scrutinizing:

a) The discourse and actions within the team of students as it constitutes the team's intersubjective meaning making,

b) The temporal unfolding of interaction and, in particular, the instrumental genesis of CSCL tools as used by the team, and

c) The team's adoption of group practices associated with the CSCL approach and resources.

A number of *ijCSCL* articles in the past address aspects of methodology appropriate to accomplish such analysis of CSCL interventions.

The focus on student discourse is perhaps the primary consideration. This is motivated by theories focused on discourse, such as the theory of "commognition" (communication-based cognition). **Sfard (2008)** proposed that human cognition (thinking) is a derivative form of communication (speaking). Young children first learn to talk in family interaction, later engaging in self-talk, which eventually evolves into silent thought (see Vygotsky, 1930/1978; 1934/1986). Language was the first step in cultural evolution, leading to cognition by nomadic hunting teams, extended-family tribes and eventually individuals (especially with the advent of written language). Commognition incorporates the response structure of interacting multiple voices even in an individual's solitary reflection (Bakhtin, 1986). In CSCL data, the sequential nature of discourse can be made visible in the structure of external-memory artifacts, including captured transcripts. Techniques of sequential analysis can be adapted to CSCL from conversation analysis, as systematized by Schegloff (2007), analyzing how utterances evoke and respond to each other in interactional processes of intersubjective meaning making, group cognition and collaborative knowledge building [Investigation 25].

Sfard's book on thinking as communicating was reviewed in *ijCSCL* by (**Stahl, 2008**) [Investigation 8]. Sfard emphasizes how mathematical cognition can be conceived of and analyzed as particular discourses. How children come to participate in these discourses and individualize the dominant social language of mathematics into their personal math thinking involves discursive social processes—not rote acquisition of memorized facts and procedures, but participation in co-construction of "realizations" (representational resources that index mathematical terms and figures). Sfard conceives this as participation in social "routines." Routines are meta-level rules that describe recurrent patterns of math discourse. Like Sfard's discussion itself, routines depict mathematical discourses rather than math objects. She describes in some detail three types of routines: deeds, explorations and rituals. Deeds are methods for making changes to objects, such as drawing and enumerating squares on a digital whiteboard. Explorations are routines that contribute to a theory, like a student's proposal. Rituals are group practices that maintain the flow of social activity, like questioning and taking turns.

Learning mathematics can be conceptualized as participation in a discourse in which people engage in the social construction of mathematical objects. In collaborative learning of math, groups of students adopt group practices that mirror social practices of the school-math tradition as they explore math problems, propose solutions and gradually employ technical terms. Through such participation, individual students can subsequently understand and personalize elements of the discourse.

Deep knowledge does not consist of memorizing discrete facts. There is not a single meaning of an equation or a theorem, but a network of interrelated potential realizations. To deeply understand the object, one must be conversant with multiple such realizations, be competent at working with them, be cognizant of their interrelationships and be able to recognize when they are applicable. This implies that evaluation of learning should not consist of testing individual memories, but of observing the application of these key practices.

Consider how students might learn the concept of perpendicular bisector and its construction in Euclidean geometry. **Öner (2016)** [Investigation 9] analyzes how a group of students enrich their collective understanding of this math object during a session in the VMT environment. She employs Sfard's commognitive framework to examine how the student team's word choice, use of visual mediators and adoption of geometric construction routines changed their character during an hour-long collaborative problem-solving session. Her findings indicate that the team gradually moved from a visually oriented discussion toward a more formal discourse—one that is primarily characterized by a routine of constructing geometric dependencies.

Öner's particular analytic focus is on the changes in: (a) the team's use of the word "perpendicular," (b) the visual mediators the team acted upon (i.e., the example perpendicular bisector in the workspace), and (c) their mathematical routines, since the shifts in these features were the most salient aspects of their changing discourse. Öner's study investigates two routines:

1. The production of the perpendicular: This routine was gradually altered from drawing by visual placement to construction by creating dynamic-geometry dependencies.
2. The verification of perpendicularity: This routine for substantiating whether a line is in fact perpendicular to another line shifted from visual judgment or

numerical measurements to use of theoretical geometry knowledge to justify proposed solutions.

Initially, the student team's notion of perpendicular referred to a visual image. It gradually evolved toward one that represented a mathematical relationship based on defining dependencies. These transformations of discourse and of construction practices took place within the context of group interaction, enacting task instructions and interacting with the VMT software. The team's shift to increasingly abstract thinking corresponds to a major development in human cognition—both in the evolution of the species and in the intellectual maturation of the group or its individual members.

The way that actions and conceptualizations shifted in starts and fits during the hour of interaction involving perpendiculars highlights the importance of temporality in learning. An article by **Reimann (2009)** [Investigation 10] addresses the need for temporal analysis in CSCL research more generally. For both the socio-cultural and the individual-cognitive views of learning, the nature of the learning process is temporal: learning unfolds over time. Because human learning is inherently cumulative, the sequence in which experiences are encountered affects how one learns and what one learns. This applies to the communication and interaction processes that take place in groups as much as in the silent reflections of individual learning.

Reimann contends that the quantitative, variable-centered method dominant in most experimental learning research makes restrictive assumptions on the kind of data useful for analysis and on the forms of causation allowed to explain change. Adapting a process-analysis approach focused on temporality and sequentiality provides an alternative, still rigorous method to analyze group processes. Temporal-event analysis can offer a methodological link between those researchers in CSCL who are producing descriptive, "thick," interpretive accounts of groups' computer-mediated interactions, and those who work experimentally and quantitatively. However, existing process models in CSCL, which predominantly describe short-term interactions, will need considerable theoretical extension to connect with theories of longer-term change.

An example of temporal analysis is provided by **Damsa (2014)** [Investigation 11] in her examination of productive interactions. For her, "productive interaction" refers to knowledge co-construction within the context of a knowledge domain, entailing joint actions directed toward mutual goals, increased intersubjective understanding of concepts, and actions that contribute de facto to the construction and progress of shared knowledge objects. The emergent epistemic (knowledge) objects are key to collaborative learning because they influence the course and productivity of interaction. The knowledge objects become both outcomes and mediating elements in the interactional process. Damsa's study finds that groups who manifest shared epistemic agency produce knowledge objects that are more complex and better suited to the problems addressed. More than technological artifacts, which are adopted as mediating instruments, a group's knowledge objects can remain problematic and open to transformation and further exploration by the group.

It is essential to define the nature of productive interactions:

a) How they are different from other types of interaction and how they lead to knowledge construction;

b) The temporality involved in the interaction; and

c) The multiple analytic layers that comprise this process—including interactions, knowledge objects, agency and their interconnections.

The unit of analysis is not the individual student's mind, but the joint action (verbal or otherwise) directed at the co-construction and elaboration of the knowledge objects involved—in other words, the mediated interaction. This leads us to the combination of the productive interaction, the objects that variously mediate this interaction and the agency of the group as a construct of multiple individual engagements and collective action. The way these are woven together is intimately related to the temporality of the longer collaborative-learning process and to how these components combine while unfolding in time.

One distinctive contribution of Damsa's empirical examination is its effort to follow, along with the unfolding interaction, the knowledge that emerges and gains shape through the interaction. This analysis focuses on the trajectory of the knowledge from the moment it enters the interaction process until it has materialized and is elaborated into the final objects produced by the groups. The productivity of the interaction manifests itself through the sequence of actions in the interaction that leads to the co-elaboration of knowledge objects. Organizing and attending to the sequential structure in which knowledge is not only generated and discussed but also taken up, elaborated upon and refined is of essential importance. (See Suthers' notion of "up-take" in Investigation 4.) Early versions of knowledge objects often play a catalyzing role in groups' extended interactions, influencing how interaction changes or adjusts with time, in order to become meaningful for the co-construction of shared knowledge objects.

Analysis of the temporal structure of interaction can take many forms. **Çakir, Zemel and Stahl (2009)** [Investigation 12] show how participants in a VMT session sequentially construct graphical animations of their shared mathematical representations in order to build intersubjective meaning. In order to collaborate effectively in group discourse on a topic like mathematical patterns, group participants must organize their activities in ways that share the significance of their utterances, inscriptions and behaviors. This case study investigates the moment-by-moment details of the interaction practices through which the students organize their chat utterances and whiteboard actions, highlighting the sequentiality of action and the implicit indexicality of the intersubjective meaning making. This is a nice example of the use of graphical inscription to take advantage of visual skills.

A student constructed the whiteboard diagram of the stack of blocks at the bottom of Figure 4 (left) by successively adding columns of blocks. The student first took the highest existing column and copied it to form an additional column, and then added an extra block at the top. The sequentiality of this construction process made the mathematical pattern clear to everyone in the group: that the number of blocks increases with each new column by one more that the amount it increased with the last column. This visual articulation of the structure to the pattern allowed the group to quickly derive its formula. Similarly, the array of hexagons in Figure 4 (right) is overlaid by one of the students with colored lines that first divide a composite hexagon like the outlined hexagon into six symmetrical sectors. Then the lines crossing one of these sectors are overlaid by lines of different colors. The sequentiality of drawing these lines makes visible and consequent the structure of short lines constituting a hexagon with sides of N units. Namely, there are 1+2+3+…+N units in each of the three directions within each of the six sectors. This visually observable structure leads the group directly to a mathematical expression for the number of unit triangles and lines for any size hexagon.

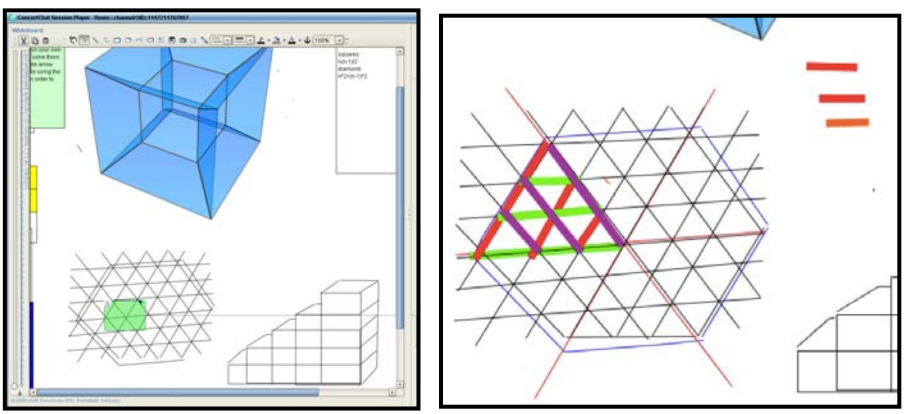

Figure 4. The VMT interface with a student construction of a hexagon array and of stacked blocks (left). Colored lines decompose a large hexagon into sequences of small triangles (right).

The sequential temporal analysis by Çakir, et al. treats the whiteboard as a kind of shared external memory or "joint problem space" (Teasley & Roschelle, 1993) where the group builds up, shares and preserves a record of agreed-upon facts, opinions, hypotheses or conclusions. Shared visible communication media like this can provide places where the group does its work—where it cognizes. Ideas, concepts, meanings and so forth can subsequently be taken up by individuals into their personal memories as referential resources for future social or mental interactions. There is no need to reduce group meaning to identical individual mental contents; the location of the group cognition and group memory is the visible discourse medium, with all its particular affordances and modes of access (e.g., graphical, representational, symbolic, spatial, highlighted, color-coded, labeled, etc.). Mental re-presentations in individual minds inaccessible to each other, when they exist, are derivative of the public presentations.

In another temporal analysis of collaborative learning in the same sequence of VMT sessions, **Medina and Suthers (2013)** identified a set of group practices that the student team adopted as representational resources. This included the use of colored lines to establish shared indexicality. The analysis was based on a detailed tracking of the development of several practices that individual students introduced to the team and that were gradually adopted as shared group practices. Interestingly, the tracking of this development was done retrospectively, by successively following the usage backwards through the group sessions (Medina, 2013). This analysis also demonstrated that the practices were collectively understood as group practices and effectively shared as personal practices (knowledge, skill) in the end because each student was ultimately able to initiate use of each practice within the group interaction.

In these papers, the problem of common ground—controversial in CSCL [Investigation 20]—is re-conceptualized from an issue of converging personal mental representations (e.g., in Clark & Brennan, 1991) to a practical matter for the group of being able to jointly relate semiotic objects to their indexed referents [Investigation 5]. The analyzed references do not reside in the minds of particular actors but have been crafted into the presentation of the chat postings and drawing inscriptions through the details of wording and sequential presentation. The references are present in the data as affordances for understanding by group participants as well as by researchers studying transcripts of the interaction. The meaning is there in the visual presentation of the communication objects and in the network of interrelated references, rather than in presumed mental re-presentations of them. The understanding of the references is a matter of normally tacit social practices, rather than of rationalist explicit deduction [Investigation 15].

The analysis of group practices in CSCL interaction data shows how the group discourse negotiates the adoption of practices (whether invented by the group or derived from personal or community practices). The group practice may be subsequently used without further explicit discussion. The use of this practice is then shared in common by the group and grounds the group's further activity. This reconceptualization of common ground overcomes the problematic tension in previous psychological versions of the concept as personal mental representations that somehow become shared by a group or community.

The practices of the group may be related to personal skills of the individual group members as well as to countless social practices established in the larger community or culture. For instance, the students in the VMT team brought in the mathematical practice of summing the sequence 1+2+3+…+N to Gauss' well-known summation expression (N+1)/2. On the other hand, the practice of overlaying colored lines on a whiteboard diagram had to be explained by one student to the others, who did not know how to select colors for lines in the interface. Both of these practices were adopted by the team, then understood and used repeatedly by all team members as "group practices" [Investigation 16] contributing to productive interaction.

The considerations about analysis and evaluation of CSCL interactions discussed in this section indicate how to address the theoretical views of collaborative learning presented in the previous section:

a) Analysis focuses on the group discourse and visible actions as contributing to intersubjective meaning making,

b) The temporal development of the group's use of tools, terminology and referential resources is followed closely, and

c) The team's adoption of group practices—which may indirectly contribute to group members' individual intellectual development—is tracked and documented.

It seems that CSCL research may be on the verge of fostering significant, urgently needed development of human cognition through the design of instruments of collaboration or external memory and by promoting the acquisition of associated group practices that exploit those tools in productive ways. Then the next question is how we can proceed to disseminate the early successes, innovative approaches and timely visions of CSCL. There is no point in waiting for some technical perfection of the field, for maturity and impact of CSCL will only come with experience meeting authentic needs in genuine educational circumstances.

Delivering the CSCL Vision

CSCL is advanced through pioneering forms of computer support, as well as theoretical and methodological innovation. Many CSCL research labs have focused on the development of new forms of computer support and/or the design of online environments to foster collaborative learning. This book is not the place to review such efforts, as important as they may be for transforming theoretical understanding of collaborative learning into practical efforts to promote and sustain knowledge building within student groups. However, three rather diverse examples of innovative pedagogical design and technological support are recommended to illustrate inventive ways to extend the CSCL paradigm. Although they could not be included in this already thick volume, they are available in back issues of *ijCSCL*:

1. **Schneider and Pea (2013)** explore the use of eye-tracking hardware in an online collaboration environment. The traces of where the participants' eyes are looking can be made available to the students themselves in real time, as well as to researchers retrospectively. When the students see where their partners are looking, they adjust their own visual attention. This can enhance joint attention. Just as we see in Investigation 12, joint attention can be required for productive collaboration. This component of shared understanding and intersubjectivity will be discussed at length in the theoretical and philosophical investigations of Part III. Access to eye-tracker traces for CSCL researchers can be useful for formulating objective measures of common ground.

2. **Chen, Scardamalia and Bereiter (2015)** provide a new feature within Knowledge Forum for classes to promote what the students identify as "promising ideas." This provides "group agency" support for the group to reflect upon directions to pursue in their own collaborative discourse. This feature appears to be a promising idea for extending this popular software, even for use by students as young as eight years old.

3. **Kapur and Kinzer (2009)** discovered one of the most intriguing results of CSCL experimental research. They determined that allowing collaborating student groups to struggle and even fail at tasks that are ill-structured and beyond their skills and abilities may be a productive exercise in failure. The "failed" groups may develop relevant group practices of conceptualization, decomposition, representation, inscription or problem solving that are useful in subsequent efforts, whether collaborative or individual. This discovery has implications

for sequencing the presentation of problems and challenges for collaborative work—an important but subtle part of CSCL curricular design. The efficacy to long-term group learning of temporary failure also problematizes the traditional emphasis on testing the short-term success of individuals.

As emphasized above, new technologies and curricular interventions need to be tested, investigated and developed in realistic settings. Designers need to see how groups of students use and enact the designed objects. Simple pre/post tests of learning effects are not generally adequate, although they may play useful roles within the larger research context. Sequential analysis is often necessary to see how interacting student teams make intersubjective meaning through the mediation of the designed artifacts and how they produce knowledge objects over time [Investigation 2]. Identifying the adoption of group practices may inform and even guide this analysis.

Design-based research (DBR) is widely recognized in CSCL as a productive approach to technology design. This provides a research structure for observing how student teams take up the intended affordances of innovative technology, pedagogy and curricular resources. In the DBR process, theory and analysis methodology co-evolve along with the design of the various components of the intervention. There is no corresponding accepted methodology for evaluating the performance of designs as they go through iterations of testing, evaluation and redesign. The methods for critiquing the design of the technology, pedagogy and curricular resources must be derived from the theory, which emerges from analysis of the student interaction during sequences of trials. This is where the focus on intersubjective meaning making, referential resources, instrumental genesis, epistemic objects, temporal sequentiality and adoption of group practices is needed.

Nevertheless, even once one has determined that a CSCL innovation has been adequately refined, there is still much to do to put it into widespread practice. One key to delivering the CSCL experience to students in a systematic way is the involvement of qualified teachers. As illustrated in the following, each of the major efforts so far to implement CSCL in schools has emphasized teacher preparation. Experience has shown that CSCL requires a classroom culture of collaboration. Establishing such a culture requires the leadership of experienced teachers, who know how to guide student discourse and encourage student agency without being invasive and interfering in the collaborative interactions themselves. It generally takes at least three years for even a motivated early adopter teacher to transition from leading a teacher-centric classroom to facilitating a collaborative-learning one.

The VMT project offered teacher-professional-development credits in teaching collaborative dynamic geometry through the Math Forum and masters level courses at Drexel and Rutgers-Newark Universities for in-service mathematics teachers. In these courses, teachers participated in the same VMT curriculum as their students would later use, although the teacher discussions included pedagogical issues as well as a more sophisticated mathematical discourse (Alqahtania & Powell, 2017).

In Singapore, Hong Kong, Canada, Finland and other countries in which CSCL has been systematically introduced into school systems, teacher training has always been the emphasis. Researchers worked with individual teachers over extended periods, and early-adopter teachers served as mentors for other teachers in their schools. The most commonly used CSCL technology in these countries has been Knowledge Forum. **(Bereiter & Scardamalia, in preparation)** comprehensively review all major aspects of this technology and pedagogy, including teacher preparation. The lead researchers in Singapore and Hong Kong have provided insightful reflections on their experiences as well, as summarized in the following review of two reports in *ijCSCL*.

In Singapore, the national government legislated transformation of schooling to meet twenty-first-century cognitive needs. They established an academic research lab to plan, spearhead and evaluate this effort. The lab recruited CSCL researchers from around the world as staff and collaborators. Some of the leaders at that lab reflected on their approach in *ijCSCL*. **Looi, So, Toh and Chen (2011)** [Investigation 13] note that research supported by individual grants to researchers has produced interesting ideas and small-scale proofs of concept. However, when one thinks about transforming school systems, one sees that the CSCL tools are fragmentary and scattered. Putting together a coherent classroom program requires a variety of work that has not yet been done for CSCL. This requires a serious commitment from all concerned.

In Singapore, the authors report, there exists a combination of strong, explicit top-down directives and bottom-up desire for transforming and improving the educational system. Looi, So, Toh and Chen argue for design-based research as the methodological framework for designing and enacting school-based research that can impact school practices, as well as for refining theoretical understandings on how beliefs about the premises of CSCL are shaped and changed in the course of research implementation. They discuss their research innovations from a systemic-change perspective that includes the micro, meso and macro levels of educational systems. Their paper reviews: policy imperatives governing Singapore's educational landscape as macro-level actions; socio-cultural factors of the school's learning ecology as meso-level considerations; and contextualized classroom-based interactions as micro-level factors.

The Singapore educational national plan (adopted in 2008) explicitly foregrounds a central role for technology-enabled learning: to develop students to be collaborative learners. Significantly, it also recognizes the need to address the curriculum and assessment conundrum in order for technology-enabled pedagogical practices to really take off in schools. This was addressed through four major phases of systemic-change processes for sustainability at the macro level: 1) creation of readiness, 2) phasing of changes, 3) institutionalization and 4) ongoing evolution and creative renewal of the policies.

The Singapore effort to bring CSCL to scale in a (relatively small, culturally homogeneous) national school system addressed the complex interrelationship among teachers, school culture, leadership and educational policies. Effectively scaling up encompassed four interrelated dimensions: depth, sustainability, spread and shift in reform ownership. Depth refers to consequential change in classroom practice, altering teachers' beliefs, norms of social interaction and pedagogical principles as enacted in the curriculum. Sustainability involves maintaining these consequential changes over substantial periods. Spread is based on the diffusion of the innovation to large numbers of classrooms and schools. Shift requires districts, schools and teachers to assume ownership of the innovation—deepening, sustaining and spreading its impact. Beyond these dimensions comes evolution, in which the innovation, as revised by its adapters, is influential in reshaping the thinking of its designers and creating a community of practice that continues the innovation process.

Design-based research was iterated in selected Singapore schools, as researchers engaged in design of technology and curriculum, worked with teachers to enact the design in classroom settings, researched the contextualized learning processes, developed or refined theories of collaborative learning, engaged in re-design, and continued the cycle of re-design and implementation. With the realization that both teachers and students initially lacked the expertise to facilitate collaborative learning, the researchers and teachers co-designed many classroom sessions using a relatively simple CSCL tool, Group Scribbles. This digital Post-It-Notes technology allowed students to compose, share and compare notes combining text and drawings.

A number of factors were key to eventual success:

- Routine use was emphasized in the classroom from the outset. In the first school worked with, the teachers were supported for a period of two years in the routine use of the technology in weekly lessons. The routine practices helped alleviate the novelty effect of experiencing a new technology and the associated innovative pedagogy.

- The technology was simple and easy to use. However, there was not a technology focus at the outset. Instead, enculturation opportunities were provided for the teachers and students to enact collaborative practices first, before using the technology.
- Face-to-face CSCL technology was used in class to mediate student-student and student-teacher conversations, increasing the bandwidth of communication.
- Design principles were adopted and refined to empower teachers to design collaborative activities. The objective was for the teachers to be ingrained with sound design principles for designing pedagogy, so that even without the use of CSCL technology, the teachers would incorporate notions of rapid collaborative idea-improvement in their teaching.
- New lessons tapped existing curriculum, and thus were integral to the learning of the curriculum.
- The lessons were co-designed by the teachers and researchers, providing ownership by the teachers of the lesson plans and resources. Toward the later part of the intervention, teachers were able to devise their own CSCL activities to share their experiences and lesson plans with teachers at other schools.
- There was extensive professional development for the teachers, especially to help them orchestrate collaborative-learning activities in the classroom.
- Going to scale involved systematic expansion, eventually leading to deeper pedagogical changes in teaching and learning practices.
- Maintaining on-going dialogues between researchers and teachers was important so that schools could ultimately benefit from the enduring and synergistic alignment of policy, practice and research.

The effort to adopt CSCL in Hong Kong had a somewhat different approach, but many parallel lessons. **Chan (2011)** [Investigation 14] reports on the establishment of classroom cultures and communities of practice among teachers in schools and systems. She draws on experiences in Hong Kong and examines research-based CSCL classroom innovations in the context of scaling up and sustaining a knowledge-building model in Hong Kong classrooms.

Classroom innovations involve complex and emergent changes occurring at different levels of the educational system. The experience of CSCL knowledge-building classroom innovations in Hong Kong schools included research, interventions and teacher support at three major levels: the macro-context of educational policies and educational reform, the meso-context of a knowledge-building teacher network, and the micro-context of knowledge-building design in classrooms. At the macro level, the Hong Kong case study begins with educational reforms and the policies of the Hong Kong government that provided a favorable context for CSCL classroom innovation. At the meso-level, its focus is on how a knowledge-building teacher network supported teachers' changing attitudes towards classroom innovation. The study also addresses the micro-level classroom design to illustrate how principles, pedagogy and technology are integrated, considering the socio-cultural context, for example, the strong emphasis on examinations in Hong Kong schools. Three interacting themes— (1) context and systemic change, (2) capacity and community building, and (3) innovation as inquiry—are proposed for examining collaboration and knowledge creation for classroom transformation.

The transition from micro-level case studies of isolated small groups using CSCL technology to *whole school systems* adopting the CSCL vision is challenging. Epistemological and cultural factors, such as student beliefs and the tradition of teachers working as individual (largely isolated and autonomous) professionals, are generally not congruent with research in learning sciences and CSCL. Organizational and school-level constraints make it very difficult for teachers to reflect collectively on their practices and engage in sustained expansive learning in CSCL environments. Furthermore, the current CSCL tools are limited and require surveying what is available; adapting it to the local conditions; setting up infrastructure; carrying out missing research; adopting long-term approaches to training and supporting teachers; and affecting a cultural change of public expectations, understanding and attitudes. These require massive funding for resources such as coordinated research, infrastructure, administrative support, training, teacher time for mentoring, textbook materials and public education.

Addressing these barriers and needs, various teacher communities emerged in Hong Kong, some spontaneously and some supported by the government and universities. Through technological advances and CSCL research, a new kind of structure—a teacher network—emerged as a type of meso-level bridge from government policy via *capacity building* to classroom implementation. The Knowledge-Building Teacher Network—organized and supported by Chan's research group with national funding and commitment—initially focused on helping teachers to reflect on their pedagogical beliefs or practices, and to contrast them with the knowledge-building model of collaborative learning associated with Knowledge Forum software.

The teacher-network community played a central role in supporting change in Hong Kong schools. Research revealed that teachers go through different phases in adopting technology and that communities of practice are useful for scaffolding and connecting technology use with principle-based understanding. One approach is to engage teachers in using technology in ways that are aligned with principles, pedagogy and assessment, thus affording them deeper insights. Teachers in the network were encouraged to contribute their reflections to community discussion-boards, to help them experience how technological affordances connect with pedagogy. Tool development for the assessment of knowledge building is not just for research analysis; the tools can be placed in the hands of teachers and students so that they might take agency to reflect on their work.

The key lesson is that researchers do not just ask schools and teachers to adopt pedagogy developed in other classrooms; they work together with teachers to create new usable knowledge, to *innovate* themselves. Co-inquiry and knowledge creation—not the imposition of ready-made innovation—is a central theme in designing and facilitating collaboration in professional communities. Within the teacher network, researchers and early adopters (often supported with paid leave from their classrooms to work with other teachers) collaborated with teachers new to the network.

One can consider such group teacher professional development as knowledge creation, with teachers working collectively to build shared knowledge. More broadly, a teacher network may provide a meso-level structure that coordinates and regulates macro-level political, institutional and cultural influences on micro-level classroom processes and student change. The Hong Kong researchers adapted CSCL discussion technologies to support the teacher network throughout Hong Kong. They also employed CSCL methods of discourse analysis to analyze the meso-level interactions among teachers reflecting on their classroom experiences.

Singapore and Hong Kong are both special cases of national school systems under pressure to prepare a workforce for leadership in a technologically sophisticated global economy. It is striking that the results of the first PISA study of collaborative problem solving (OECD, 2017) ranked Singapore number one and Hong Kong number three out of 51 countries tested in 2015, a couple years after the interventions reviewed here. PISA uses a very different methodology than what has been discussed here. However, the case studies by the Singapore and Hong Kong researchers demonstrate that propagation of CSCL approaches is possible in mainstream classrooms. Furthermore, their thoughtful reflections on the efforts in these countries provide multiple important lessons and recommendations.

Propagating the CSCL Vision

In this Investigation, we have considered a vision of collaborative learning, illustrated by the VMT prototypical research effort. The scope of CSCL was then extended in response to contemporary theory and current social issues, clarifying the distinctiveness and priority of intersubjective meaning making, instrumental genesis, epistemic objects and other theoretical and analytic constructs. These conceptualizations suggested approaches to evaluation of CSCL interventions in terms of sequential analysis of discourse and adoption of group practices mediated by appropriated artifacts—filling a need for a methodology appropriate to CSCL theory. Examples of efforts in Singapore and Hong Kong to bring the CSCL vision to scale in educational practice were then reviewed. Now we need to consider how to realize this vision of CSCL more generally.

We begin by considering how our prototypical example of CSCL could be scaled up for routine use in schools around the world. After 15 years of grants and collaboration with many international researchers (see Stahl, 2009), VMT had been developed to the point at which it establishes a proof of concept for the VMT vision and the associated theory of group cognition (Stahl, 2006), applied to collaborative dynamic geometry. The software is robust enough for classroom usage—in both desktop and mobile versions. A core concept of the domain has been identified: dependencies in geometric constructions. Corresponding to this concept, curriculum for introducing dynamic geometry has been developed through numerous iterations and has been used in trials with researchers, math teachers and students in and outside of school (Stahl, 2013). Teacher professional development has also been offered, using the same curriculum, supplemented with resources for teaching using collaborative learning.

It seems clear that the VMT prototype could be scaled up. Dynamic-geometry software like SketchPad and GeoGebra are already used in many math classrooms worldwide, although without support for online collaboration or a systematic curriculum. The VMT Project ported the free, open-source GeoGebra software to the VMT multi-user collaboration platform. The developers of GeoGebra would be willing to adopt and support this kind of multi-user version if they saw a broad demand for it. Their software is already used in 190 countries and translated into 52 languages. The Math Forum subsequently became part of the National Council of Teachers of Mathematics, an ideal dissemination center in the US. Thus, the technical infrastructure and access to individual teachers seems to be at hand.

The VMT curriculum was largely based on Euclid's original, orderly presentation of geometry and on the US Common Core geometry curriculum. It could now be further elaborated to tie in to major textbooks so that online collaborative sessions could be held in conjunction with traditional lectures, textbooks, YouTube videos and homework assignments. Teachers could orchestrate the collaborative learning to serve different functions within math courses: exploration, challenge problems or roles that are more central. Teacher guides could be prepared, directing teachers how to modify, excerpt, extend or adapt the session presentations to their classroom contexts. The curriculum could also be developed for use in different cultures or countries, translating the approach as well as the language. Finally, additional curriculum could be written for other math topics—GeoGebra is designed for all middle-school, high-school and early college mathematics, not just geometry. Once students, teachers and schools have positive experiences with collaborative learning or with a given CSCL technology in one course, they are much more prepared and motivated to use it in other areas.

Similarly, other proven CSCL interventions—bearing family resemblance to the VMT prototypical example—could be scaled up to global adoption. One could, for instance, identify a core underlying concept of a selected domain to target or specify certain social practices that would be important for groups to adopt to facilitate their knowledge building in that domain. For example, just as (Stahl, 2013) identified dependency as fundamental for geometric thinking, (Roschelle, 1992) identified acceleration as fundamental for physics, and analyzed student discourse for signs of adoption of group practices associated with this concept. Then, carefully sequenced and articulated topics could be presented for collaborative exploration, with guidance to stimulate productive interaction and knowledge-building discourse.

This could be coordinated with related course materials and instructional approaches, and accompanied by support for teachers to adapt and orchestrate the various resources. Researchers would need to collaborate with teachers over extended periods, as adoption of the CSCL intervention spread gradually and systematically through school systems. Given new educational networking platforms like MOOCs, collaborative curricula could be made available to students globally to learn together. This could both establish personal international cooperation among students and share curricular resources among developed and developing nations.

Such an envisioned scaling up of CSCL would require significant long-term commitment from government agencies to finance the research, dissemination, training, evaluation and support—as began to take place in Singapore and Hong Kong. CSCL research labs involved in such efforts would need to pool expertise in domain knowledge, learning theory, educational practice, teacher training, discourse analysis, software design, research expertise, grant management and other skills. Ideally, this would involve global networks of researchers. The Kaleidoscope funding during the late 1990s in the European Community might be considered the golden age of CSCL, where networks of researchers across Europe collaborated, resulting in some of the research reviewed above. Now a broader worldwide initiative is required, eventually including an emphasis on dissemination in school systems.

Advancing the CSCL vision is feasible today. CSCL theory can be refined and integrated to provide a unified conceptual framework for understanding collaborative learning as distinctive and as foundational for all learning. CSCL methodology can incorporate the sequential analysis of adoption of group practices. CSCL curriculum can be extended to meet worldwide needs. CSCL can play a driving role in evolving humanity to meet the challenges of the 21st century.

Significant progress in CSCL, especially including propagation to regular classrooms, is not a task for an individual researcher or even a single lab. It requires too many advanced professional capabilities and too great a long-term commitment. The CSCL community cannot manage this on its own. However, if the CSCL field is not centrally involved in setting the agenda and designing the direction, then the power of the CSCL vision to advance human cognition is unlikely to reach fruition. If the vision of CSCL can be maintained and exert a broad impact, then the discourse of humanity might be able to evolve a more complex understanding of phenomena like ecological sustainability, world peace, economic equity, informed political involvement. That would profoundly advance the CSCL vision and benefit the world.

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Investigation 2. A Theory of Group Cognition in CSCL

Gerry Stahl

Abstract. The digital age of computer support has transformed human cognition. Although thinking always had social origins in the small-group interaction of family units, tribes, work teams and friendships, cognition is now enmeshed in networks of social media, technological infrastructure, online knowledge sources, global production. Computer-supported collaborative learning (CSCL) stands at the crossroads of this historic transformation. CSCL research provides a laboratory for studying the nature of collective intelligence or group cognition. It explores how collaborative learning by small groups can become a foundational form of knowledge building—including for the individual group members and for the society in which the groups live. This introductory Investigation presents a paradigmatic CSCL setting and highlights the role of group practices as vehicles for collaborative learning. It addresses the dual questions of how intersubjectivity is possible and what the preconditions are for establishing, supporting and maintaining intersubjectivity—providing central pillars of a theory of group cognition and suggesting implications for educational practice. It then delves into the structure of collaborative discourse, analyzing data from exemplary CSCL sessions. The analysis of group interaction points to a multilayered structure, in which individual, small-group and cultural cognition are intertwined.

Keywords. CSCL theory, group practice, paradigm example, social practices, co-experienced world, intersubjectivity, discourse structure, multi-layered analysis.

A Theory of Extended Cognition

The notion of group cognition proposes that human thinking and learning is at root interactional; the origin, influence and effect of human cognition extend essentially beyond the skull. We acquire our ability to think and to learn by adopting practices that arise within small-group interactions, such as in our family, work teams or collegial circles. Our thinking is responsive to and conditioned by our embeddedness in a physical, interpersonal and cultural environment—particularly the immediate discourse or action context.

Group cognition theory poses an alternative to psychological theories of mental phenomena in individual minds as well as to sociological theories of societal structures existing independently of the people who inhabit those structures. According to group cognition theory, thinking and learning take place in the interactions among people and across the small groups of interacting individuals.

The theory of group cognition emerged from study of student discourse in CSCL settings. It is aligned with the writings of Vygotsky, Lave, Bereiter, Koschmann, Engeström and Hutchins as well as with socio-cultural, distributed-cognition and embodied-cognition approaches generally. However, it maintains a systematic focus on the small-group unit of analysis, which others often lose to a psychological or sociological priority for the individual or society. It is also in keeping with 21st century post-cognitive philosophy, such as that of Marx, Heidegger and Wittgenstein—who critique mentalism and individualism.

Theoretical Investigations of Group Cognition

The theory emerged in the writing of *Group Cognition* (Stahl, 2006, MIT Press). Aspects of the VMT research project and technology were developed and described in *Studying Virtual Math Teams* (Stahl, 2009, Springer). Various perspectives on the research were extended and explicated in *Translating Euclid* (Stahl, 2013, Morgan & Claypool). A detailed longitudinal study of a team of students engaged in successful collaborative learning of dynamic geometry was analyzed and presented in *Constructing Dynamic Triangles Together* (Stahl, 2016, Cambridge). The theory of group cognition has strong implications for the methodology of the learning sciences and for educational practice, as well as for CSCL technology design and design-based educational research.

The implications of these studies of the VMT Project (2002-2016) for the theory of group cognition are taken up in the Investigations of Part III of this volume. Investigation 2 presents an introduction and overview of the following essays, which represent my most important recent theoretical reflections on group cognition:

- Investigation 15. *A Paradigmatic Unit of Analysis*. A specific example of CSCL research is presented as a useful prototype for thinking about the field of CSCL: The Virtual Math Teams (VMT) Project was designed to develop a technology platform and pedagogy for sustaining collaborative learning within small groups of students discussing mathematics and solving problems together. The VMT online environment was instrumented to collect data of student interactions. Using this example, it is argued that CSCL can offer a distinctive and timely new vision of educational research, focused on the small-group unit of analysis.

- Investigation 16. *Group Practices*. Using multiple examples from VMT sessions, it is suggested that the adoption of shared practices by student teams is central to the collaborative learning that takes place in these groups. Group practices may or may not be derived from or related to either individual or cultural practices (such as rules from school mathematics), but they are adopted by the group in its collaborative work. Effective curriculum can be designed to encourage adoption of strategic group practices that contribute to skilled behavior in the contemporary world. Collaborative learning can be defined, designed, supported, fostered and evaluated in terms of the adoption of specific relevant sets of group practices.

- Investigation 17. *Co-experiencing a Shared World*. CSCL raises the question of how multiple individuals can “share” practices, learning or thinking as proposed by the concept of group cognition. In this Investigation, examples of discourse data from several VMT sessions show how the members of an effective group participate together within a shared world. This experience of interacting within a co-experienced world provides a basis for their shared understanding. There are many ways that group members negotiate and sustain joint attention to objects, experience them together, negotiate their shared understanding and repair potential misunderstandings.

- Investigation 18. *From Intersubjectivity to Group Cognition*. The question of how people can share understandings and understand each other is a philosophical issue. It has been discussed by a series of philosophers and social scientists. This Investigation tracks an evolving analysis of this discussion through about a dozen stages, culminating in the theory of group cognition.

- Investigation 19. *The Constitution of Group Cognition*. The analysis of three VMT examples of interaction shows typical mechanisms used to achieve intersubjectivity. In particular, groups engage in extended sequences of dialogical responses to each other, building longer argumentation structures, such as informal derivations of mathematical conclusions. They remain involved in persistent co-attention to shared objects of interest. By co-experiencing these micro-worlds, they establish and maintain shared understanding.

- Investigation 20. *Theories of Shared Understanding*. The usual view on how shared understanding among multiple minds is possible involves the notion of "common ground." This Investigation considers several prevalent, competing theories, including that of common ground. They are subjected to analysis in terms of evidence of how small online groups of students develop, check and maintain shared understanding, thereby constituting group cognition.

- Investigation 21. *Academically Productive Interaction*. The recent pedagogical theory of "academically productive discourse" or "accountable talk" is primarily oriented toward individual cognition. Accordingly, it adopts the approach of cognitive convergence, guiding individual students to converge their own understandings with the understandings of other students, the teacher or the community. In the alternative paradigm of group cognition, one tries to guide groups of students to maintain and build on their co-presence and intersubjective shared understanding to articulate their largely tacit shared group understanding.

- Investigation 22. *Supporting Group Cognition with a Cognitive Tool*. CSCL is motivated by the potential to design technologies to support collaborative learning. In this Investigation, the use of a pointing tool in the VMT environment is explored. The tool permits a student to point with a graphical connecting line from a current chat-text posting to a previous post or to an object or area on the shared whiteboard. This supports deixis, the ability to direct the attention of others to an object of interest. Pointing is a ubiquitous means for supporting joint attention; this tool provides an effective digital analog of physical pointing.

- Investigation 23. *Sustaining Interaction in a CSCL Environment*. Interaction in groups takes place through sequences of actions, such as text-chat postings, spoken utterances, drawing movements or bodily gestures. These can often be analyzed in terms of pairs of actions, such as the posing of a question followed by the offering of an answer. Here, the question elicits an answer, setting the stage on which a respondent is encouraged to provide an answer. In turn, the answer confirms by its responsiveness that the preceeding action was taken as a question and completes the meaningful question/answer interaction. While a question encourages interaction to continue with an answer, the answer could end the interaction. To sustain interaction, such response pairs must themselves be combined in larger structures.

- Investigation 24. *Viewing Learning and Thinking in Groups*. This Investigation proposes a systematic approach to revealing larger structures in CSCL settings. It provides a view of group interactions such as those analyzed in the VMT Project as hierarchically structured, with events (like VMT Spring Fests), composed of sessions, covering multiple themes, built out of sequences of discourse moves, consisting of adjacency pairs, linking utterances, including references. This hierarchy provides a framework for analyzing student interaction with a view toward structurally understanding group cognition.
- Investigation 25. *Structuring Problem Solving*. An extended interaction in VMT is here analyzed in some detail to show how a sequence of discourse moves is built up out of adjacency pairs, eventually carrying out a mathematical derivation by a small group. It is common to consider mathematical derivation the work of an individual thinker; however, here we see a group construct a result that no one of the individual students involved would have been able to do. The analysis of the derivation must be conducted at the unit of analysis of the group interaction.

This set of studies raises several central problematics of a theory of group cognition. These are particularly germane to CSCL, which focuses on small groups of students communicating over networked computers. The theory of group cognition claims that "groups can think." This is a new idea, reflecting that our era of digital technology has changed the nature of knowing, understanding, thinking. Now there can be various collective levels of interacting groups, networks or communities who interact across computer-based media. For researchers, the collective basis of cognition raises many issues, necessitating a rethinking of how to generate, collect and analyze data for studying collaborative learning and group cognition. The following aspects of these issues will be discussed in the remainder of this Investigation:

a. *The nature of intersubjectivity*, including the conditions necessary to establish and maintain it. Investigations 15, 16 and 17 take different approaches to conceptualizing the collective group and understanding the ways intersubjectivity can be established within the groups. See "Conceptualizing the Intersubjective Group" below.

b. *A methodology for studying and understanding intersubjectivity*. Investigations 18 and 19 trace theories of intersubjectivity and common ground. A key to the analysis of intersubjectivity is derived from ethnomethodologically informed conversation analysis, which developed the analysis of adjacency pairs in discourse. The theory of group cognition draws upon this for its analysis of the response structure of group interaction. See "Ethnomethodologically Informed" below.

c. *The relationship of group cognition to artifacts*, including tools to support collaboration. Investigations 20, 21 and 22 consider ways to foster collaborative learning through pedagogical and technological systems of support. This is critical for effective CSCL learning, which is necessarily mediated by technological artifacts (e.g., communication media and/or subject-domain representations). We need to consider how such mediation takes place. See "Artifacts and Collective Minds" below.

d. *The interrelationships among multiple levels of description*, such as individual, small group and community. Investigations 23, 24 and 25 are concerned with the structure of interaction at the small-group unit of analysis. It is important to consider this within the larger context of the relation of the small-group level to the individual and social layers of thinking and learning. See "Traversing Planes of Learning" below.

These thematic areas are discussed here based on editorial introductions to issues of *ijCSCL* that emphasized these areas (Stahl 2012a; 2012b; 2012c; 2013; 2015), and some of the papers published in those journal issues.

These editorials and papers are reviewed here to motivate reading the Investigations of Part III, which delve into these themes in greater depth and build on them as foundations of the theory of CSCL. They also serve to point to relevant discussions published in *ijCSCL* that were not able to be reprinted in full in this volume. They indicate the wealth of theoretical considerations in the back issues of the journal that could contribute to elaborating the vision of CSCL sketched in Investigation 1 and the theory outlined in this Investigation.

This set of themes defines central questions for a theory of CSCL, conceived as an attempt to understand and support learning at the small-group unit of analysis. They provide an understanding of intersubjectivity and of the group level of description in relation to non-human artifacts, individual subjects and encompassing communities or cultures. This Investigation thereby clarifies the conceptual background for the theory of group cognition in the Investigations of Part III.

Conceptualizing the Intersubjective Group

Intersubjectivity may be considered the defining characteristic of CSCL [Investigation 15] because it is what makes collaboration possible. *Intersubjectivity* is a concept that indicates *shared understanding* among people [Investigation 18]. This "sharing" is not a matter of individuals having similar understandings, but of them participating productively in a joint meaning-making discourse within a communal world.

Collaborative learning cannot take place if group participants do not have a shared understanding of what they are talking about or working on. On the other hand, "cooperative learning" can take place because this approach that was popular prior to CSCL involved individuals dividing up learning goals into small tasks that group members could accomplish individually, without extensive shared understanding and without engaging in processes of intersubjective meaning making. Many groups and team members opt for cooperation rather than collaboration because it is easier in that it does not require the establishment and maintenance of extensive intersubjectivity. However, it also lacks the power of collaboration to build knowledge across individual understandings. In cooperation, thinking takes place primarily in individual minds and the conclusions may then be collected by the group. In such cases, the meaning of the conclusions is relative to the individual understandings of the group members and may or may not be similar. Cooperation typically lacks the thinking together that involves intersubjective meaning making and results in shared knowledge resulting from joint activity.

Many experimental interventions and published analyses of learning involving groups of students lack the focus on intersubjectivity, but still use the term CSCL. They may in fact be studying cooperative learning or even individual learning. In this volume, CSCL refers to learning that takes place at the group level of interaction and intersubjectivity. Unfortunately, such learning is relatively rare, even in the field of CSCL.

A group has achieved intersubjectivity if the members of the group interact well enough to pursue the group's aims together. Intersubjectivity must be built up gradually through interaction and repaired frequently. CSCL research should explore the conditions and processes that are conducive to the establishment and maintenance of intersubjectivity among groups of learners. CSCL pedagogies should be structured to promote the intersubjectively shared understanding that makes collaborative learning possible. CSCL technologies should be designed to support intersubjectivity by providing media of communication and scaffolds for meaning making within specific domains of learning.

When CSCL theories discuss "groups," they are not referring to arbitrary gatherings of multiple learners, but to functional groups that have achieved a degree of intersubjectivity. The concept of collaborative learning in CSCL does not refer to a sum of individual learning that takes place among a group's members, but to the increase in inter-subjective understanding or collaborative knowledge building within the group that results from joint meaning making in a shared context. It involves the understanding expressed in the group discourse and the knowledge encapsulated in group products, such as texts or artifacts produced by the group. The group's understanding may differ from what any individual member might say, write or think when not interacting within the group.

This focus on the intersubjective group differentiates CSCL from other approaches to the study of human learning and educational instruction. It implies a research paradigm that prioritizes the group unit of analysis and studies groups that have achieved intersubjectivity. Analyzing an utterance (or chat posting) as part of a group interaction involves seeing how its meaning is constructed sequentially through its response to previous actions and elicitation of future behavior by other group members. The meaning of the utterance is inherent in the working of that utterance within the shared world of the group, not to be explained in terms of some purported individual mental thoughts accompanying the utterance. As in Ryle's (1968) thick description of a wink, the meaning of an utterance (or wink) is expressed by the utterance (wink) itself as an interactional action, not by assumed additional mental intentions of the speaker (winker).

Despite the centrality of the notion of intersubjectivity to CSCL, this concept has not often been explicitly discussed in the CSCL literature. Newcomers to CSCL therefore have difficulty determining the boundaries of the field. They may assume that CSCL is the same as traditional educational psychology or instructional design, except that it involves small groups and online technology. However, the importance of analyzing intersubjectivity at the group unit of analysis has become increasingly clear to many established CSCL practitioners. For instance, the *ijCSCL* Mission Statement now specifies that the journal "features empirically grounded studies and descriptive analyses of interaction in groups, which investigate the emergence, development and use of practices, processes and mechanisms of collaborative learning." The central research questions are no longer what experimental conditions produce the most valued learning experiences or outcomes at the individual unit, but how intersubjective meaning making and understanding is established, maintained and increased within the *interaction in groups*, by social practices, small-group processes and interactional mechanisms analyzed at the group unit.

The shift of research from assessing individual student outcomes to analyzing group-level phenomena has been slow coming and is still difficult to implement consistently. In the cooperative learning of the late 1900s, educational researchers like Johnson and Johnson (1999) or Slavin (1980) explored the effects of group interaction on learning outcomes of individual students. The focus was on individual cognition, but in cases where the individual was somehow influenced by being in a group. With the advent of CSCL, interest changed to the group processes that could be supported with networked-computer technologies. In their report on the evolution of research on the new approach of *collaborative* learning, Dillenbourg, Baker, Blaye and O'Malley (1996) noted that new methods were now necessary to study group phenomena. Although Koschmann (1996) proposed that this involved a paradigm shift, it has not been widely recognized what a radical change in perspective and methodology this shift to the group level implied.

Subsequently, Koschmann (2002) defined CSCL in terms of "joint meaning making." The centrality of intersubjective meaning making to the concerns of CSCL as a research field have been stressed programmatically in scattered proposals and examples, for instance in Investigations 4, 5, 12. Multiple attempts to define new methods corresponding to this agenda of group-level analysis were also proposed, as in Investigations 6, 7, 8, 9, 10 and 11 as well as several other *ijCSCL* articles.

After 20 years, CSCL researchers are just beginning to work out group-level conceptualizations, such as group cognition, group knowledge construction, group agency, group engagement, group metacognition, group practices and so on. Some researchers now see CSCL as pursuing a post-cognitive paradigm distinguished from the cognitivism of traditional learning sciences based on cognitive psychology [Investigation 15].

Co-Operative Action

Intersubjectivity goes by many names. Goodwin (2013; 2018) has recently developed an analysis of the essential intersubjectivity of human cognition within a post-cognitive perspective, grounded in a number of ethnomethodologically informed analyses of interactional data, including family members conversing, children playing and arguing, anthropologists or chemists learning analytic skills, and videotaped evidence being presented in a courtroom. As diverse as these settings are, they all involve small groups of people in face-to-face interaction. In this volume, we will adopt much of Goodwin's perspective, but apply it within computer-mediated scenarios.

Like group-cognitive theory, Goodwin's post-cognitive approach is not focused on psychological states as making human cooperation possible, but rather on "public social practices that human beings pervasively use to construct in concert with each other the actions that make possible, and sustain, their activities and communities" (2018, p.7). In place of a model of the speaker that takes as its point of focus mental phenomena within the individual actor, Goodwin identifies how people constitute their participation in discourse through their ability to "engage in appropriate but differentiated ways in a field of interactively sustained action constituted through the public organization of language use" (2013, p.15). These ways correspond to what Investigation 16 calls group practices.

The accumulation of group practices is central to the organization and evolution of human culture, collective knowledge and social life (Investigations 7, 16); the adoption and use of these practices by small groups pervades social life. Goodman, citing the philosopher and semiotician Pierce, highlights in particular the central importance of diagrammatic reasoning in human thought, noting that geometry provides a perspicuous example. The historic role of Euclidean geometry as a training ground for cognitive practices which integrate visual, logical, gestural and representational practices was a motivation for the VMT Project's focus on collaborative dynamic geometry as a subject domain.

Goodman provides an analysis of co-operative action that can usefully be applied to the study of VMT interaction data. He chose the term "co-operative" because people typically perform specific operations in coordination with each other. For instance, a speaker will decompose materials provided by a previous speaker and then reuse them with transformations. Goodman takes this process of decomposition and reuse of resources to be a very general and unique characteristic of human cognition. Mankind innovates by analyzing (taking apart) ideas and tools accessible in the social world and synthesizing (recombining) them in transformed ways. In particular, our speech is generated by decomposing, and reusing with transformation the resources made available by the preceding speech of others. So, their language becomes ours, and ours is a form of theirs: "We inhabit each other's actions" (Goodman, 2018, p.1).

The operations of decomposition and transformation take place in multi-dimensional settings, resulting in what Goodman calls environmentally coupled gestures. These require for their understanding not only a gesturing hand, but also the environment being pointed at and co-occurring language that formulates what is to be seen there in a specific way. For instance, anthropologists train their students to see and discuss subtle shades of ground at an excavation by pointing to color charts and using technical terminology. Similarly, students in VMT learn to construct challenging geometric dependencies by highlighting or making salient specific graphical elements on the computer screen and chatting about them using geometric terminology. This kind of co-operative action or group cognition is a fundamental way in which groups accumulate group practices, group members become more skilled, and the community builds knowledge. As Goodman puts it,

> The accumulation and differentiation through time within local co-operative transformation zones of dense substrates create a multiplicity of settings for action. Each of these must be inhabited by competent members who have mastered the culturally specific practices required to perform the activities that animate the lifeworld of a particular community. Through the progressive development of, and apprenticeship within, diverse epistemic ecologies, communities invest their members with the resources required to understand each other in just the ways that make possible the accomplishment of ongoing, situated action. (2013, p.21)

Not only does this process make possible new professions and realms of knowledge, but it also recursively forms the basis of intersubjectivity, required for all mutual understanding.

Translating Goodman's view of co-operative action to the concrete situation of the VMT paradigmatic case of CSCL, we can observe student teams decomposing and transforming each other's contributions. Trausan-Matu (Trausan-Matu, Dascalu & Rebedea, 2014; Trausan-Matu & Rebedea, 2009) has analyzed VMT transcripts using Bakhtin's notion of polyphony, showing how students build on each other's word use to inhabit an inextricably interwoven shared world.

Perhaps even more striking would be an analysis of how a team working on geometry decomposes and transforms each other's construction efforts. Interaction in VMT includes graphical (geometric construction) actions as well as linguistic (chat postings). Although it would be tricky to present a detailed study of this concisely, the construction data is now available in the recordings of the Cereal Team in their Winter Fest 2013 interaction, especially Session 6 (Stahl, 2015). Here, the three students took turns extensively exploring how to construct points, lines, circles, triangles, squares and polygons with specific dependencies. There was lots of trial and error, but an adequate analysis would show that it was by no means random efforts. Each student closely observed the deadends that the others ran into. They decomposed the false starts by erasing the shared workspace and then reconstructing the effort with key transformations, which eventually led to success. The successes were immediately recognized by the whole group and adopted into the future work of the group and of its members. This resulted in a shared understanding of their intersubjective meaning making in the shared VMT world.

The Conditions of the Possibility of Intersubjectivity

Several articles in the 2015 *10*(3) issue of *ijCSCL* focused on intersubjectivity; they illustrate and further develop a group-level focus of CSCL research. For instance, the first article provides a discussion of Habermas' philosophy as it relates to CSCL issues and introduces to the CSCL audience the work of the contemporary author who has written the most on the concept of *intersubjectivity*. Then, three papers analyze the intersubjectivity of small groups of students in different ways. One looks at how *groups learn how to learn together* with support from specific CSCL tools. A second transforms the concept of engagement to the group unit of analysis as collaborative *group engagement*. The final one makes a parallel move for *formative feedback and metadiscourse*, applying them at the group level. Together, they offer stimulating glimpses of CSCL theory, technology, meta-learning and analysis focused on the group as agent.

In his introduction of Habermas' philosophy of communicative action to the CSCL community, Hammond (2015) translates from Habermas' application of this theory in the public sphere of traditional media to the online world of CSCL. For Hammond, Habermas is relevant because he brings a fresh, well-considered and critical perspective to the discussion of joint knowledge building. In particular, Habermas' writings provide a framework for judging the evidence we bring to the analysis of collaborative learning as well as for valuing the evidence that our student subjects provide in their argumentation. Habermas defines the conditions necessary for the establishment of intersubjectivity, such as the inherent assumption of an ideal speech situation underlying communicative action. What Kant's *Critique of Pure Reason* did for the individual mind, articulating the conditions of the possibility of human knowledge, Habermas translated to the group level, explicating fundamental discourse conditions necessary for intersubjective meaning making in social collectivities.

Consider a student chat, a discussion forum or a medium like Wikipedia. How should we judge the quality of the knowledge building that takes place there? Moreover, how should one judge the quality of researchers' analysis of that knowledge building? Habermas provides a standard for judgment that is grounded in the nature of human discourse. He argues that effective communication would be impossible without the underlying postulation of an ideal speech situation—even if this ideal is never in fact fully achievable (Habermas, 1981/1984). The act of communicating with the aim of establishing intersubjectivity, making shared meaning and building knowledge together assumes that there is no other force of persuasion at work than that of the better argument and no other motivation than the cooperative search for truth. Enlightened discourse is only possible under the assumption of this goal. Of course, there always are other forces and motivations present. But the character of the ideal speech situation that underlies collaborative dialog provides a basis for critiquing those systematically distorting forces. For instance, if knowledge building assumes that no one can impose his or her views through force rather than through supported reasoning, then appeals to authority or intimidation can be soundly censured.

Habermas' theory is, additionally, more complex and nuanced. A major contribution of his work was to distinguish realms with different criteria within the public sphere (Habermas, 1967/1971). There is, as Hammond puts it, the objective world (of nature and labor), the social world (of institutions and interaction) and the subjective world (of personal experience). Each has very different criteria of validity. The objective world follows the laws of physics and involves human mastery over nature through technical, goal-oriented, instrumental calculation; the social world, in contrast, involves normative rules reached through negotiation; while the subjective world is a matter of one's self-narrative.

Consider the research task of analyzing an online team of students collaborating on a geometry construction. Certainly, this involves comparing the team's work with mathematical knowledge developed in the objective world of mathematical relationships. However, it also involves tracking the development of the team's adoption and mastery of its own group practices of collaborating and of working on geometry in the team's intersubjective world. Furthermore, it may be possible to assess individual learning by team members as a personal-world spin-off of their teamwork. Each of these dimensions has quite different methodological criteria. Seeing how each is accomplished with the mediation of specific CSCL pedagogical approaches or CSCL technological tools can feed into design-based research for improving support for collaborative knowledge building.

Habermas' distinction between the objective, social and subjective realms gives him leverage for his critiques of modernism and other popular philosophies, extending the critical social theory of the Frankfurt School. As cited by Hammond, Habermas' concern with mutual recognition led him to criticize classical liberalism for reducing ethical liberty to a "possessive-individualist reading of subjective rights, misunderstood in instrumentalist terms." There are many analogous examples in the CSCL literature, where social phenomena are inappropriately reduced either to individual subjective criteria or to instrumental objective criteria. Hammond suggests that a focus on intersubjectivity could provide a corrective in such cases and open up new perspectives for design and research. It is important to distinguish different levels of analysis carefully and to apply the appropriate evaluative criteria or analytic methods to each.

Intersubjective Learning to Learn

As a foundation of all communication and cognition, intersubjectivity applies to education specifically. Teaching students to learn how to learn or to develop "thinking skills" has long been considered important—particularly in the information age, where knowledge evolves rapidly (e.g., Investigation 13; Wegerif, 2006). In their research report in the same *10*(3) *ijCSCL* issue, Schwarz, de Groot, Mavrikis and Dragon (2015) extend this goal to the group level with their construct of learning-to-learn-together. A core component of this approach is supporting groups of students to engage in argumentation as a form of intersubjective meaning making. Schwarz and colleagues situate computer support for argumentation in an innovative dual-interaction space.

The authors take an iterative design approach to developing a software environment, curricular tasks and teacher roles for supporting learning-to-learn-together. They hypothesize that mutual engagement, collective reflection and peer assessment may be three critical group processes to encourage and to investigate. To explore these, they design a prototype with two primary components: a construction space and an argumentation space. The construction space includes a selection of domain-specific modeling applications to support student inquiry in specific topics of mathematics or science. This provides a mutually visible "joint problem space" for collective reflection by the group on the progress of its inquiry. The software creates a shared world for mutual engagement, as opposed to individuals trying to solve a challenging problem on their own. As one group member performs an action in the space, the others assess that action in the argumentation space, either affirming it or questioning it. This prompts the students to build on each other's actions, producing a joint accomplishment.

In some dual-interaction systems, like VMT, a text-chat feature accompanies an online construction space. This provides the possibility of engaged discourse, group reflection and peer assessment when group members are not situated face-to-face. However, the described argumentation system goes beyond this with a sophisticated planning/reflection tool. Even if the students are sitting together around a shared computer, this tool prompts, guides and supports team efforts at planning steps for the group to take (collective agency) and it facilitates team reflection on the current state (collective responsibility).

While the software mainly displays advice and ideas from the teacher or from individual students, its persistent visibility and its manipulable structure allow it to influence group agency and meta-learning. The potential power of this approach seems to come from the integration of the support for argumentation and reflection by the group with the inquiry activity itself in the shared inquiry environment. As always in CSCL, success also depends on a culture of collaboration: appropriate motivations/rewards, careful training in collaboration and subtle mentoring. The emphasis of the pedagogy and the support throughout is on the group as meta-learner. Group learning here is a form of intersubjective meaning making, incorporating group agency and group responsibility.

Intersubjective Engagement

In the next presentation of the *10*(2) issue, Sinha, Rogat, Adams-Wiggins and Hmelo-Silver (2015) provide a multi-faceted conceptualization and operationalization of intersubjectivity based on aspects of what they term "group engagement." Using this approach, they provide a clear illustration of a team of students that does not form an intersubjective group contrasted by one that does. The construct of group engagement developed in this paper allows the authors to identify this contrast and to analyze it using both quantitative and qualitative methods. The quantitative approach includes statistical correlations based on ratings of several aspects of group engagement, measured in five-minute intervals. The qualitative approach involves thick descriptions of illustrative excerpts of group discourse. The descriptions relate the interactions within the groups to their work (or lack thereof) of meaning making in establishing the engagement of the group as a whole in its problem-solving task.

A major achievement of the paper is to shift the analysis of engagement—which is increasingly popular in CSCL—from the psychological individual to the intersubjective group unit of analysis. The authors are explicit about this. Their observational protocol is designed to situate engagement within the collaborative group, its joint problem and its shared situation. For instance, the dimension of social engagement reflects group cohesion, or evidence that the task is conceptualized as a team effort, rather than as an individual activity. The contrast of one group's use of the subject "we" versus the other's use of "I" reflects in the details of the discourse the distinction documented in the ratings—showing that the distinction is actually one made by the group.

The paper is an impressive response to the cited prior research on engagement. According to the literature review, earlier studies generally operationalized engagement as consisting of a single dimension, as a stable state and as a characteristic of the individual learner. In addition, the cited work decontextualized engagement from concomitant conceptual and disciplinary tasks. By contrast, this study proposes a differentiated, evolving, multi-faceted and group-based model of engagement and applies this model to explore an insightful example from actual classroom practice. The paper's mixed-methods analysis reflects a careful attention to the unit of analysis, operationalizing engagement at the group level. Thereby, it adds in a rich way to our conceptualization of intersubjective meaning making.

Intersubjective Metadiscourse

Like the preceding paper, the one by Resendes et al. (2015) also uses mixed methods, with both quantitative and qualitative analysis. While collecting data at both the individual and group units of analysis, its focus is also at the group unit. In fact, it goes a step further than the previous paper and most other CSCL reports by capturing the outcomes at the group level. Here, because the main data source is a Knowledge Forum database, the group product of shared notes responding to each other within the group is the most important object for examination in response to the primary research question. Thereby, the correlation of the experimental condition with resultant collaborative learning or knowledge building can be conducted at the group level.

The social-network analysis of the Knowledge Forum data shows the effect of experimental feedback tools on the group process and the degree of intersubjectivity established by each group. The paper's analysis strikingly indicates that in the control condition most students are not strongly connected to other students, whereas in the experimental condition everyone is strongly connected to everyone else. Because the social-network connections here represent sharing of vocabulary terms—such as those displayed in the experimental condition's feedback tool—this means that there is a higher degree of intersubjective, shared understanding in the experimental groups. Shared understanding at the group unit of analysis is not dependent upon individuals' cognitive states, internal representations or personal understandings, but is visibly displayed in the team's unproblematic use of shared language.

We are shown further evidence of increased group metadiscourse through the analysis of group discussion in a number of propitious interaction excerpts. While these demonstrate the experimental group's comprehension of the visualizations of their group discourse (displays of its use of domain vocabulary and of Knowledge Forum epistemic markers), the primary metadiscourse moves (prompting the group to plan, question, analyze, explain) were made by the teacher, rather than by the student group. The experimental intervention at the group level led to productive metadiscourse, but this was not at all independent of the teacher. Thus, the study merely indicates a potential for the design of formative assessment visualizations that represent group-level behaviors and that support group metadiscourse. It does not demonstrate that the implemented tools led to student metadiscourse on their own. The students may need more experience with this approach or more maturity to take on this form of agency within the student group. Nevertheless, the paper offers stimulating design suggestions: group-level formative feedback can represent group vocabulary; support the group to evaluate its own progress; give feedback on secondary processes (like vocabulary building, rather than directly on learning or task accomplishment); suggest positive steps (rather than just identify deficiencies); facilitate self-assessment by the group; and guide individual students to become more effective group members.

Together, the papers in issue *10*(3) of *ijCSCL* suggest the centrality of intersubjectivity to a theory of CSCL and provide inspiring examples of how to explore and articulate aspects of our conceptualization of group intersubjectivity.

Ethnomethodologically Informed

The research field of CSCL is ethnomethodologically informed, or at least ethnomethodologically influenced. This has not always been the case, although there is a logic to this growing tendency.

Ethnomethodology (EM) is an approach to conducting research in the human sciences founded by Harold Garfinkel and largely defined by his *Studies in Ethnomethodology* (Garfinkel, 1967; Garfinkel & Rawls, 2012). EM addresses the "methods" that members within a given linguistic community use to establish and maintain intersubjective understanding. Since CSCL can be characterized as being focused on joint meaning making, the analysis of prevalent meaning-making methods seems particularly relevant to the methodological quandaries of CSCL research.

Ethnomethodology has been slow to catch on in CSCL, in contrast to its role in allied fields like CSCW, where it seems to be a dominant research paradigm. There are a number of theoretical and historical reasons for this. For instance, as discussed below, practitioners of EM eschew research questions and theoretical framings because these could obscure the meaning-making perspective of the people whose interactions are under investigation. This injunction against guiding theory makes it difficult to integrate EM studies into the educational and design agendas of CSCL investigators. In addition, the case-study approach of EM to analyzing naturally occurring events is at odds with the traditional emphasis in educational and psychological research on controlled experiments and statistical generalizations. CSCW is based more in social sciences, in contrast to the psychology foundations of much CSCL.

On the other hand, there are strong arguments for viewing the ethnomethodological approach as especially appropriate for analyzing computer-supported collaborative learning. In particular, a major stream of research within EM has been conversation analysis. This is the analysis of talk-in-interaction, as pioneered by Sacks (1965/1995) and other colleagues of Garfinkel. An early finding of conversation analysis was the system of turn taking in face-to-face informal conversation. While this system does not apply directly to such CSCL interactions as online text chat about an academic topic (Zemel & Çakir, 2009), the underlying techniques of sequential analysis (systematized by Schegloff, 2007) seem highly applicable to the analysis of meaning making in CSCL settings. Such sequential analysis explicates the evidence embodied in instances of discourse that reveal meaning-making processes taking place in small groups [Investigation 25]. It looks at the semantic, syntactic and pragmatic details of how utterances respond to each other and elicit new responses in the flow of group cognition.

The Historical Traditions of CSCL Research

To a large extent, early CSCL investigators turned from inspirations in computer science and artificial intelligence to the fields of educational psychology and sociology to find methods of studying the effects of using CSCL systems in classrooms or in laboratories. The theories and research paradigms that they brought in from these established fields focused on either the individual student or the larger society as the unit of analysis. Educational theory operationalizes learning as a hidden change in mental state of student knowledge from before an intervention to after, as measured indirectly by pre- and post-tests of individual students. At the other extreme, social science approaches hypothesized societal forces that could not be observed directly but could be inferred and measured by controlled experiments using statistically significant numbers of randomly selected subjects.

Ethnomethodology—drawing on philosophical influences from phenomenology and reacting against functionalist approaches to sociology—takes a different tack, centered on what is made visible in the interactions between people. EM argues that one can observe the meaning-making processes at work by carefully studying the discourse between people; one does not have to make inferences about hidden changes in mental models or invisible social structures. Furthermore, EM studies can focus on the small-group unit of analysis, which seems most appropriate to analyzing collaborative learning. While other areas of education and of sociology may seem centrally concerned with individual or societal units of analysis and while collaborative learning may also involve processes and phenomena at those levels, the meaning making in contexts of joint activity which is definitive of CSCL takes place primarily at the small-group level, even if a complete understanding will need to tie all the levels together.

The ability to conduct microanalysis of interaction was historically made possible by recording technologies, which allowed utterances to be replayed and slowed down. Conversation analysis arose in the age of the tape recorder. That technology made it possible to hear exactly what was said and how it was articulated. It allowed the production of detailed transcripts, which represent intonation, pauses, emphasis, restarts and overlaps so that the mechanisms of verbal interaction could be studied. Subsequent development of video recording led to analysis of gesture, facial expression, gaze and bodily posture as important but generally unnoticed aspects of interpersonal interaction. For online communication typical of CSCL, computer logs and even the ability to replay synchronous interaction can provide adequate data sources necessary for the study of how students actually engage in computer-supported collaborative learning.

Applied to CSCL, the approach of EM implies that we can observe and report on the ability of given technologies and pedagogies to mediate collaborative interactions between students in concrete case studies. EM suggests ways to do this systematically, with intersubjective validity, and to generalize the findings. Insights from this can be used to critique the designs of interventions and to suggest redesign criteria. To make these claims about EM plausible, we will need to review some of the principles of EM.

The Theoretical Framing of CSCL Research

There is a prevailing notion that EM is atheoretical or even anti-theoretical, that it rejects all theorizing. Yet Garfinkel and Sacks (1970) were highly theoretical thinkers, influenced by philosophy, sociology and communication theory. In fact, EM represents a strong theoretical position about the nature of human reality and the possibilities of comprehending it. EM claims that human social behavior is structured by a large catalog of "member methods"—patterned ways of making intersubjective sense with other members of one's linguistic community. Furthermore, these member methods are "accountable" in the sense that they provide an observable account of their own character. People's actions are designed so that the meaning of the actions will be recognizable by others within the given discourse situation. This accountability is necessary for intersubjective understanding among members. But it has the secondary consequence that researchers can understand the methods as well (given certain conditions). The theory of EM thereby explains how EM is possible as a scientific enterprise.

The member methods of a linguistic community contribute significantly to the social order of activities within the community. The social structure is enacted in the very interactions of the members by virtue of their use of these methods; the accountability of the methods, as they are realized, reveals to the other participants (and potentially to researchers) evidence of what is being enacted. As Garfinkel put it, “any social setting [should] be viewed as self-organizing with respect to the intelligible character of its own appearances as either representations of or as evidences-of-a-social-order” (Garfinkel, 1967, p. 33). There is reflexivity at work between the meaning of an elemental interaction (e.g., an utterance/response pair) and the local context of the on-going discourse, in which the utterances are situated within a context whose significance they interpret in a continuously emergent way. The theory of EM is formulated in its concepts of member methods, accountability, reflexivity, etc.

The reason that EM is often considered to be atheoretical is that it systematically rejects the kind of theoretical framing that is associated with many other research approaches. For instance, in other paradigms an experiment and its analysis are motivated and structured by a theory or conceptualization of the phenomena to be studied. There may be a specific research question that the researchers have in mind. There may even be hypotheses about how the experiment will turn out based on preconceptions. While scientific researchers must remain open to their hypotheses being disproven by the evidence, the posing of research questions and hypotheses define a research perspective within which the evidence is pre-interpreted. For instance, CSCL discourse data might be coded according to a set of codes designed to make distinctions relevant to this perspective, experimental conditions will be structured to test these distinctions and coders will be trained to categorize their data from this perspective—all before the students even interact or produce their utterances.

EM, in explicit contrast, wants to understand the data from the perspective of the participants in the study (e.g., students). Because the analysis of discourse is a human science, it must take into account what the discourse means for the speakers and audience. The participants are viewed as people engaged in meaning making, and EM researchers want to understand the meaning that the participants are making. EM researchers do not want to impose a perspective on the data analysis that is based on their own preconceived theories about the interaction. Rather, they want to engage in “thick description” (Ryle, 1949) of the discourse to explicate the meaning making that is taking place in the discourse and that is displayed in the accountability of how it is formulated. The fact that the discourse is accountably intersubjectively understandable allows the researcher to analyze the meaning that is implicit in the discourse as it sequentially unfolds.

This is the sense in which EM rejects theory: that it adopts the participant perspective on understanding the meaning in the data, rather than imposing a perspective based on a theoretical research framing. There has been considerable debate within CSCW about how EM analysis can be used to guide design of collaboration systems if it cannot be directed toward theoretical issues (e.g., see

Crabtree, 2003). But the stricture against theory in EM is only against imposing an *a priori* analysis framework, not against drawing theoretical consequences from case studies. So, one can, for instance, study the discourse of students embedded in a computer-supported interaction, and analyze the nature of the methods they use—which they enact, adapt or create—for achieving their collaborative tasks. The details of these methods can have design implications, such as addressing technical barriers that resulted in unnecessarily cumbersome behaviors. Thus, EM can contribute to the analysis phase of design-based research (DBR Collective, 2003), which is a widespread approach in CSCL to the design of effective collaboration technologies.

The Ubiquity of Methods

Ethnomethodology posits the existence of member methods pervading all of social life. EM research for the past fifty years has documented many such methods, for instance in informal conversation, in doctor-patient discussion, in mathematical proof, in criminal interviewing and in workplace communication. These methods are often sedimented in the traditional design of the tools we use and in the clichéd turns of speech within our vernacular. They constitute our myriad overlapping cultures.

Sacks (1965/1995) argued that the pervasiveness of member methods meant that one could profitably study almost any interaction and learn from it about the nature of social existence. He argued that the universal application of these methods was necessary if people were to understand each other. In the CSCL literature, one often talks about the establishment and maintenance of "common ground" (Clark & Brennan, 1991) as providing the foundation for intersubjective understanding. But, according to EM, it is not a matter of the participants having corresponding mental models of propositional knowledge; rather, intersubjectivity is founded on co-experiencing a world through using shared methods of communication [Investigation 17]. These methods provide "resources" for engaging in specific domains of the social world. According to the EM viewpoint, collaborative learning does not consist in the storing of propositional knowledge as mental contents in individual minds, but in the increasing ability to enact relevant resources or shared practices in interactions with others.

By looking carefully at interactions in CSCL settings, we can analyze the methods being applied. Because the acceptance of these methods is widespread within a culture, the results of a single case study can have quite general ramifications. Of course, to accept the implications of a single case study—or even a small catalog of case studies analyzing variations on a method—as valid and of general applicability, we need to ensure lack of bias or idiosyncrasy. This is usually addressed in EM by "data sessions" and other mechanisms to involve multiple analysts (Jordan & Henderson, 1995). If discourse under analysis displays an account of itself, then a group of experienced analysts who share the relevant cultural understanding with the discourse participants should be able to reach a consensus about the meaning being created in the discourse. EM case-study publications frequently include very detailed transcripts of the relevant discourse excerpts to enable readers to confirm the analysis based on their own cultural understanding.

Group Practices

The identification of group practices—their adoption and use by groups—seems central to analyzing intersubjective meaning making and collaborative knowledge building in CSCL. Investigation 16 delineates a theory of group practices and proposes that CSCL methodology be centered on this.

Group practices are routinized behaviors that a group adopts and that ground intersubjectivity by providing shared understanding. They may mirror established social practices or member methods, such as procedures commonly used by experts in their work but as yet unknown to the students. The theory argues that the analysis of group practices can make visible the work of novices learning how to inquire in science, mathematics and other fields. These ubiquitous social practices are invisibly taken for granted by adults in their professional lives but can be observed as they are brought into usage, and rigorously studied in adequate traces of online collaborative learning.

The analysis of the enactment of group practices by teams in CSCL contexts can systematically inform the design, testing and refinement of collaborative-learning software, curriculum, pedagogy and theory. Applied to the evaluation of trials of CSCL innovations, the analysis of how student teams adopt or fail to adopt desirable group practices contrasts with traditional pre/post comparisons that miss sequential interactional processes or that reduce group phenomena to either individual or social factors. Investigation 16 concludes by proposing that CSCL can be re-conceptualized as the directed design of technology to foster the adoption of targeted group practices by student teams.

The theory of group practices emerged from a longitudinal case study of a team learning the basics of dynamic geometry in eight hour-long VMT sessions. This data provides the prototypical example for the vision of CSCL being offered in the present volume. The interdisciplinary VMT research team at the Math Forum conducted a year of weekly data sessions on this data, resulting in a book-length analysis of the collaborative learning that took place (Stahl, 2016). A daylong workshop on the data was also held involving international researchers, and findings were discussed during visits by the author to European research labs.

During eight hours of chat and manipulation of geometric representations, the group employed countless social practices, most of which were intuitive, tacit and non-problematic for the students. However, over sixty group practices were also identified in the analysis as practices that had to be explicitly negotiated and adopted through group interaction processes.

The catalog of these adopted group practices agrees well with lists of social practices enumerated in the research literature. For instance, it includes online analogues of group practices ("member methods") defined by face-to-face conversation analysis: sequential organization (response structure), turn taking, repair, opening and closing topics, indexicality, deixis, linguistic reference, and recipient design. Other group practices correspond to practices CSCL has previously investigated as providing foundations for intersubjectivity: joint problem spaces, shared understanding, persistent co-attention, representational practices, longer sequences and questioning. As observed in various VMT studies, practices in mathematics education include: mathematical discourse and technical terminology; pivotal moments in problem solving; and the integration of visual/graphical reasoning, numeric/symbolic expression, and deductive narrative. In addition, there were group practices that are necessary for constructing figures with specific dependencies in dynamic geometry.

It is likely that the VMT team picked up many group practices unproblematically, without having to go through an explicit negotiation process because the available resources—including the curriculum texts or classroom presentations before the online collaboration—guided smooth, tacit adoption of the practices. The curriculum, software environment and teacher guidance were based on careful study of what sorts of practices are involved in productive interaction related to collaborative dynamic geometry. This involved the researchers and the teachers developing personal experience with, for instance, constructing figures in Euclidean and dynamic geometry. They also read research reports about how students learn this domain. There are many physical practices involved in constructing different geometric elements on the computer screen and additional practices involved in dragging them to make sure they behave as desired. There are practices involving physical dexterity, computer manipulation, geometric relationships, communication, terminology, problem solving, explanation and so on. In a collaborative setting, these must often be shared as group practices.

The identification of group practices has substantial implications for the design and evaluation of CSCL software, curriculum, pedagogy and experimental intervention. According to the theory of instrumental genesis described in

Investigations 6 and 7, it is not sufficient for a CSCL designer to have good ideas and honorable intentions; one must develop an initial prototype environment and try it out with groups of students. Based on observation of problems, the prototype must then be iteratively re-designed and refined. By observing breakdowns in group interaction and the gradual enactment of new group practices in response to the breakdowns, a designer can identify problem areas and constructive processes that need additional support. The analysis of group practices provides a systematic analytic method for driving CSCL design.

The analysis of adoption of group practices can be conducted either informally or rigorously. For instance, in browsing through the just completed online interaction of student groups one day, I noticed that one group had accomplished something impressive in their geometrical construction. However, they had not had time to reflect on what they had done in terms of negotiating new group practices or engaging in discourse about the "dependencies" that they had established in their construction. I had designed the tasks with the goal of deepening the students' understanding of mathematical dependencies, so I wanted the students to spend more time interacting around their accomplishment. I emailed the teacher and suggested that she extend her groups' work on this task the next day. Because I knew that I had designed the intervention with the intention of facilitating the adoption of group practices of discourse and construction related to the concept of dependency, I was oriented to scanning for this when replaying the student sessions. Informal analysis could drive design, altering the sequencing of topics and changing the wording for the next iteration of the course.

By contrast, to develop a deep understanding of what the student team accomplished in that session and how they built their knowledge interactively, I had to go over the data many times, in slow motion, and analyze it with other researchers experienced with mathematics learning. Eventually, we developed a nuanced sense of the development of the team's group cognition. We saw how its shared understanding of mathematical concepts like dependency had developed significantly, but was still not robust. We catalogued the repertoire of group practices the team now shared, which provided it with an initial fluency in collaborative dynamic geometry, as intended by the design of the eight-session curriculum. We could then document the longitudinal development of mathematical cognition at the group level and observe the articulation of that newly acquired understanding by the team members' discourse. We could specify the vaguely characterized cognitive evolution from concrete visual to abstract conceptual thinking in terms of the accumulation of adopted group practices, which we could observe and document.

Analysis of how the Cereal Team developed their mathematical understanding as a group illustrates the working of intersubjective meaning making through the interaction among team members. However, it is also important to take into account the role of artifacts, such as geometry constructions and labeled diagrams in this computer-supported collaborative learning.

Artifacts and Collective Minds

The age of simple objects like well-designed artifacts, minds confined inside of skulls, and cultures cloistered in the tacit background has been left in the fading past according to current socio-cultural theory [Investigation 3]. We are now enmeshed in dialectical processes of social enactment, whereby designed objects continue to evolve well after they enter into the structuring of our thought patterns [Investigation 6].

Biological human evolution has long since transformed itself into cultural evolution, proceeding at an exponential pace [Investigation 7]. Along the way, thought overcame the limits of individual minds to expand with the power of discourses, inscriptions, digital memories, computational devices, technological infrastructures, computer-supported group cognition, and virtual communities [Investigation 8]. Both human cognition and its mediation by technological artifacts morph from fixed nouns into process verbs [Investigation 10], like "cognizing mediating" (Stahl, 2012a)—where human cognition and technological media shape each other in ways we are just beginning to conceptualize.

The owl of Minerva flies only at night, according to Hegel's (1807/1967) metaphor: theory—which is one's time grasped in concepts—lags behind the continuous unfolding of practice. As today's viral software successes rapidly outstrip our design theories, we must try to understand the ways in which new generations of users adopt and adapt their digital tools, thereby defining and redefining their conceptual, social and pragmatic ties to their worlds. Hegel theorized the dialectic between subject and object, proposing that the identity of the human subject is formed when a subject subjects an object to goal-oriented design (Stahl, 2006, p. 333f), creating an artifact within the effort to forge intersubjectivity and its spin-off, the individual's self.

Vygotsky (1930/1978) recognized the role of double stimulation in mediated cognizing: that the subject's access to an object is mediated by tools such as hammers, names and physical-symbolic inscriptions, so that in higher-order human cognizing we are stimulated by both an intentional object and a cognizing-mediating tool. It is this mediation of cognition by artifacts and via other people that opens the zone of proximal development, allowing the individual mind to first exceed and then later extend its limits. Engeström's (1987) concept of expansive learning added the cultural dimensions from Marx' social theory to Vygotsky's simple triangle of subject-artifact-object. Henceforth, socio-technical understandings of artifacts have to situate them culturally, historically, politically.

We have considered the labyrinthine nature of the artifact's affordances previously within theories of human-computer interaction (Hutchins, 1999; Norman, 1991), cognitive science (Gibson, 1979; Hutchins, 1996) and CSCL [Investigations 3, 4, 5 and 11]. For instance, based on Merleau-Ponty's (1945) philosophy, Bonderup Dohn (2009) argued that the affordances of an artifact were potentials realized in response to human behaviors.

The 2012 *7*(2) issue of *ijCSCL* focused on the role of artifacts in CSCL. The issue opens with Investigation 6, which explores the nature of artifacts by comparing the theory of affordances with the theories of structuration and of instrumental genesis. Structuration (Giddens, 1984; Orlikowski, 2008) is a well-known theory developed to account for the dialectic between social structures and the local interactions, which are both constrained by these structures and reproduce them. Instrumental genesis is a recent theory developed in France by Pierre Rabardel and his colleagues. Investigation 6 introduces the theory of instrumental genesis to the CSCL community and explores how the theory might impact work in CSCL, at methodological, technological, and theoretical levels.

Investigation 6 compares the three major recent theories about the interaction between artifacts and people, using a concrete case study of a typical CSCL setting. It argues in favor of the general approach of instrumental genesis as an analysis of the micro-genesis of artifacts and as the best available description of the nature of tools, particularly for CSCL. The theory of affordances tends to focus on the individual, for instance with Gibson's biological perspective, Norman's use of mental models, or Piaget's schemas in individual minds. In contrast, the sociological theory of structuration focuses on the societal or cultural level. The theory of instrumental genesis can more naturally be applied to the small-group collective level central to CSCL, as Investigation 6 does in discussing how triads of students enacted a feature of an argumentation-support software system.

Investigation 6 presents a "theoretically grounded" conception of the artifact-agent connection. A next step would be to explore an empirically grounded analysis of the connection. While Investigation 6 referred to data from a CSCL experiment, it simply used high-level descriptions of the data to illustrate aspects of the theories being described. It will be important to also analyze such data in detail to see if the connections of groups of students to computer-support systems follow the contours of one or more of the three theories, or whether they display different lines of development. Furthermore, it will be useful to consider more complex technologies, whole meso-level infrastructures [Investigation 3] rather than isolated functions. For instance, in an online course, small groups may have to negotiate the coordinated use of hundreds of functions in Blackboard, Google search, Wikipedia, Facebook, Google Docs, iChat, Gmail, Word and PowerPoint in order to produce a one-week assignment. Such an undertaking invokes the use of individual experience or expertise, established social practices in the school culture, consideration of course requirements and project goals, as well as collaborative discourse and trials by the small groups. The resultant computer-supported effort assembles and interprets a complex technical infrastructure, increases the expertise of the group participants, and provides a medium for group knowledge building. The connection of the collaborative group with the technical infrastructure continuously evolves through use during an academic term.

Having glimpsed the potential relevance of the theory of instrumental genesis to CSCL, issue *7*(2) of *ijCSCL* turns next to a discussion of that theory within the context of CSCL system design. Lonchamp (2012) argues for applying Rabardel's theory by expanding Engeström's (1987) Activity Theory triangle of mediations, to explicitly represent both the processes of mutual shaping of agent and artifact and the specific role of the teacher in CSCL classrooms: He pictures the various mediated interconnections among tool, designer, teacher, student, peer and tutor. Furthermore, he discusses how the agent-artifact connection—embodied in Rabardel's conception of the instrument—evolves over time through usage and re-design.

Lonchamp's paper concludes with a review of CSCL system-design approaches to supporting "instrumentalization" by teachers and students. Although it comes close to describing design-based research (Brown, 1992; DBR Collective, 2003), this review does not name it. DBR is a dominant approach within CSCL research to integrating system design, usage analysis, educational research and practical classroom interventions. It was developed in response to the need to conduct user-centered design of innovative educational software for collaborative groups—a realm lacking in detailed theories, specific analysis methods, adequate software or design guidelines. Perhaps an explicit combination of Rabardel's theory with data from DBR projects could provide empirically grounded insights into the mutual shaping of CSCL software and group cognition in on-going design and usage processes.

The third paper in *ijCSCL 7*(2) is Investigation 7. It situates Rabardel's theory within the context of knowledge-building practices, as these are conceptualized in recent work at the Scandinavian-led Knowledge Practices Laboratory (KP-Lab). This context is populated with social practices grounded in knowledge-building artifacts (Hakkarainen, 2009) and structured in space and time by chronotypes (Ligorio & Ritella, 2010). The knowledge-building artifacts are instruments in Rabardel's sense; they provide for advanced forms of Vygotskian double stimulation (Lund & Rasmussen, 2008). The whole context is the result of the cultural evolution (Donald, 1991; 2001) that led up to our involvement with digital information and communication technologies in an increasingly powerful, distributed, and mediated cognitive universe.

From prehistoric times to the present, the proliferation of forms of inscription (Latour, 1990) transformed the human cognitive architecture as profoundly as earlier leaps in biological evolution, allowing radical externalization and collectivization of cognition. In a sense, CSCL aims to push this further, designing collaboration media to foster group cognition that can lead to new forms of individual learning, team knowledge building, and community social practices. To the extent that this is true, we need to design new tasks for computer-supported teams, aiming for cognitive achievements beyond the reach of individual team members without computer supports. The goal of CSCL research should not be to simply demonstrate repeatedly that individuals learn better in online groups, but to design and investigate tasks that go beyond traditional instruction. Recent findings concerning "productive failure" (Kapur & Kinzer, 2009) illustrate how groups with challenging tasks may be learning in ways that defy standard testing indicators, but that contribute to increased problem-solving skills of the groups and ultimately of their members.

The analysis of instrumental genesis within the framework of knowledge building points to both the potentials of CSCL and the barriers to widespread dissemination. The historical evolution of tools as "epistemic artifacts" can itself be seen as a knowledge-building accomplishment of the greatest cognitive consequence, related to Vygotsky's—perhaps misleadingly named—notion of "internalization" by individuals of skills germinated in intersubjective circumstances. On the other hand, the complexity involved in successful instrumental genesis translates into severe barriers when, for instance, one tries to promote adoption of CSCL technologies, pedagogies, chronotypes and educational philosophies in established school communities and institutions. Parallel to the difficulties of the students struggling to enact the technological affordances are the difficulties of the researchers, trying to document, analyze and conceptualize the tortuous paths of instrumental genesis in CSCL.

While research on CSCL focuses on the small-group unit of analysis to understand the collaboration, this does not mean that it should ignore processes at the individual or the community levels. Group cognition theory does not ignore individual learning or cultural influences. While many educational researchers inside and outside of the CSCL field have investigated processes at the individual and social levels, few have systematically delved into the relations and influences between these levels, beyond hypothesizing relationships based on common sense presuppositions.

Traversing Planes of Learning

Planes of Learning in CSCL

Learning, cognition and knowledge building can be studied at multiple units of analysis. For instance, analyses of CSCL are often conducted on one of three levels: individual learning, small-group cognition or community knowledge building. One can identify and analyze important processes taking place at each of these levels of description. This tri-partite distinction is grounded in the practices of CSCL. With its focus on collaborative learning, CSCL naturally emphasizes providing support for dyads and small groups working together. In practice, CSCL small-group activities are often orchestrated within a classroom context by providing some initial time for individual activities (such as background reading or homework drill), followed by the small-group work, and then culminating in whole-class sharing of group findings. Thus, the typical classroom practices tend to create three distinguishable levels of activity. Often, the teacher sees the group work as a warm-up or stimulation and preparation for the whole-class discussion, facilitated directly by the teacher. Conversely, the importance of testing individual performance and valuing individual learning positions the group work as a training ground for the individual participants, who are then assessed on their own, outside of the collaborative context. In both of these ways, group cognition tends to be treated as secondary to either individual or community goals. By contrast, the role of intersubjective learning is foundational in Vygotsky (1930/1978), the seminal theoretical source for CSCL. Regardless of which is taken as primary, the three planes are actualized in CSCL practice, and the matter of their relative roles and connections becomes subsequently problematic for CSCL theory (Dillenbourg et al., 1996; Rogoff, 1995; Stahl, 2006).

While these different units, levels, dimensions or planes are intrinsically intertwined, published research efforts generally focus on only one of them and current analytic methodologies are designed for only one. Furthermore, there is little theoretical understanding of how the different planes are connected. To the extent that researchers discuss the connections among levels, they rely upon commonsensical notions of socialization and enculturation—popularizations of traditional social science. There are few explicit empirical analyses of the connections, and it is even hard to find data that would lend itself to conducting such analyses.

The individual student is the traditional default unit of analysis. This assumed approach is supported by widespread training of researchers in the standard methods of psychology and education. In the era of cognitive science, analysis made heavy usage of mental models and representations in the minds of individuals (Gardner, 1985). With the "turn to practice" (Lave & Wenger, 1991; Schatzki, Knorr Cetina & Savigny, 2001), the focus shifted to processes within communities-of-practice. Group cognition lies in the less-well-charted middle ground. It involves the semantics, syntax and pragmatics of natural language, gestures, inscriptions, etc. The meaning-making processes of small-group interaction involve inputs from individuals, based on their interpretation of the on-going context (Stahl, 2006, esp. Ch. 16). They also take into account the larger social/historical/cultural/linguistic context, which they can reproduce and modify.

Computer technologies play a central role in mediating the multi-level, intertwined problem-solving, content-acquiring and knowledge-building processes that take place in CSCL settings. From a CSCL perspective, innovative technologies should be designed to support this mediation. This involves considering within the design process of collaboration environments how to prepare groups, individuals and communities to take advantage of the designed functionality and to promote learning on all planes—e.g., through the provision of resources for teacher professional development, scripted collaboration activities and student curriculum.

The Theory of Interconnected Planes

How are the major planes of learning connected; how can we connect investigations at different units of analysis? To consider a more intuitive physical case initially, a highway ramp or bridge often creates a possibility that did not otherwise exist for going from one level to another at a given point. To traverse from a local road to a limited-access expressway, one must first find an available on-ramp. To cross a river from one side to the other, one may need a bridge. This is the individual driver's view. From a different vantage point—the perspective of the resource itself—the ramp or the bridge "affords" connecting the levels (Bonderup Dohn, 2009).

By "affords" we do not simply mean that the connecting is a happy characteristic or accidental attribute of the bridge, but that the bridge, by its very nature and design, "opens up" a connection, which connects the banks of the river it spans. In his early work, Heidegger analyzed how the meaning of a tool was determined by the utility of the tool to the human user, within the network of meaning associated with that person's life and world; in his later writings, he shifted perspective to focus on things like bridges, paintings, sculptures, pitchers and temples in terms of how they themselves opened up new worlds, in which people could then dwell. In considering the intersubjective world in which collaboration takes place on multiple connected levels, we might say that the work of artifacts like bridges is to contribute the spanning of shores within the way that the world through which we travel together is opened up as a shared landscape of places and resources for meaningful discourse and action.

This transformation of perspective away from a human-centered or individual-mind-centered approach became characteristic for innovative theories in the second half of the 20th Century. It is a shift away from the individualistic, psychological view to a concern with how language, tools and other resources of our social life work. It is a post-cognitive move since it rejects the central role of mental models, representations and computations [Investigation 15]. The things themselves have effective affordances; it is not just a matter of how humans manipulate mental models in which the things are re-presented to the mind. In phenomenology, Husserl (1929) called for a return to "the things themselves" (*die Sache selbst*) and Heidegger (1950) analyzed "the thing" (*das Ding*) separate from our representation of it. In ethnomethodology, Garfinkel and Sacks (1970) followed Wittgenstein's (1953) linguistic turn to focus on the language games of words and the use of conversational resources (Stahl, 2006, Ch. 18). In distributed cognition, Hutchins (1996) analyzed the encapsulation of historical cognition in technological instruments. In actor-network theory, Latour (1990) uncovered the agency of various kinds of objects in how they move across levels in enacting social transformations. Vygotsky (1930) used the term "artifact" to refer to both tools and language as mediators of human cognition. The broader term "resource" is frequently used in sociocultural analysis (Furberg, Kluge & Ludvigsen, 2013; Linell, 2001; Suchman, 1987) for entities referenced in discourse. Such artifacts or resources are identifiable units of the physical world (including audible speech and physical gesture) that are involved in meaning-making practices—bridging the classical mind/body divide.

A central research issue for CSCL is how collaborative knowledge building takes place. The main problem seems to be to understand the role of individual cognition and of societal institutions in small-group meaning-making processes. Figure 1 indicates (without claiming to explain or model) some typical processes on each of the primary planes of learning in CSCL and suggests possible paths of influence or connection, as events unfolding on the different planes interpenetrate each other. This figure is not meant to reify different levels or activities, but to sketch some of the constraints between different phenomena and possible flows of influence. The distinctions represented by boxes and arrows in the chart are intended to operationalize an infinitely complex and subtle matter for purposes of concrete analytic work by CSCL researchers.

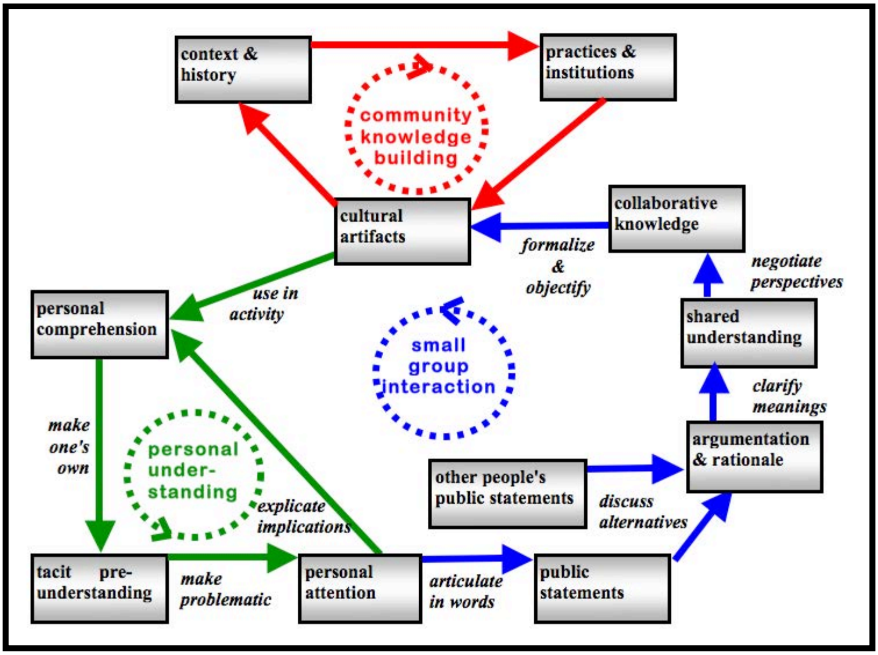

Figure 1. A model of collaborative knowledge building. Adapted from (Stahl, 2006, Ch. 9).

Some researchers, such as many ethnomethodologists, argue against distinguishing levels. For instance, in their description of conversation analysis, Goodwin and Heritage (1990, p. 283) open their presentation with the following claim: "Social interaction is the primordial means through which the business of the social world is transacted, the identities of its participants are affirmed or denied, and its cultures are transmitted, renewed, and modified." Social interaction typically takes place in dyads and small groups, so interaction analysis may be considered to be oriented to the small-group unit of analysis. However, CSCL researchers also want to analyze the levels of the individual and the culture as such—e.g., the individual identities and learning changes or the social practices and institutional forces: How do the identities of participants get affirmed or denied as a result of social interaction? How are cultures transmitted, renewed and modified through social interaction?

In general, the sequential small-group interaction brings in resources from the individual, small-group and community planes and involves them in procedures of shared meaning making. This interaction requires co-attention to the resources and thereby shares them among the participants, who co-experience the shared resources. Such a process may result in generating new or modified resources, which can then be retained on the various planes. The resources that are brought in and those that are modified or generated often take the form of designed physical artifacts and sedimented elements of language. We would like to study how this all happens concretely within data collected in CSCL settings.

Resources Across Levels in CSCL

The question of how the local interactional resources that mediate sequential small-group interaction are related to large-scale socio-cultural context as well as to individual learning is an empirical question in each case. There are many ways these connections across levels take place, and it is likely that they often involve mechanisms that are not apparent to participants. In the following, we explore one way of thinking about how such connections can occur: thanks to interactional resources.

In his study of how social institutions can both affect and be affected by small-group interactions, Sawyer (2005, p. 210f) argues that we can conceptualize the interactions between processes at different levels as forms of "collaborative emergence": "During conversational encounters, interactional frames emerge, and these are collective social facts that can be characterized independently of individuals' interpretations of them. Once a frame has emerged, it constrains the possibilities for action." The frames that emerge from small-group interactions can take on institutional or cultural-level powers to influence actions at the individual unit. This interplay among levels involves both *ephemeral* emergents and *stable* emergents. Sawyer's theory of emergents suggests a relationship among different kinds of resources along the lines pictured in Figure 2.

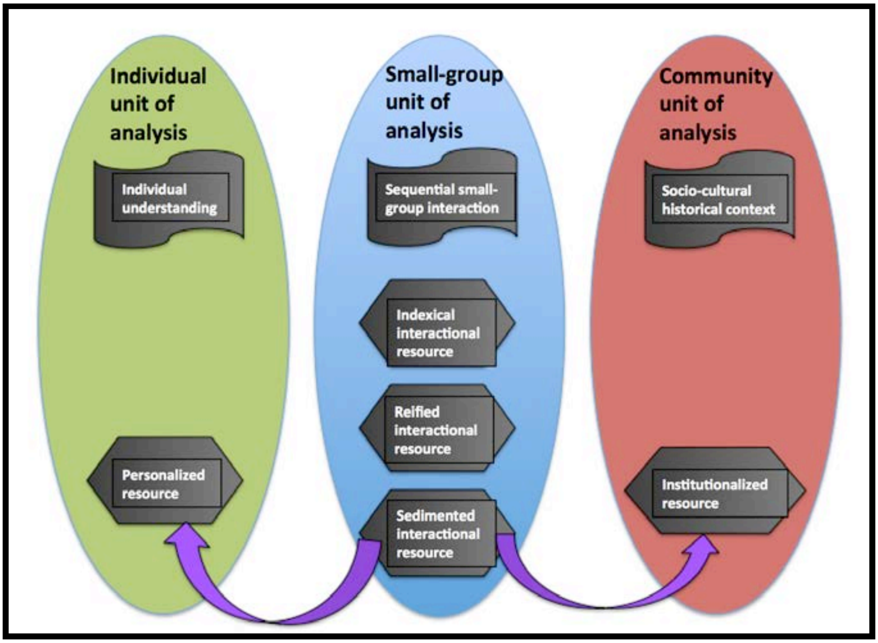

Figure 2. A diagram of emergent interactional resources bridging levels of analysis

While Sawyer's analysis addresses a broad "sociology of social emergence," it can be confined and adapted to the concerns of CSCL. What is most relevant in his theory is the view of emergence arising out of the subtle complexities of language usage and small-group interaction—rather than from the law of large numbers, the interaction of simple rules or the chaotic behavior of non-linear relationships. He thereby rejects the relevance of most popular theories of emergence for CSCL and shifts the focus to the discourse at the small-group unit of analysis. The vast variety of interactional emergents form an intermediate level of analysis between the level of individuals and that of community structures, providing a dynamic and processual understanding of social structures and infrastructures. Analysis focused on these emergent artifacts can deconstruct the reifying processes of emergence that span the group level to both the individual and the social.

The small-group interaction represented in the center of Figure 2 can be theorized as being based on an "indexical ground of deictic reference" (Hanks, 1992). This means that the "common ground" (Clark & Brennan, 1991)—which forms a foundation for mutual understanding of what each other says in conversation—consists of a shared system of *indexical-reference resources*, such as deictic pronouns, which are used to point to unstated topics or resources. The coherence of the interaction and its comprehensibility to the group participants is supported by a network of references, each of which is defined indexically, that is by a pointing within the on-going discourse context (“here,” “it,” “now,” “that point”). Interactional resources, which can be indexically referenced in the interaction, can typically only be understood within their discourse context, but they facilitate meaning making within that context [Investigation 5 and 19].

Interactional resources can undergo a process like Rabardel’s instrumental genesis [Investigations 6 and 7]. They may initially be constituted as an object of repeated discussion—an interaction frame (Goffman, 1974)—which we might call a *reified resource*, something capable of being picked out as having at least an “ephemeral-emergent” existence. Through repetition within a group discussion, a term or the use of an object might take on a settled significance within the group’s current work. Over time, continued usage can result in a *sedimented resource*, something whose existence has settled into a longer-term “stable-emergent” form, which retains its meaning across multiple group interactions.

A sedimented resource is susceptible to being taken up by a larger community as an *institutionalized resource* within a structured network of such resources, as in Latour’s (2007) social-actor networks, contributing to the socio-cultural-historical context surrounding the interaction. Thus, the institutional resource not only references the social context, but also partially reproduces it in a dialectical relationship of mutual constitution by contributing a new element or revitalizing an old set of resources.

On the other hand, interactional resources at various degrees of reification can also be taken up into the individual understanding of community members as *personalized resources*, integrated more or less into the intra-personal perspective of one or more group members. The personalization of previously inter-personal resources by individuals renders them into resources that can be referenced in activities of individual understanding—corresponding to processes of micro-genesis in Vygotskian internalization.

The various components of this view of interactional resources have been hinted at in previous theoretical contributions grounded in empirical examples. The progressively emergent character of resources can be seen even in fields of mathematics and science, as documented in Investigations in this volume.

The term “reification” goes back to Hegel’s dialectical philosophy of mediation (Hegel, 1807). Sfard (Sfard, 2000; 2008; Sfard & Linchevski, 1994) has applied it to the formation of mathematical concepts. Husserl (1936) argued that the ideas of the early geometers became “sedimented” in the cultural heritage of the field of geometry. Livingston (1999) differentiated discovering a mathematical proof from presenting a proof; a transformational process takes place, in which the byways of exploration and possibly even the key insights are suppressed in favor of conforming to the "institutionalized" template of formal deductive reasoning. Netz (1999) (see also the review by Latour, 2008) documented the important role of a controlled (restricted and reified) vocabulary to the development, dissemination and learning of geometry in ancient Greece. Analogously, Lemke (1993) argued that learning the vocabulary of a scientific domain such as school physics is inseparable from learning the science. Vygotsky (1930, esp. pp. 56f) noted that the micro-genetic processes of "personalizing" a group practice into part of one's individual understanding—which he conceptually collected under the title "internalization"—are lengthy, complex, non-transparent and little understood. These seminal writings name the processes of reification, sedimentation, institutionalization and personalization of interactional resources; their empirical investigation poses a major challenge for CSCL research.

Among the theories influential in CSCL—such as activity theory (Vygotsky), distributed cognition (Hutchins) and actor-network theory (Latour)—artifacts play a central role as resources for thought and action. In the foundations of activity theory, Vygotsky (1930) conceives of artifacts as including language as well as tools. In his seminal study of distributed cognition, Hutchins (1996) analyzes how the complex of navigational tools, naval procedures for trained teams of people and specialized language work together to accomplish cognitive tasks like ship navigation. He even analyzes data to show how an indexical phrase becomes reified within a dyad's interaction to take on significance that could have led to intra-personal and/or institutional usage. In a witty essay, Latour (1992) shows how a common mechanical door-closer artifact can act to fill the role of an individual person (a doorman), to participate in the politics of a group and to enforce institutional rules. He also argues (Latour, 1990) that an inscription artifact like a map on paper can traverse levels from a local discussion in ancient Asia to the social niveau of imperial Europe. However, studies like these have not often been duplicated in the CSCL literature.

Reviews of CSCL research (Arnseth & Ludvigsen, 2006; Jeong & Hmelo-Silver, 2010) show that few papers in our field have bridged multiple levels of analysis. Yet, the desired CSCL research agenda (Krange & Ludvigsen, 2008; Stahl, Koschmann & Suthers, 2006; Suthers, 2006) calls for a study of representational artifacts and other resources that traverse between individual, small-group and community processes to mediate meaning making. The preceding sketch of a theory of emergent forms of evolving resources could be taken as a refinement of the research agenda for the field of CSCL: a hypothesis about how levels in the analysis of learning are connected; and an agenda for exploration. A number of Investigations in this volume can be read as beginning such an undertaking. They present examples of interactional resources in small-group discussions and indicate how the resources can be seen as bridging levels of analysis.

Resources for Collaboration and for Mathematics

The idea of viewing interactional resources as central to mathematical discourse around dynamic geometry is proposed in Investigation 9, the first article in *ijCSCL* 2013 issue *8*(3). It argues that rather than focusing on the "coordination of interaction" [Interaction 12], collaborative activity should be analyzed in terms of the "coordinated use of resources." Participants rely on two major categories of resources when working on a geometry problem within a computer-based dynamic-geometry environment: (1) mathematical and tool-enabled resources (math-content-related) and (2) collaboration resources (relational or social). In Investigation 9, Öner proposes a focus on the coordination of these resources—which characterize collaborative dynamic-geometry problem solving—for understanding what goes on in such productive math learning.

The combination of social and content resources brought to bear on geometric problem solving often bridges levels. Social resources—such as greetings, invitations to speak, checks on discourse direction—function to cohere the group out of its individual members, drawing upon community standards and institutional routines. Uses of math resources—such as manipulating visual representations, referencing recent findings, expressing relationships symbolically—move fluidly between individual perceptual behavior, group problem-solving sequences and the cultural stockpile of mathematical knowledge. Perhaps the incessant traversal of levels is particularly visible in collaborative math discourse because of its explicit use of multiple layers of reality: a physical drawing, the intended figure, a narrative description, a symbolic expression, the conceptualization, the mathematical object.

Öner's methodological proposal is to trace both the math-content-related and the social/collaborative/relational resources used by students solving dynamic-geometry problems. Math resources may come from graphical, narrative and symbolic representations or expressions of the math problem or from previous math knowledge of culturally transmitted concepts, theorems, procedures, symbolisms, etc. Social resources include communication practices, such as the rules of conversational discourse (transitivity, sequentiality, shared attention, argumentation, turn taking, repair, etc.).

Öner's Investigation cites a number of distinctions drawn in the CSCL literature for contrasting social/collaborative/relational resources with content-related resources:

- An inter-personal-relations space versus a content space (Barron, 2000);
- Building a joint problem space (JPS) versus solving a problem (Roschelle & Teasley, 1995);

- Temporal dimensions of the JPS versus diachronic content (Sarmiento & Stahl, 2008);
- Text chat versus shared-whiteboard graphics (Çakir, Zemel & Stahl, 2009);
- Project discourse versus mathematical discourse (Evans, Feenstra, Ryon & McNeill, 2011);
- Spatio-graphical observation (SG) versus technical reflection (T) (Laborde, 2004).

The "space" that a group builds up and shares is a structured set of resources gathered by the group (JPS, indexical field, common ground). The resources are "indexical" in the sense that they are only defined within (and thanks to) this constructed space of the specific problem context. Through their discourse, the group compiles these resources as potentially relevant to the problem. In turn, the resources help to define the emergent problem, dialectically.

Öner generated data to explore the interaction of the contrasting dimensions by having two people work together face-to-face in front of a shared computer on a particular dynamic-geometry problem, whose solution required a mix of spatio-graphical observation and technical reflection involving mathematical theory—a mix of SG and T resources, to use the distinction she adopts from Laborde. She uses this distinction among resources to structure her analysis. In doing so, she shows how these various resources bridge the different units of analysis. Resources of *individual* perception (during dragging of geometric objects on the computer screen) feed into the *group* problem solving, just as do references to classical theorems passed down through *cultural* institutions. They make possible and stimulate the group interaction. This analysis provides examples of interactional resources at work in CSCL settings.

By analyzing both social and content resources, Öner shows how interrelated these can be. For instance, at one point in the data, one student says, "now two isosceles, oops, equilateral triangles are formed here." This utterance is deeply indexical. It is pointing to the "here" and "now" of the geometric construction. The student is narrating his work, intersecting two circles to locate the vertices of the desired equilateral triangle (see Figure 3). The method he is using refers back over 2,500 years to Euclid's first proposition, which teaches this construction. It also notes that one could use either of two potential intersections to construct alternative triangles. This leads his partner to see first one of the intersection points and then the other. Öner notes that the two students collaboratively accomplished this construction; in the doing of it, they collectively recalled the procedure, which they had performed in the past but forgotten. She also emphasizes that this utterance includes a self-repair, in which the speaker substitutes a correct term ("equilateral") for an incorrect one—a move she considers social. Repairs are conversational moves aimed at avoiding or correcting potential misunderstandings.

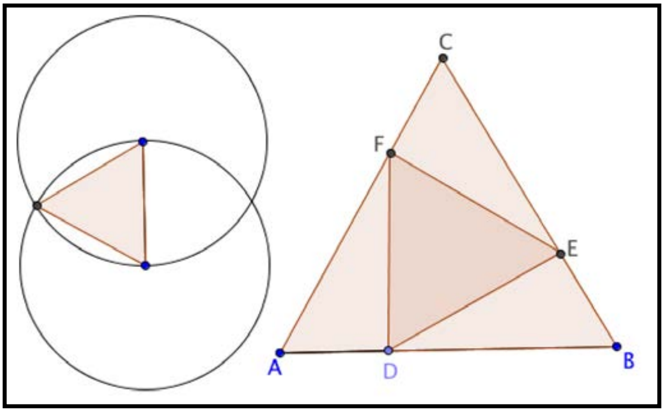

Figure 3. Constructing an equilateral triangle inscribed in an equilateral triangle.

This raises a key theoretical point. Should this utterance be analyzed, categorized or coded as a social resource or as a mathematical one? What is the resource here? Is it the generic conversational resource of self-repair as a "member method" (Garfinkel, 1967), or is it the word "equilateral" in the shared language, or is it the geometric concept of equilateral polygon? I.e., is it a conversational move, a linguistic term or a mathematical concept? This is a matter of level of analysis, because one could characterize it in any of these ways. Alternatively, one could argue that the interactional resource that exists here spans multiple levels of analysis, providing an object for analysis at the conversational, linguistic and mathematical levels of the interacting group, the speaking individual and the cultural conceptualization. In other words, such a resource can serve as a boundary object (Star, 1989), which can be discussed from different perspectives, focused on different units of analysis.

Öner succeeds in analyzing how her students collaborated on their geometry problem by focusing consistently on the interplay between social and content resources. It may be that we can often follow the movement of discourses across different levels by keeping our eyes on consequential resources. However, other CSCL researchers interpret the theme of resources differently from Öner. This leads them to different insights about their data. Perhaps we can use the concept of resource as a methodological boundary object to bring together the disparate theoretical voices. Too often, they seem to talk at cross-purposes, emphasizing differences when they might well be seeing the same phenomenon from different angles.

Scientific Representations across Levels

Even if analysts agree in identifying a certain object as a pivotal interactional resource, that does not mean that the nature or meaning of that resource is self-evident to students using it for collaborative learning—as the second article in *ijCSCL* 8(3) by *Anniken Furberg*, *Anders Klug* and *Sten Ludvigsen* (2013) makes clear. They turn to look at how students make sense of scientific diagrams to support their collaborative learning of physics. The implications of a diagram of a photoelectric cell only emerge gradually for a group of students striving to understand and explain the scientific processes represented there.

The central case study of this paper illustrates how the students gradually produce the meaning of the scientific representation. It is the sense-making process—mediated by the representational resource—that spans levels: The individuals, each with their own approaches and each bringing in different other resources, contribute to the group's collaborative effort, resulting in a group understanding, expressed however awkwardly and partially in their written report. The representation—first from their textbook and then complemented with a second diagram from the Internet—is a contribution from the larger scientific or science-education community.

The paper characterizes the science diagram as a *structuring resource*. It argues that the representation, as it becomes meaningful to the students, structures the group's sense-making work. The structuring takes place on various levels: Interactionally, the group uses the diagram as a deictic resource, pointing to its features either gesturally or linguistically to support the verbal accounts. Individually, the students refer to the diagrams to monitor their own understanding. At the level of science norms, the students attempt to use canonical language to express the sense they are making of the diagram.

Student discourse generally halts in articulation of an idea at the point when everyone seems to understand each other adequately for all practical purposes of the conversation. Even adding a third person to the discourse can extend the discussion somewhat, because the third person brings new questions and needs for understanding. However, when students go to write up a point, they must attain a much higher standard of articulation. They must make their written statement comprehensible and persuasive for a general audience or for people not present to indicate their understanding or agreement. This audience might, for instance, include the teacher, other students in the class or even an audience of unknown potential readers. The audience might require a scientific formulation, using the vocabulary and stylistic genre of physics. Furthermore, since the reading audience is not co-present with the speakers, physical gestures and deictic references to times, places, people and objects present are no longer effective. While the diagram still helps to structure their articulation of the description, the description can no longer rely so heavily on the diagram to help convey their meaning.

It is always true that there is a dialectical circularity or recursive character to the relationship of the discourse context and the utterances that are made within that context; this becomes even clearer in the relationship of the diagram as a structuring and interactional resource to the students' understanding of this resource. The (tentatively understood) diagram helps to structure the students' (increasing) understanding of the diagram itself. The paper nicely shows how the introduction of a second diagram enriches the dialectic by shedding light on the first diagram's meaning through the tension created by the differences between the two representations.

Referential Resources for a Math Problem

In the third paper of *ijCSCL* 8(3), Investigation 5 takes an ethnomethodologically informed look at the role of resources, representations, referential practices and indexical properties in the mathematical problem-solving interactions of students within a CSCL setting. Viewed in the context of the 8(3) issue of *ijCSCL*, Investigation 5 develops further some of the central themes of the two previous papers. It concurs with the first paper on the importance of tracking the use of resources, and it further emphasizes that it is the on-going specification-in-use that determines the significance of a given resource. It concurs with the second, in adopting a concern with representations, and it makes even more explicit the extent to which the representational practices—how the representation was built and worked with—contribute to the problem clarification and problem solution.

In theoretical terms, this paper develops the discussion of *indexical reference resources* by Hanks (1992). It considers two groups of students who were presented with the same problem statement involving combinatorics. The two groups identified completely different sets of "indexical properties," which allowed them to formulate implicitly, share collaboratively and solve mathematically the "same" problem, which, however, had been specified quite differently. In the first team, Bwang8 specified the stair-step pattern of squares in terms of two symmetric sets of lines. Each set of lines followed the pattern: 1, 2, 3, …, n, n. In the second team, Davidcyl specified the problem initially as: "the n^{th} pattern has n more squares than the $(n-1)^{th}$ pattern."

Ethnomethodologists are keen to observe the "work" that people do to accomplish what they do. Both teams engaged in intricate coordination of text understanding, sequential drawing, retroactive narrative and symbolic manipulation to make sense of the problem statement they faced and to arrive at a mathematical solution. The work involved in this can be characterized as discovering, proposing and negotiating successive determinations of indexical properties of the problem they were working on. The indexical properties are ways in which the team members can reference aspects of the problem, such as in terms of sets of lines arrayed in specific identifiable patterns. These indexical properties are tied to the local problem-solving context of the respective team. They specify the problem for the team in practical terms, which allow the team to make progress in both understanding and solving the problem.

This approach is appropriate for what Rittel and Webber (1984) called "wicked problems." These are non-standard problems, for which the approach to problem solving is not obvious and turns out to be a matter of coming to understand the problem itself. One can imagine Bwang8 entering a completely unknown territory. He was not familiar with the online environment, had never seen the kind of problem statement that was displayed, did not know the other team members and was unclear about what was expected of him. He spotted (visually) an interesting symmetry in the problem and started by stating it as an initial specification about how to view (perceptually and conceptually) the problem. Then he started to draw the problem, so specified, on the shared whiteboard. Davidcyl entered a similarly unknown territory. He started drawing the pattern for N=4, as suggested in the text. In so doing, he developed some copy-and-paste practices, which he presented (in the sequentiality of his drawing process as well as in his accompanying description) as tentatively mathematically relevant.

Starting from *individual* suggestions of indexical properties (by Bwang8 or Davidcyl, respectively), each group developed a growing shared indexical ground of deictic reference. The work of building that space of possible references led the *group* to make sense of a problem and to discover a path to a solution in mathematical terms. The ground itself is a set of shared interactional resources that allows the team to refer to its object of concern in mutually intelligible ways. By gradually moving from purely deictic terms like "it" or "this," to mathematical terms or abstract symbols, the indexical resources incorporated cultural knowledge and contributed to a less locally situated store of understanding that could be relevant in a larger classroom or *culture* of school mathematics (including standardized tests). The analysis of how these groups successively and collaboratively re-specify their referential resources suggests approaches to studying how groups make sense of problems and artifacts whose indexical properties are initially unknown or underspecified. This is a foundational concern for CSCL, as "a field of study centrally concerned with meaning and the practices of meaning making in the context of joint activity, and the ways in which these practices are mediated through designed artifacts" (Koschmann, 2002).

Roles as Interactional Resources for Community Meaning Making

If the previous studies take interactional approaches, the next paper in *ijCSCL* 8(3), Hontvedt and Arnseth (2013), can be considered to be largely at the community-of-practice level. Like the apprenticeship cases of Lave and Wenger (1991), this one is concerned with how novices take on the practices of a professional community. Situated in a simulator for training Norwegian sailors, the apprentices role-play at navigating a ship. To bring a ship up the fjord to Oslo, they must bring aboard a local expert. This master pilot helps to establish the professional navigational practices with the apprentices. Interestingly, the pilot insists on using the international language of shipping, English. At times, the trainees slip into Norwegian to reflect on their role-playing, thus marking linguistically the duality of their realities. On the one hand, they are playing the roles of professional sailors interacting in English on the bridge with the local pilot; on the other, they are Norwegian students discussing their educational activities.

Through their role-playing, the participants—whether newcomers or established members of the sailing community—co-create interactionally the context of their learning. Much of the learning consists in this subtle process, which includes integrating interpersonal relations, language constructs, physical artifacts, a designed setting and nautical tasks. Together, this constitutes what the authors call an *activity context*. Building on the theoretical framework of activity theory, an activity context is closely related to Goffman's (1974) concept of frame.

The roles taken on by the students are resources for their apprenticeship meaning making. Like roles in a play on stage, they require a willing suspension of disbelief. The analysis in the paper nicely shows how the students fluidly move in and out of their roles and negotiate when to do so, often through code switching between the languages of the two cultures. Never taking the simulation fiction too seriously—as though it were an immutable reality—the analysis reveals how the participants themselves achieve the tenuous existence of the activity context interactionally.

The interactional resources of this learning community are ephemeral emergents—which also means they can collapse. The action can call for a role or an artifact that is missing from the simulation, resulting in improvisation, chaos, laughter. This carries a lesson for all of us: an assemblage of resources for learning cannot foresee all uses. Even the most rehearsed experiment in complex learning is likely to run afoul of glitches. In the best cases, the participants laugh off the troubles … and the analysts discover insights in the breakdowns.

Annotations as Resources for Individual Learning

In the final paper of issue *ijCSCL 8*(3), Eryilmaz et al. (2013) take a controlled-experiment approach to evaluate the effect of a promising annotation-support tool as a resource for individual learning. While acknowledging that online asynchronous discussion in a university course is a group activity in an educational social setting (with an instructor, discourse standards, canonical texts, grading, etc.), the authors systematically focus on the learning of individual students as evidenced by their individual postings and isolated pre-/post-tests. In contrast to the qualitative analysis of interaction in the preceding papers, this one codes individual posts and analyzes them with a battery of quantitative methods. Even the analysis of sequentiality is done without reference to interactional context. The group and social setting are considered controlled for, and only the presence of the software function distinguishes the treatment from the control condition.

By methodologically focusing on the individual student and the individual posting as the units of analysis, this study is able to isolate and quantitatively assess the role of context on these units. For instance, the paper asserts that, "collaborating students are able to use one another as a *resource for learning*" (emphasis added). That is, while learning is conceptualized as a process that primarily takes place in individual heads, it is enhanced by the interactional level of individuals formulating ideas as posted text and receiving feedback as posted responses from others. Asynchronous discussion forums seem like good media for supporting such enhancement, except that their use apparently causes excessive "cognitive load," reducing the ability to engage in the cognitive processes required for deep learning and therefore counteracting the potential benefits of social interaction.

The complex socio-cultural and interactional processes analyzed in the previous papers are here viewed as likely sources of unwelcome cognitive load. In order to communicate one's ideas about a text in annotations that might make sense to other students, one must engage in the sorts of collaborative meaning making analyzed in the other papers. For instance, one must construct explicit indexical references, such as "the third sentence in the conclusion," which can be used to coordinate co-attention.

To make it easier to establish joint reference, the authors of this study provided students with a software indexing function, which graphically connects annotations with relevant selections in the provided educational text. The treatment group uses this software tool as an *interactional resource*, which is not made available to the control group. The research then studies the effect of the resource on learning with the rigor of its chosen methodology. The study shows that the treatment group produces more posts coded as "assertions" and "conflicts." It also does better than the control on the post-test, confirming experimental hypotheses. The conclusion is that the software resource reduced the cognitive load needed to co-construct effective shared interactional resources, like indexical descriptions of target text passages. This allowed the students more cognitive ability—or perhaps just more time-on-task—to engage in interactive assertions and conflicts. So, the focus on the individual unit of analysis allowed this study to evaluate interactions between individual learning, group interaction and socio-technical setting.

Of course, one can always question a study's assumptions and operationalization. The recent findings in CSCL research about "productive failure" (Kapur & Bielaczyck, 2012; Kapur & Kinzer, 2009; Pathak et al., 2011) problematize the purely negative view of what is here characterized as cognitive load, as well as the way of assessing deep learning. Positive findings about productive failure suggest that group processes can underlie learning in ways that may not show up immediately. The effort (cognitive load) to build a joint problem space about a text through interpersonal interaction may confer learning benefits that are not achieved when that task is delegated to software. The benefits may also not show up in measurements taken immediately at the individual unit of analysis.

This final paper of the *ijCSCL* issue, taken together with the preceding four, illustrates how different methodologies can be adopted for analyzing resources and their relations to different levels of analysis. What can be taken as a resource for purposes of CSCL research is open to a broad range of approaches and theoretical frameworks. One can find resources for individuals, groups and communities. Often, those resources can be seen as traversing across or mediating between levels. Analysts can fruitfully focus on one aspect or another of this; or they can strive to follow resources across multiple levels.

The CSCL Agenda on Levels of Analysis

The time has come for CSCL to address the problem of traversing levels of analysis with exacting research. Attempts to research a given level in isolation have run into fundamental limitations. Although it is clear to most researchers that the levels of individual, small-group and community phenomena are inextricably intertwined, opinions differ on how to respond analytically. Religious wars between adherents of different methodological faiths are often based on misunderstandings: people agree on the need to comprehend the levels together but articulate that need in incommensurate-seeming locutions.

Multiple-method approaches, multi-level statistics and multi-vocal analyses are too limited, because they do not explicitly address the interrelationships among different levels. Some researchers claim that the apparent levels are all reducible to one fundamental level—whether individual cognition, group interaction or the social—while others assume that they can be studied independently. Some say that there are no such thing as different levels, but only different kinds of analysis, although they generally end up talking of individual understandings, group interactions and community practices. There are vague theories that one level is emergent from another or dialectically coupled with it, but these ties are not well worked out or evidenced with CSCL data.

The contributions in this issue provide examples of the kinds of studies and analyses that are needed. In order to comply with one or another standard of rigor, most research focuses on specific relationships within a single unit of analysis. We now also need to generate, compile and analyze data that sheds light on relationships across levels. The idea of tracking *interactional resources* as they mediate across levels offers one suggestive approach. The different papers discussed here and other referenced theories show that there are many ways to conceptualize, analyze and theorize resources. We do not mean to define or defend a particular tack, but to suggest interactional resources as a candidate boundary object for discussion across competing approaches. We do not claim to have proposed a consistent position, but rather to raise some questions about what can be meant by resources for computer-supported collaborative learning, in the hope of stimulating thinking for CSCL research in the future.

This Investigation has tried to prepare the way for the more detailed considerations of a theory of group cognition in this volume, especially the essays of Part III. After tracing the historical expansion of the concept of cognition–especially in twentieth-century philosophy—from individual minds to group and collective cognition, it focused on the concept of intersubjectivity as central to analyzing and designing collaborative learning. Intersubjectivity is the ability of multiple subjects to understand each other by interacting within a shared world. A number of approaches to intersubjective meaning making were reviewed, including by CSCL researchers, philosophers, ethnomethodologists and activity theorists. The intersubjective processes at the small-group unit of analysis were seen as intimately connected with the adoption and use of artifacts and social practices. This led to consideration of the inherent integration of multiple planes of learning and the role of resources that span the individual, group and cultural levels. These themes are explored at length by the Investigations of Part II and Part III, which follow. They provide detailed arguments and clarifications for the vision of CSCL proposed in Investigation 1, with its theoretical, methodological and pedagogical focus on the intersubjective small group.

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Part II: A Vision of CSCL

Investigation 3. A Relational, Indirect, Meso-level Approach to CSCL Design in the Next Decade. By C. Jones, L. Dirckinck-Holmfeld, & B. Lindstrom

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Looi, C.-K., So, H.-J., Toh, Y., & Chen, W. (2011). The Singapore experience: Synergy of national policy, classroom practice and design research. International Journal of Computer-Supported Collaborative Learning. 6(1), 9-37. Doi: 10.1007/s11412-010-9102-8.

Investigation 14. Bridging Research and Practice: Implementing and Sustaining Knowledge Building in Hong Kong classrooms. By C. K. K. Chan

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http://ijcscl.org/download.php?file=chan_6_2.pdf&path=_preprints/volume6_issue2

Chan, C. K. K. (2011). Bridging research and practice: Implementing and sustaining knowledge building in Hong Kong classrooms. International Journal of Computer-Supported Collaborative Learning. 6(2), 147-186. Doi: 10.1007/s11412-011-9121-0.

Part III: A Theory of Group Cognition

Investigation 15. A Paradigmatic Unit of Analysis

Gerry Stahl

Abstract. This essay poses the question: Can CSCL represent a new paradigm of educational research within the Learning Sciences? It begins by looking at the historical relationship of the two related research communities: Computer-Supported Collaborative Learning and the Learning Sciences. It presents them from the perspective of the author as a participant in those communities during 20 years (1995-2015). It reviews the institutional history of their relationship within the International Society of the Learning Sciences. Trends in the history of philosophy and social theory are then reviewed to motivate an innovative contemporary paradigm. A "post-cognitive" educational paradigm is proposed that focuses on group interaction as the unit of analysis that is most central to CSCL. Finally, the author's CSCL research agenda is described as an illustration of a candidate approach. In conclusion, it is proposed that CSCL research should focus on the analysis of group processes and practices, and that the analysis at this level could be considered foundational for the Learning Sciences.

Keywords. Post-cognitive, educational paradigm, research community, shared understanding, collective intentionality, group agency.

A Participant's View of LS and CSCL

The Learning Sciences (LS) and Computer-Supported Collaborative Learning (CSCL) are not easy to clearly distinguish. There are no objective or fixed definitions of these two fields. They are best understood as communities of researchers. Despite their fluidity, they do seem to evolve in a certain direction over time. The shifting nature of the communities appears differently to different participants and is often negotiated in discussions among them. In this essay, I discuss the CSCL and LS communities from the perspective of my own participation in them. I start with some personal and community background because research is always oriented in response to personal and political commitments of the people and communities involved. This does not contradict the "objectivity" of the research—for it must still meet accepted standards of rigor and accept the implications of scientific findings—but provides a context of significance necessary for relevance, motivation and shared understanding. Theory is always situated.

CSCL is trans-disciplinary, requiring a mix of academic backgrounds. I came to CSCL from philosophy and computer science. In the 1960s and early 1970s, I studied twentieth-century continental philosophy and social theory at MIT, Northwestern, Heidelberg and Frankfurt, but supported myself as a math teacher and computer programmer. In the early 1990s, I studied computer science academically, specializing in AI, design theory, HCI and CSCW at the University of Colorado in Boulder. Upon graduation in 1993, I decided to apply computer science to educational innovation. When Tim Koschmann spent a year at Boulder during 1997/98 while I was starting my career as a research professor, I participated in his course on CSCL and he introduced me to local conversation analysts, whose courses I also attended. Koschmann was instrumental in organizing the first seven CSCL conferences and editing the seminal CSCL book (Koschmann, 1996). I participated in all the CSCL conferences, starting in 1995, and also the ICLS conferences from 1998 on. During 2001/02, I lived in Germany for a year and worked on a European Union CSCL research project. That year, I met many of the Europeans active in the CSCL community and visited their labs, workshops and conferences.

Koschmann convinced me to be program chair of CSCL 2002 in Boulder. At the closing session of CSCL 2002, those present agreed to found a new organization, the International Society of the Learning Sciences (ISLS), to provide an institutional framework to bring together the CSCL and ICLS conference series and also the *Journal of the Learning Sciences* (*JLS*). It was decided that Timothy Koschmann, Janet Kolodner and Christopher Hoadley would share leadership of the society. I agreed to be on the founding board, to draft the by-laws, to set up the website and to design a logo.

The contested relationship of CSCL to LS soon flared up at CSCL 2003 in Bergen, when the legal incorporation of ISLS was announced there. The central participants in the CSCL community were largely European members who had been active in the AI-in-Education community. They felt that Roger Shank had betrayed the AI-in-Education community when he hosted their conference at Northwestern in 1991 and used that occasion to proclaim himself the leader of a new field, which he called "the learning sciences." Kolodner was seen as his protégée, who had extended his technical contribution in AI models of case-based reasoning and was the founding editor of *JLS*, the journal of LS. At the time, virtually all articles in *JLS* had been by North American authors and represented a strongly cognitivist approach. The International Conference of the Learning Sciences (ICLS), the conference series for LS, was held exclusively in the US until 2008, and had been dominated by a few American schools, primarily prestigious departments of education at US universities (e.g., Northwestern, Georgia Tech, Michigan, Washington, UCLA, Indiana, Berkeley, Stanford, Vanderbilt, Pittsburgh).

So, at the Bergen conference, a group of European CSCL researchers raised harsh questions about whether ISLS was an attempt by American LS leaders to take over the field of CSCL and its conference series, which was finally being held in Europe in 2003—after Euro-CSCL 2001 in Maastricht was retroactively recognized as an official CSCL conference. Kolodner, Koschmann and Hoadley were unable to satisfy the concerns raised. There was lively discussion among the conference attendees, and a smaller group of us drafted a position paper overnight. The outcome was to proceed with the establishment of ISLS, but to set up a CSCL Committee within ISLS to represent the CSCL community. The CSCL Committee would exercise control over CSCL matters, like the CSCL conference series. During the same conference, the idea of a CSCL journal (*ijCSCL*) was proposed; Hans Spada suggested that I found it with the co-editorship of Friedrich Hesse. Pierre Dillenbourg had already established a CSCL book series published by Springer. These initiatives helped to form links and establish parity between LS and CSCL.

ISLS gradually became established. Hoadley was the first President, and subsequent presidents included several prominent European and American CSCL researchers, including some who had raised the original critical questions at the 2003 conference in Bergen, Norway. Kolodner served as Executive Director of ISLS throughout its formative years. The tension between CSCL and LS gradually dissipated; the CSCL Committee lingered on, primarily playing a symbolic role. ISLS, ICLS and *JLS* gradually made concerted efforts to become more international and to broaden their leadership. Although the assumption has generally been that the two communities have largely merged, my sense is that the theoretical differences between them and between the two conference series have not much altered during the intervening decade. The conflict between CSCL and LS may have been more than a political clash between overlapping communities; it may also represent a rejection by many CSCL researchers of the extreme commitment to cognitivist theory and methodology by prominent LS proponents at that time.

It is hard to define the difference between CSCL and LS other than, perhaps, in terms of the people involved. This is because both communities profess openness to the same range of theoretical and methodological frameworks, although both promote certain preferred orientations in subtle and unspoken ways. For instance, most researchers in both fields claim to accept the situated nature of learning and the sociocultural perspective, but if you look closely at their analyses, you find that they both often rely on methods and approaches that predate and may contradict these theoretical positions. While many researchers publishing in CSCL venues still employ cognitivist methods—such as interpreting isolated utterances as expressions of mental representations, empiricist controlled experiments manipulating objective conditions or coding along predetermined categories—it may be that the CSCL vision calls for a different research paradigm.

Did CSCL or LS Adopt a New Paradigm?

In the introduction to his edited volume of CSCL studies, Koschmann (1996) proclaimed that CSCL provided a new paradigm of research on instructional technology. He used Kuhn's principle that a paradigm must be "sufficiently unprecedented to attract an enduring group of adherents away from competing modes of scientific activity" (Kuhn, 1972, p. 10). A few years later, as Koschmann saw that there was no consistency in method among actual CSCL studies and that the vast majority of CSCL and LS studies had, in fact, not moved away from traditional approaches to measuring individuals' learning outcomes, he reconsidered that claim. He argued that:

> Traditional theories of learning treat learning as a concealed and inferred process, something that "takes place inside the learner and only inside the learner" (Simon, 2001, p. 210). CSCL research has the advantage of studying learning in settings in which learning is observably and accountably embedded in collaborative activity. Our concern, therefore, is with the unfolding process of meaning making within these settings, not so-called "learning outcomes." It is in this way that CSCL research represents a distinctive paradigm within IT. By this standard, a study that attempted to explicate how learners jointly accomplished some form of new learning would be a case of CSCL research, even if they were working in a setting that did not involve technological augmentation. On the other hand, a study that measured the effects of introducing some sort of CSCL application on learning (defined in traditional ways) would not. (Koschmann, 2001, p. 19)

In his keynote talk at CSCL 2002, Koschmann proposed that "CSCL is a field of study centrally concerned with meaning and the practices of meaning making in the context of joint activity, and the ways in which these practices are mediated through designed artifacts" (Koschmann, 2002, p.17). It is important to note that "meaning making" is here taken as an intersubjective or "joint" practice or small-group process. Meaning is not conceived as a mental model somehow existing in the brains of individuals.

Koschmann then reviewed what he took to be a seminal CSCL paper by Jeremy Roschelle (1992) as an early instance of the CSCL paradigm, because Roschelle focused on the analysis of meaning-making practices (such as conversational moves in a dialog) in a context of joint activity (dyads working collaboratively on challenges) mediated by a designed activity (a software simulation of the relationship of velocity and acceleration).

Koschmann focused on the version that Roschelle published in *JLS*—which Koschmann himself later republished in his CSCL edited volume (Roschelle, 1996). However, in terms of the relationship of CSCL and LS, the situation was rather more complicated as well as more interesting than what Koschmann reported. First, Teasley and Roschelle (1993) presented an analysis involving the co-construction of a "joint problem space" (JPS) by students, using Roschelle's dissertation data. The JPS was an explicit transformation of the cognitivist conception of a mental problem space in (Newell & Simon, 1972) into the intersubjective realm of situated interaction. Newell and Simon's notion of cognitive production rules (mental mechanisms) was re-conceptualized by Teasley and Roschelle as socially distributed, turn-taking, collaborative completions (discourse moves). The unit of analysis was transformed from the individual mind to the small group interaction (dialog and joint attention through pointing).

Stephanie Teasley was instrumental in bringing a post-cognitive framework to this analysis in her collaboration with Roschelle, while they were both interns at the Institute for Research on Learning (IRL) in Palo Alto. IRL was a hotbed of post-cognitive innovation, inspired by theories of conversation analysis, ethnomethodology, activity theory, situated action and situated learning. Teasley (then named Behrend) and Roschelle first presented their analysis with co-author Janice Singer at the CSCW 88 and ITS 88 conferences (Behrend, Singer & Roschelle, 1988; Singer, Behrend & Roschelle, 1988). These papers grew into the version later published as (Teasley & Roschelle, 1993), presented at a NATO-sponsored workshop in Italy in 1989 (the first event ever to use the term "CSCL"!).

It was these early versions of the paper that really emphasized the intersubjective practices of meaning making in the context of joint activity. The authors explicitly juxtaposed their perspective to cognitivism: "Thus, in contrast to traditional cognitive psychology, we argue that collaborative problem solving takes place in a negotiated and shared conceptual space, constructed through the external mediational framework of shared language, situation and activity—not merely inside the cognitive contents of each individual's head" (Roschelle & Teasley, 1995, p. 70).

In the *JLS* article reporting on this research, Roschelle argues that the ability of the dyad to "share" knowledge in a cognitive sense (as convergent mental contents) could be demonstrated by an analysis of the collaborative sense in which the students "share" a joint meaningful world (are engaged with co-constructed meanings and artifacts). Tying the analysis of intersubjective meaning making to the problematic of cognitive convergence (as encouraged by *JLS* editor Kolodner) had the potential of appealing to the *JLS* audience, because it put the argument in cognitive terms they could relate to without disrupting their paradigm. However, this made the argument more complex and detracted from its ability to stand as a clear example of a post-cognitive paradigm.

Koschmann concluded that CSCL could be a new paradigm if studies would maintain a focus on how groups of learners collaboratively achieve new understandings in the presence of computational artifacts. However, in most CSCL studies (as in LS studies) there is a conflict between the espoused and the applied theories of learning or between the motivating theoretical concerns and the bottom-line methods of analysis. After Roschelle and Teasley's publications, most actual instances of research by the CSCL and LS communities fall back on old traditions in educational psychology or other forms of measuring and correlating learning outcomes of individuals—sometimes despite the researchers' best stated intentions and even the inherent needs of their research questions. (This observation is based on ten years of reviewing all the submissions to *ijCSCL* as well as searching the broader literature for instances of analysis at the group unit of analysis.)

Measuring the effectiveness of dialog or collaboration is never a straightforward affair. It is highly dependent upon the details of the setting and the group practices. Methodological concerns related to this were expressed early in founding documents of CSCL, for instance by Dillenbourg, Baker, Blaye and O'Malley (1996, p. 189). Here they distinguish the CSCL vision of research on "collaborative" learning from the cognitivist tradition of research on "cooperative" learning (e.g., Johnson & Johnson, 1989; Johnson & Johnson, 1999; Slavin, 1980):

> For many years, theories of collaborative learning tended to focus on how *individuals* function in a group. More recently, the focus has shifted so that *the group itself has become the unit of analysis*. In terms of empirical research, the initial goal was to establish whether and under what circumstances collaborative learning was more effective than learning alone. Researchers controlled several independent variables (size of the group, composition of the group, nature of the task,

> communication media, and so on). However, these variables interacted with one another in a way that made it almost impossible to establish causal links between the conditions and the effects of collaboration. Hence, empirical studies have more recently started to focus less on establishing parameters for effective collaboration and more on trying to understand the role that such variables play in mediating interaction. In this essay, we argue that *this shift to a more process-oriented account requires new tools for analyzing and modeling interactions*. (Italics added)

In the first volume of the *International Journal of CSCL* (*ijCSCL*), Suthers (2006, p. 321) [Investigation 4] proposed a research agenda for CSCL: "To study the accomplishment (a post hoc judgment) of intersubjective learning we must necessarily study the practices (the activity itself) of intersubjective meaning making: how people in groups make sense of situations and of each other." He agreed on the need for CSCL research to focus on analysis of group processes. He immediately noted, however, that few studies published in the CSCL literature have addressed *intersubjective* meaning making directly; all but a few analyze data taken as related to individual minds.

The motivating vision of CSCL presented here suggests that research in this field should focus on the small-group unit of analysis. CSCL is a response to the potential of computer technology networked though the Internet to bring learners together and to support their collaborative knowledge building. The potential is not just to provide innovative tools and broadened sources of information to individual learners, but to allow cognition itself to evolve from individual efforts to group efforts [Investigation 7]. The rigorous analysis of computer-supported collaborative learning requires a new research paradigm oriented primarily to the small-group unit of analysis as the locus of intersubjective meaning making.

There are many pressures against research adopting a new paradigm and new tools for analyzing interactions. For one, the study of interaction processes and group practices requires analytic skills that are not generally taught in standard college courses on research methods and statistics. There are also external influences: The public wants stories that meet common-sense images of science based on popular notions of traditional science, such as mechanistic Newtonian physics. Politicians and funding sources want simple numeric results that they can cite as clear measures of return on government or grant investments in education. Academic hiring and promotion committees want publications in well-established conferences and journals to justify their decisions. Conferences and journals rely on peer review by scholars trained in traditional notions of rigor. Systems of social rewards—which largely define behaviors in academic research communities—

militate against methodological innovation, even as they reward superficial adherence to the latest trends.

It is hard to determine how many publications in CSCL or LS break free of the cognitivist paradigm's stronghold on publication. For instance, studies of CSCL publications (e.g., Akkerman et al., 2007; Jeong & Hmelo-Silver, 2010; Jeong, Hmelo-Silver & Yu, 2014; Kienle & Wessner, 2006; Lonchamp, 2012; Tang, 2014) bring their own paradigmatic blinders or filters. They sometimes eliminate from consideration any paper that does not focus on "empirical" data analysis, often excluding ethnographic case studies and certainly theoretical articles. They generally miss many of the most influential papers or more innovative approaches. Many highly rated journals in the educational field advertise that they only publish papers that conform to traditional empiricist methodological standards. The stances of these journals in turn influence the attitudes of reviewers for other journals and conferences. Attempts to categorize publications in CSCL and LS often succumb to a similar fate, imposing implicit or explicit criteria on the selection of papers to be categorized.

We have seen that it is hard to determine the extent to which a post-cognitive paradigm is making headroads in CSCL and/or LS research. What would a CSCL paradigm look like that systematically thematized the mutual engagement of small groups in meaning making and problem solving, as suggested by Koschmann; Roschelle and Teasley; Suthers; Dillenbourg, Baker, Blaye and O'Malley?

The following sections of this Investigation explore the implications of the post-cognitive theories that are so often espoused within the CSCL and LS communities, but relatively rarely carried through in the published analyses. First, recent post-cognitive theories are traced back to their roots in the history of philosophy, noting the historic junctures that provide the ontological and epistemological motivations for various alternative methodologies. Then, Vygotsky's argument for the foundational role of intersubjective collaborative learning for educational theory is summarized. Next, the nature of a post-cognitive paradigm is illustrated by the example of the VMT CSCL research project. Finally, it is concluded that CSCL should more consistently focus on analysis of group cognition, which, moreover, may be considered foundational for learning generally.

The Post-cognitive Philosophical Paradigm

The post-cognitive CSCL paradigm studies *meaning making as a joint (or group) activity*. For instance, the analysis by Teasley and Roschelle (1993) in terms of the collaborative activity of constructing a joint problem space was an early instance of this new paradigm. However, the analysis of the same data in terms of "cognitive convergence" reduced the meaning making to measures of traditional individual mental phenomena—externally influenced by computer images and internally involving increasingly similar mental representations of those images in the heads of the students.

To grasp the significance of this distinction between cognitive and post-cognitive, consider the schematic history in Figure 1 of a strand within Western philosophy and social theory that contributed to the theoretical foundation of this paradigm shift.

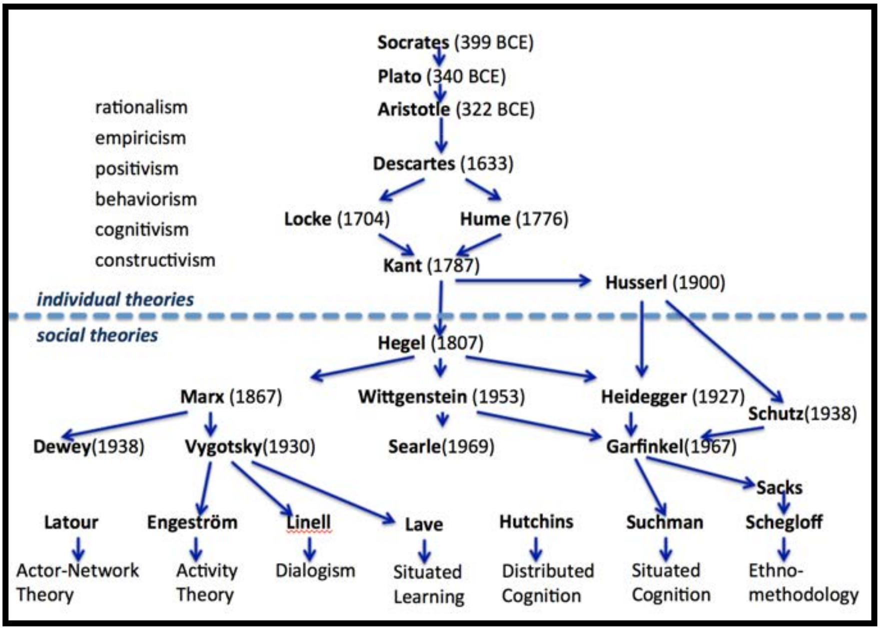

Figure 1. The evolution from individualistic to social theories in philosophy and social science. A major paradigm shift in theory occurred two centuries ago but has still not affected most CSCL and LS analyses.

Philosophy began with the classic Greeks locating knowledge in eternal ideas, rather than in the social norms of the *polis* or the traditions of mythology. Descartes relocated these ideas in the individual mind, and thereby created the epistemological problem: how can ideas in the mind correspond to valid knowledge of the non-mental world? Locke and Hume gave opposing views in response to Descartes, emphasizing individual human reason or individual human experience. Various mixtures of these philosophies motivated scientific paradigms of rationalism, empiricism, positivism and behaviorism—with their objectivist methods. Kant overcame the conflict between rationalism and empiricism by arguing that the human mind constructs what it can know of the world by structuring sense perception with categories of space, time and causality. Thus, Kant provided the philosophic basis for the paradigms of constructivism and cognitivism: People construct knowledge, so an analysis of human behavior and learning must take into account the role of cognition in making sense of the world (vs. positivism and behaviorism in the human sciences).

Note that up to this point, human nature and human cognition were posited as based in the individual person, as fully determined from birth ahistorically or universally—not dependent on one's biography, growth-related processes or social context. Remember that the views that minds develop (Freud), that social relations transform (Marx) or that humanity evolves (Darwin) all came after Hegel—inspired by his dynamic philosophy. The outmoded pre-Hegelian, ahistorical view survives in our culture as common sense and as a pervasive ideology of individualism. It also survives in the empiricist and rationalist assumptions about science and research methodology, which persist in positivist notions of objectivity and reductionism to individual cognition.

Hegel (1807/1967) argues that human consciousness emerges through productive activity in the social and physical world: Individuals are formed as such (i.e., as self-conscious individuals) through the interaction with each other and with artifacts (tools and products of work) in the world. Hegel describes the emergence of self-consciousness from within the process of mutual recognition of self and other.

Marx (1867/1976) builds on this analysis of social interaction. He situates Hegel's idealist analysis in the historical context of capitalism. For Marx, individuals in capitalist society are analyzed as results of their interactions as wage laborers, owners of the means of production or consumers of commodities. The "cell form" of social analysis is the interaction between worker and owner that produces artifacts for the market. Marx critiques the traditional notion of the abstract individual as an ideology that obscures concrete human reality as fundamentally social.

In the cognitive paradigm, one assumes that an interaction such as takes place in a CSCL setting can be analyzed in terms of individuals, who can be characterized independently of the interaction context, for instance by characterizing their mental states and internally stored knowledge. The sense making that takes place is attributed to the individuals, who then may compare their private understandings, personal opinions or mental representations. By contrast, in the post-cognitive theories listed across the bottom of Figure 1, interaction is primary. For instance, Linell (2009) describes his post-cognitive dialogical approach as follows:

> In the analysis of sense-making as it occurs in communication and interventions into the world, as well as in solo thinking or the reading of texts, etc., we must start out from the encounters, interactions, events etc. as the basic phenomena; they are primary, not secondary or derived. This idea makes dialogism different from mainstream psychology, which is based on the assumption—self-evident for its adherents—that individuals are there first, and then they sometimes interact with other individuals. Interaction for them is "external," that is, of a secondary nature. Dialogists, by contrast, assume that individuals have become what they are in and through interaction.

Toward a Post-cognitive Educational Paradigm

A related set of attempts to propose contemporary approaches to education, sociology and psychology embody new paradigms of research in keeping with the post-cognitive philosophical paradigm. Some of them are included in Figure 1. They focus methodologically on group interaction and study dynamic processes rather than just pre and post-test learning outcomes of individuals. Most of them are inspired by Vygotsky or, more generally by Marx, Heidegger and Wittgenstein. They include Bruner (1990), Cole (1996), Engeström (1987), Garfinkel (1967) and their colleagues or followers, each of whom emphasizes different aspects of the paradigm.

Vygotsky adopts Marx's ontology: the primary unit of analysis is the interaction among people situated in social relationships and mediated by artifacts. Artifacts are both physically present in the world and meaningful to people (overcoming the physical vs. mental cleavage of Descartes). Vygotsky's notion of artifact includes both tools and language. Their meaning is not projected from individual minds, but is intersubjectively emergent from social interactions, as in the dialectical analyses of Hegel and Marx. Consider Vygotsky's programmatic attempt to show how the individual mind is grounded in activity within the physical and social world. His description of the genesis of the pointing gesture illustrates a typical early experience of meaning for a small child; it shows how the meaning of this artifact (the pointing gesture) is created in the intersubjective world and only subsequently incorporated (internalized) in the child's personal sense-making repertoire:

> We call the internal reconstruction of an external operation *internalization*. A good example of this process may be found in the development of pointing. Initially, this gesture is nothing more than an unsuccessful attempt to grasp something, a movement aimed at a certain object, which designates forthcoming activity…. When the mother comes to the child's aid and realizes this movement indicates something, the situation changes fundamentally. Pointing becomes a gesture for others. The child's unsuccessful attempt engenders a reaction not from the object he seeks but from another person. Consequently, *the primary meaning* of that unsuccessful grasping movement *is established by others*…. The grasping movement changes to the act of pointing. As a result of this change, the movement itself is then physically simplified, and what results is the form of pointing that we may call a true gesture. (Vygotsky, 1930/1978, p. 56, italics added).

Here we see the *genesis of the meaning* of a pointing gesture. The meaning of the pointing gesture is not some mental schema stored in the mind of an interpreting subject, but an intersubjectively understood practice originally generated through interaction and subsequently repeatedly applied during interaction as a resource for maintaining joint attention of group members. The recognized, practical and formalized gesture becomes an artifact: it embodies meaning in the physical world. The meaning is a reference to that which is pointed at. The baby intended some object; the mother recognized that the baby intended that object; the baby recognized that the mother recognized this. The multiple mutual recognition entails that the baby and the mother recognize each other as people who can have intentions and who can recognize intentions of other people. This is a first glimmer of self-consciousness, in which the baby becomes conscious of his own and other people's intentionality. (Of course, the baby cannot represent or express this self-consciousness in any mental, verbal or conceptual sense, but only adopt it behaviorally.)

The key point for us here is not so much the birth of intentionality, social recognition or self-consciousness. It is the analysis of an artifact, such as the pointing gesture, a ubiquitous form of reference or deixis. In the origin of this gesture, we already see the basis for intersubjective, *shared understanding* of an artifact's meaning. The subsequent usage of this pointing gesture is premised upon the mutual recognition of an underlying relationship of shared attention, which emerged within the mother-child interaction.

The view of shared intention as co-constructed in the world stands in sharp contrast to the rationalist assumption that individuals first have private personal intentions—as though produced by logical calculations of self-interest by a homunculus in their heads—which they subsequently express in speech or action.

Marx, Wittgenstein and Heidegger—and their successors—soundly reject this cognitive assumption (see, e.g., Dennett, 1991; Dreyfus, 1992; Suchman, 2007). Heidegger (1927/1996), for instance, replaces Descartes' dichotomy of mental and physical with a philosophy of human being-there-together-in-the-world. One's comportment in the world precedes one's reflection upon objects in the world. People understand the shared world through their involvement with and their care for the world with other people who also inhabit that world, not initially through mental representations and logical plans. Human involvement is fundamentally processual or temporal: we aim at our projects for the future, based on having been thrown by our social past into our shared situation in the present.

In their seminal post-cognitive analysis of agency, drawing on contemporary philosophy and social science, Emirbayer and Mische (1998, p. 962) conceptualize agency in Heideggerian temporal terms. Applied to the group unit of analysis as "group agency," their post-cognitive concept could inform CSCL analysis (as in Charles & Shumar, 2009; Damsa, 2014). It is important to reconsider the notion of agency (and causality)—as Latour (1990; 1992) does by extending it to other people and artifacts in networks of actors. The traditional, mechanistic conception of agency contributes to the difficulty of overcoming cognitive habits of thought. A post-cognitive paradigm could include group cognition, collective intentionality and group agency.

The Need for a Post-cognitive CSCL Paradigm

A paradigm shift can be motivated by anomalies in established theories (Kuhn, 1972; Lakatos, 1976). Anomalies are findings that cannot be easily accounted for in a prevailing theory. They suggest the need for a change in theory. Consider two anomalies in the cognitive paradigm of measuring learning outcomes of individual minds: one from the research of Vygotsky and one from recent CSCL research.

In Vygotsky's well known discussion of the "zone of proximal development," he cites a study in which children "could do only under guidance, in collaboration and in groups at the age of three-to-five years what they could do independently when they reached the age of five-to-seven years" (1930/1978, pp. 86f). CSCL can be seen precisely as such an effort to stimulate students within their zones of proximal development—on tasks they cannot yet master individually but are close to being ready to learn—under guidance, in collaboration and in groups. If the desired results of this do not show up as learning outcomes measurable in individuals (outside of their group context) for several years, then the key effect will be systematically missed by traditional methods of testing individuals. The failure of the cognitive paradigm of instructional research to account for processes in the zone of proximal development—so central to learning—should be considered an anomaly, suggesting the need for a paradigm shift.

In his less quoted section on "Problems of Method," Vygotsky (1930/1978, pp. 58-75) called for a new paradigm of educational research almost a century ago. Arguing that one cannot simply look at visible post-test results of an experiment, he proposed a method of "double stimulation" where a child is confronted by both an object to work on and an artifact to mediate that work. Vygotsky does not call for a controlled experiment that compares learning outcomes with and without the furnished artifact. "The experiment is equally valid," he points out, "if, instead of giving the children artificial means, the experimenter waits until they spontaneously apply some new auxiliary method or symbol that they then incorporate into their operations." Taking this approach in a collaborative setting requires an attention to the children's interaction and the sense making that is involved in creative, unanticipated collaborative accomplishments. It involves understanding the unique trajectories of different *groups*, which cannot be statistically aggregated or sorted into standardized categories. This suggests the need to analyze interaction at the group level, rather than just the individual as the cognitive subject.

Relatedly, a number of CSCL studies have repeatedly documented "productive failure" (Barron, 2003; Kapur & Kinzer, 2009; Pathak, Kim, Jacobson & Zhang, 2011; Schwartz, 1995). This is one of the most intriguing findings of CSCL to date. However, it has so far been analyzed in terms of individual student learning outcomes, rather than group practices within zones of proximal development. When a number of small groups of students work on a challenging problem, the groups sometimes fall into two categories: (a) Groups that fail to solve the immediate problem but excel at solving future related problems. (b) Groups that succeed at solving the immediate problem but are less successful than the first groups at solving subsequent related problems. The robust and repeatable result of these experiments presents an anomaly for traditional educational theory. One could speculate that in the "failure" groups students are further developing their zone of proximal development or that these groups are co-constructing helpful new meanings, whereas the groups that solve the immediate problems are focused on efficiently applying their existing skills. The analysis of group processes effecting outcomes this way would require a post-cognitive perspective, focused on the cognitive accomplishment of the group as such.

A CSCL Researcher's Agenda

As an example of a CSCL research project conducted within a post-cognitive paradigm, I describe my own work during the past decade and a half. It is post-cognitive in that it analyzes the group processes that constitute collaborative learning in a computer-mediated setting. It neither defines learning in terms of outcomes nor interprets utterances in terms of mental phenomena. Without denying the reality of either individual consciousness or societal practices, it nevertheless focuses on the temporal sequentiality of small-group interaction.

The Virtual Math Teams (VMT) Project has been a collaborative effort with researchers from the Math Forum, Drexel University and Rutgers University at Newark, as well as with visiting scientists and colleagues abroad. The project is extensively documented in four books (Stahl, 2006; 2009; 2013c; 2016), nine doctoral dissertations and many other presentations (http://gerrystahl.net/vmt/pubs.html), including in a number of Investigations in this volume.

VMT is a design-based research (DBR) project, intended to develop theory and research methodology through exploring and designing technology and pedagogy for supporting online collaborative learning of mathematics. As a research prototype, the VMT environment has been used in over a thousand student-hours at the Math Forum (http://mathforum.org), as well as independently by researchers in Turkey, Israel, Singapore, Brazil and New Jersey. The final version of VMT's software and curriculum features a collaborative version of GeoGebra (http://geogebra.org), a popular dynamic-mathematics application. The VMT Project converted the GeoGebra app to a multi-user approach, so that actions by one student are synchronously shared with others in the group.

In a typical session, three to five middle-school or high-school students collaborate synchronously online for about an hour. Often, the same group will work on a series of challenging problems during five to ten weekly sessions in an after-school or in-class setting organized by a teacher who has completed the Math Forum's teacher-professional-development program associated with VMT. Students interact through text chat and GeoGebra actions (see Figure 2). A VMT Replayer shows researchers or teachers what everyone in the group saw and allows a viewer to step through an entire session with the controls added across the bottom. In this screenshot, the group is in the midst of constructing a solution to a challenging problem of inscribed polygons, repeatedly used in the VMT Project. (Interestingly, the researchers had not previously seen or thought of the particular solution collaboratively developed by the group in Figure 2.)

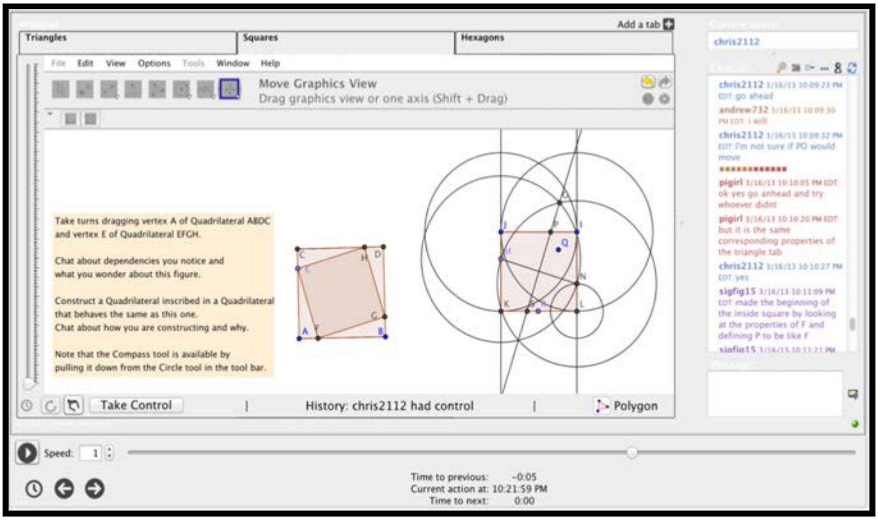

Figure 2. The VMT Replayer, with VCR-type controls below the VMT environment.

The VMT environment is instrumented to provide the data necessary for analysis of group processes. In order to track a group's meaning making, one must have a complete record of all group interaction. Otherwise, one does not know if unrecorded events contributed in unknown ways to the shared understanding. This requirement involves two aspects: (a) controlling the interaction so that no group communication takes place outside of the recorded setting and (b) recording the interaction in a complete, detailed and undistorted manner. Technologies of recording data can make possible new paradigms of research. For instance, conversation analysis only came into existence with the tape-recorder for capturing and replaying speech.

Recording group collaboration in a face-to-face classroom is "messy" and often impractical: There is so much noise that clear speech capture is difficult; transcription is laborious; and non-verbal communication through action and gesture is impossible to capture completely. Traditional analyses generally proceed by coding and counting. Recordings of speech utterances are transcribed as sentences. Then sentences attributed to the individual speaker are categorized according to some standardized schema. The number of sentences falling into each category is compared for different individuals, groups or experimental conditions. In the process of recording, transcription and aggregation, many researcher interpretations are introduced (Suchman & Trigg, 1991) and any sense of temporal or dialogical process is lost. In particular, given an a priori coding scheme, it is unlikely that any unforeseen and surprising results (like causes of productive failure or creative group moves) will still be identifiable. Due to established methodologies and other research habits, there have been practical barriers to LS making the paradigm shift to studying group interaction. CSCL can overcome these barriers because the computer-mediated collaborative setting makes problem-solving processes observable and automatically fully recordable.

Since students collaborate online in the VMT environment, all communication and action are mediated by the VMT technology. It is therefore possible to capture a complete record of everything that is visible to the student group itself. The same technology is used to replay the session for researchers, who can then slow it down or proceed posting-by-posting and action-by-action, viewing exactly what the students in the group all viewed (as in Figure 2). In addition, a convenient summary log is automatically generated in spreadsheet formats (see Figure 3). The spreadsheet automatically logs all text chat postings and dynamic-geometry actions of each student. It can easily be filtered by event type or re-formatted for log excerpts in publications. Columns for each student give a visual impression of the interactional flow.

	A	B	C	D	E	F	G	H	I	J
1	Line	Date	Start Time	Post Time	Duration	EventType	andrew732	pigirl	sigfig15	chris2112
742		3/16/13		23:49.6	0:00:01	Geogebra:Squares			tool changed to Move Graphics View	
743		3/16/13		23:49.7	0:00:00	Geogebra:Squares			release control	
744	155	3/16/13	23:50.9	23:52.3	0:00:01	chat			it works	
745	156	3/16/13	23:56.7	23:57.3	0:00:00	chat		nice		
746	157	3/16/13	23:54.2	23:58.7	0:00:04	chat	wow, great			
747	158	3/16/13	23:59.3	24:08.1	0:00:08	chat		can you simply explain how you made it		
748	159	3/16/13	24:14.3	24:26.6	0:00:12	chat				If you notice on the example, Length CE is equal to GB
749	160	3/16/13	24:27.6	24:51.3	0:00:23	chat				so you can use to compass tool to make a circle around B with point G on it such that as you move E, G moves correspondingly
750	161	3/16/13	24:53.5	24:59.4	0:00:05	chat				Then make a line going through EG
751	162	3/16/13	24:59.6	25:08.0	0:00:08	chat				and a circle with cenger E point G
752	163	3/16/13	25:08.2	25:13.3	0:00:05	chat				and a circle with cetner G point E
753	164	3/16/13	25:13.5	25:32.7	0:00:19	chat				so that a line perpendicular to EG and through the intercepts of the circles is the midpoint line
754		3/16/13	25:33.5	25:34.3	0:00:00	awareness				[fully erased the chat message]
755	165	3/16/13	25:34.5	25:35.3	0:00:00	chat				of EG
756	166	3/16/13	25:35.5	25:58.1	0:00:22	chat				and whatever points on that line intercept the outside sqaure would then make up the other two points of the inner square
757	167	3/16/13	26:03.4	26:05.8	0:00:02	chat			okay good	
758	168	3/16/13	26:34.4	26:36.5	0:00:02	chat		that makes sense		
759	169	3/16/13		26:55.2	0:00:18	system			Now viewing tab Hexagons	
760	170	3/16/13	26:49.0	26:55.6	0:00:06	chat		can we move to "hexagons"?		
761	171	3/16/13		27:00.0	0:00:04	system				Now viewing tab Hexagons
762	172	3/16/13		27:06.4	0:00:06	system		Now viewing tab Hexagons		
763	173	3/16/13		27:46.6	0:00:40	system	Now viewing tab Hexagons			
764	174	3/16/13	27:57.8	27:59.8	0:00:01	chat			ill drag	
765		3/16/13		28:02.2	0:00:02	Geogebra:Hexagons			take control	
766		3/16/13		28:02.3	0:00:00	Geogebra:Hexagons			tool changed to Move	
767		3/16/13		28:10.3	0:00:08	Geogebra:Hexagons			updated Point A	
768		3/16/13		28:30.6	0:00:20	Geogebra:Hexagons			updated Point G	
769		3/16/13		28:32.3	0:00:01	Geogebra:Hexagons			tool changed to Move Graphics View	

Figure 3. A spreadsheet of a segment of VMT interaction among four students.

The text chat is reproduced just as posted by the students, and the GeoGebra actions are listed in detail. The data of the actual interaction is available, and the process of interpretation begins with the analysis, not with the data capture and transcription. Researchers can share the replayer files and spreadsheets, so that others can check any analytic descriptions for plausibility.

The VMT system's ability to generate data, which (a) provides an automatic record of the actual interaction and (b) documents the complete group interaction, has made it useful to a number of researchers. Using this data source, they have been able to analyze group processes, rather than just individual actions or outcomes. Here are some examples of theoretical insights and methodological innovations emerging from the VMT Project before the integration of GeoGebra, when a generic shared whiteboard was used for mathematical figures:

- Sarmiento and Stahl (2008) extended the notion by Teasley and Roschelle (1993) of a Joint Problem Space, observing how students co-construct such a shared conceptualization and how it incorporates a temporal structure, integrating past collaborative sense-making results into current discussions aimed at a projected future problem solution.
- Çakir, Zemel and Stahl (2009) [Investigation 12] observed how a student group integrated their visual/graphical reasoning, numeric/symbolic expression and mathematical discourse in their problem-solving work within the VMT chat and whiteboard media—moving successively from one discourse to another.
- Zhou, Zemel and Stahl (2008) looked at the important role of questioning as a common driving force in collaborative interaction, eliciting responses and providing a guiding group agency.
- Zemel, Çakir and Stahl (2009) analyzed "reading's work" as a contribution to the analog of conversational turn taking as it is materialized in online text chat.
- Zemel and Koschmann (2013) [Investigation 5] studied how deixis and linguistic reference work within interactions in the VMT environment.
- Koschmann, Stahl and Zemel (2009) examined the nature of several key group practices in VMT collaboration.
- Wee and Looi (2009) investigated pivotal moments in group processes of mathematical knowledge building in VMT chats.
- Medina and Suthers (2013); Medina, Suthers and Vatrapu (2009) probed the nature of representational practices in a series of one VMT group's sessions, observing how practices primarily contributed by one student are later associated with the other students, as they become adopted as group practices.
- Trausan-Matu, Dascalu and Rebedea (2014) analyzed the polyphonic nature of VMT chats, graphing the intertwining of dialogical voices in a number of groups.

More generally, the concept of group cognition emerged from the early analyses of the VMT Project, as did the importance of interaction analysis (Stahl, 2006). In turn, the theory of group cognition clarified the importance of incorporating a domain-oriented shared workspace like GeoGebra and the need to record all interaction for analysis. These issues were reflected upon in (Stahl, 2009). The co-evolution of theory, research and design characteristic of design-based research (DBR) was highlighted in (Stahl, 2013c), with chapters on eleven aspects of the Project, including theory, methodology, pedagogy, technology, subject domain, history and philosophy.

The idea of focusing on the group unit of analysis or group cognition does not exclude analyses at either the individual or the community units of analysis. There are important and different phenomena and processes at each of these (and other) levels. In fact, it is often most fruitful to analyze cognition on multiple levels and to see how the processes at the different levels work together. However, the simultaneous and integrated study across levels is a current challenge for CSCL [Investigation 2]. A variety of *interactional resources* are typically at work bridging the levels (Stahl, 2013a; 2013b; Stahl & Öner, 2013). Since incorporating GeoGebra into VMT, our research has included designing sequences of curricular resources to guide collaborative exploration and bridge levels by tying individual and group knowledge to established knowledge of the mathematics community (Stahl, 2012; 2015).

In VMT case studies, topics in mathematical combinatorics or dynamic geometry centrally figure as interactional resources that bring together individual, small-group and community cognitive processes. *Sequentiality*, *co-attention* and *shared understanding* have emerged from these studies as fundamental theoretical categories for understanding and studying collaborative learning [Investigation 19]. By observing group interaction in VMT, we can see how student groups enact these mechanisms and thereby integrate individuals into groups, adopting community practices as *group practices*. For instance:

- In (Stahl, 2011) [Investigation 23], two students solve a high-school math problem that has stumped them for some time. The problem-solving steps that the dyad goes through as a team are strikingly analogous to how proficient math students solve problems individually. In the discourse captured in this case, one can see how the *group* integrates contributions from the two *individual* participants to accomplish a task in accordance with *community* standards of practice—illustrating the productive interplay of cognitive levels. A sequence of ten discourse moves (similar to extended adjacency pairs in Schegloff, 2007) by the group details their *sequential organization* of the problem [Investigations 23 and 25].

- In (Stahl, Zhou, Çakir & Sarmiento-Klapper, 2011) [Investigation 17], three students develop techniques for helping each other to see what they are seeing in the diagram they have drawn for a math problem. This *persistent co-attention* to a shared object of analysis allows the team to solve their problem as a group.
- Similarly in, (Çakir & Stahl, 2013) the students are able to work together because they effectively manage their *shared understanding* of the problem.
- (Stahl, 2016) follows a group of three young girls longitudinally through eight hour-long sessions in the VMT chat room with a multi-user version of GeoGebra. It describes the display of mathematical reasoning by the team discussing the dependencies of a series of dynamic-geometry figures. By analyzing the network of mutual responses, it tracks the intersubjective meaning-making process and observes how the team develops its abilities by adopting *group practices* of collaboration, mathematical discourse and dynamic geometry.

When a group enters the VMT environment, it is presented with a challenging math problem, which is designed to guide the group interaction in an academically productive direction [Investigation 21]. The problem acts as a problem-solving resource for the group. The group must interpret the problem statement, elaborate the way in which they want to conceive the problem and determine how to proceed [Investigation 5].

A math problem, for instance, can serve as an effective interactional resource for bridging across cognitive levels. Typically, it introduces content—definitions, elements, procedures, principles, practices, proposals, theorems, questions—from the *cultural* traditions of mathematics and from school curriculum. In so doing, it recalls or stimulates *individual* cognitive responses—memories, skills, knowledge, calculations, deductions. It is then up to the *group* interaction to bring these together, to organize the individual contributions as they unfold in the on-going interaction in order to achieve the goals called for by the community, institutional, disciplinary and historical sources. In this way, the group interaction may play a central role in the multi-level cognition, interpreting, enacting and integrating elements from the other levels, producing a unified cognitive result and thereby providing a model for future community practice or individual skill.

Group cognition is not the same as individual cognition. Certainly, it relies upon individual cognition to make essential contributions; however, one cannot say that all of the cognition is reducible to the individual unit, because the work of assembling the high-level argumentative structure typically occurs at the group unit of analysis. Surely, putting together problem-solving arguments (incorporating planning, synthesis, deduction) must be considered a cognitive activity as much as the memory or computation that goes into making the detailed contributions to individual steps. This group cognition may be considered to involve students in their zone of proximal development, with the expectation that they may later be able to conduct such extended problem-solving argumentation individually, based on their group experiences.

In addition, the individual discourse contributions are not meaningfully separable from the group processes. They are largely responses to what has gone before in the group interaction. These contributions are expressions that would not have occurred without the preceding opening for them and the elicitation of them by the group process. Many of the contributions are largely reactions at the group level, which reference and inter-relate resources available in the discourse context more than they introduce new elements from the personal perspective and individual background of the actor. They are also prompts for reactions by others. The important knowledge-building achievement is emergent in this give-and-take at the group level, rather than a simple collection of expressions of isolated individual cognitive accomplishments.

Note that the emergence of group cognition is quite different from the emergence of complexity from the non-linear interaction of simple rules in chaos theory; group cognition emerges primarily through the intertwining of subtle linguistic phenomena of indexicality and sedimented shared meaning inherent in sequentially organized utterances of multiple voices [Investigation 2].

Of course, coherent and impressive examples of group cognition—such as solving a math problem that the group members would not have been able to solve on their own—do not occur every time that people come together in conversation. In fact, the research field of CSCL has repeatedly documented that desirable forms of collaborative knowledge building are disappointingly rare. The VMT research summarized above indicates some reasons for this. First, it is difficult to set up a group interaction where everything relevant to the cognition at the group level of analysis is captured in a form adequate for detailed analysis. It took years to iteratively design, develop and deploy the VMT group sessions to successfully generate adequate data of successful group cognition. Secondly, the group interaction must be directed and guided to focus on an appropriate cognitive task. Certain challenging math problems, carefully presented, seem to provide effective interactional resources for stimulating interesting episodes of group cognition. Additionally, groups must work consistently to ensure the presence of certain preconditions of effective group cognition. They must persist in building *longer sequences* of responses to each other, they must maintain continuous *co-attention* to a shared focus of discussion, and they must build and sustain a *shared understanding* of the topic of conversation [Investigation 19, Investigation 23].

The VMT studies listed above are focused on the small-group unit of analysis. This is consistent with other contemporary attempts to shift away from an exclusive concern with individual cognition, for instance in actor-network theory, ethnomethodology, distributed cognition and activity theory. In the VMT project, most analysis has focused on the under-researched unit of the small group (Stahl, 2006; 2009). However, recent work on VMT looks at the interactions among the individual, small-group and community units of analysis (Stahl, 2013a; 2013c). This has the potential of bridging to other analytic approaches in LS and CSCL, although it raises new methodological issues about studying the interrelationships of the different levels.

The Foundational Relationship of CSCL to the Learning Sciences

The post-cognitive educational paradigm assigns an analytic priority to group cognition, as the level at which fundamental processes of learning take place. Applying this to the study of learning is motivated by Vygotsky's developmental principle:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (*interpsychological*), and then *inside* the child (*intrapsychological*). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher [human mental] functions originate as actual relations [interactions] between human individuals. (Vygotsky, 1930/1978, p. 57)

Cognitive phenomena such as specific forms of learning develop first in group interaction and then only subsequently—through complex and extended transformations—emerge as individual skills or outcomes. In this sense, collaborative learning may be considered not just an optional and rare mode of instruction, but rather a foundation of all learning. Group cognition is a basis of human cognition: individual, small-group and community.

Already in the Introduction to *Group Cognition*, it was proposed that:

> Small groups are the engines of knowledge building. The knowing that groups build up in manifold forms is what becomes internalized by their members as individual learning and externalized in their communities as certifiable knowledge. At least, that is a central premise of this book. (Stahl, 2006, p.16)

Accordingly, the analysis of group processes is foundational for the study of learning because these processes mediate between individual and community learning. One can argue, as Vygotsky and his followers often do, that group cognition underlies and grounds all cognition. Human thinking—including planning, conceptualizing, narrating, analyzing and problem solving—involves language to identify, distinguish and express itself. Vygotsky studied the development of children's thinking and speaking, concluding that spoken language (learned in dialog within family groupings) gradually over years led to silent self-talk, which then eventually morphed into mental thought. Accordingly, thinking individuals evolved (both as a species and in each case) from group participants, adopting as their own many practices that were acquired in interaction with others. While it is difficult to know about the early origins of cognition, the priority of group cognition seems apparent in the kinds of learning targeted by most CSCL research projects.

While the existence of many small groups may be ephemeral and teams may form for limited time spans, groups are typically the units within which cognitive advances are made, particularly the kinds of challenging, ill-structured, complex forms of learning aimed at by CSCL interventions. For instance, the collaborative learning of dynamic geometry by the Cereal Team and the solving of tricky problems of mathematical combinatorics by other VMT student groups were cognitive accomplishments that the individual team members often would not have been able to achieve on their own. The analysis of their interaction documented that the individuals ended up with deeper understandings as a result of their participation in the team's collaborative learning.

Other CSCL projects explore similar results in fields of scientific conceptualization, ecological interdependencies or argumentation about controversial social issues. In such realms of learning, the challenge is beyond the ability of most individual students, but can be approached with well-designed CSCL resources, tools and supports, as well as the interaction of multiple student perspectives and skills. Experiences of collaborative learning can direct the cognition of participants as well as provide models of reflection for subsequent individual thinking.

For a different kind of example, consider how knowledge building took place within the communities of CSCL and LS themselves, as illustrated in the discussion of their history at the start of this Investigation. A sociocultural description of how they changed might argue that individual researchers apprenticed to the research communities and gradually adopted the prevalent social practices of data analysis and publication (Lave & Wenger, 1991). As the researchers innovated, the communities were modified in turn. However, theories at the level of the individual and the community run into the problem of structuration (Giddens, 1984)—how these two levels influence and respond to each other. If one considers what took place at the CSCL 2003 conference, one sees that community-level institutions like ISLS and *ijCSCL* were structured through processes of group interaction. There were tensions between political groupings representing different constituencies, and each of these groupings engaged in internal and external interaction. Ad hoc groups sprung up and committees were established to define the community structures. As people interacted within small groups in this context, they changed their individual identities and modified social practices of their field. Through such mechanisms, the group processes mediated and transformed both the individual and community processes, helping to define individuals' identities and institutional practices. In this sense, as the engines of structuration, the group unit of analysis is foundational for the study of change and learning at multiple levels.

The group cognition paradigm argues for scientific study at the small-group unit of analysis (Stahl, 2010) in addition to the individual and community units. Too often, traditional educational researchers reduce group-level phenomena either to individual-psychological constructs or to societal institutions and practices. But, as we have documented in VMT case studies, there are often important practices and processes taking place at the small-group unit of analysis that are not reducible to the mental behaviors of an individual or to the institutions or established practices of a community.

The term "group cognition" does not mean there is some kind of "group mind" at work or anything other than the interaction of the students in a group. Rather, it means that the analysis of cognitive achievements may be most appropriately conducted at the group unit of analysis—in the VMT context, in terms of the collaboration recorded in the interplay of the text-posting and geometric-drawing actions shared by the group.

For instance, a specific student utterance or text posting is not to be ascribed to hypothesized mental processes of the student (what one assumes the student may be thinking, feeling or trying to accomplish). Rather, it is taken as a situated response to what came before (such as actions or utterances by other students) and as an invitation to subsequent group action [Investigation 23]. That is, it is analyzed as a dialogical move situated in ongoing group interaction. Methodologically, a study of group cognition aims to understand interactional responses within the group rather than cognitive motives of individuals' actions.

CSCL could fully embrace the group cognition paradigm, as illustrated in the VMT Project. This would be appropriate since CSCL was created in response to the potential of networked computers to bring people together into functional groups to collaborate. More specifically, CSCL is oriented to the potential of bringing students together to learn collaboratively. This effort should focus on the group-level interactions in which problem solving, knowledge building and other forms of collaborative learning take place.

CSCL is not the science of some existing, objectively observable phenomenon, like physics or psychometrics are often assumed to be. It is the search for a new form of learning—taking advantage of technologies that are yet to be developed and group processes that are difficult to observe and have largely gone unnoticed. Therefore, it cannot be studied in the manner of a summative assessment, by comparing measurable learning outcomes. It is more of a design science, using design-based research to transform "existing situations into preferred ones" (Simon, 1981). In DBR, the vision of CSCL itself is a product of research.

To guide redesign in iterative research projects, it is not sufficient to "predict" the percentage increase in outcomes that is attributable to a particular, currently available technological condition. What is needed is insight into how groups of students in realistic situations may actually make sense of and take advantage of possible technologies, as well as what barriers groups may encounter in trying to use them. This means looking at how groups of students actually interact with various technological artifacts and observing their intersubjective meaning-making processes, their enacting of the technologies and their collaborative problem solving as mediated by the technologies.

Of course, not all groups of students will act the same way under similar conditions. Groups are unique—with students at different zones of proximal development for different skills and with interactions highly situated within un-reproducible discourse trajectories. Therefore, statistical generalization is not a relevant goal in such research. What one seeks, rather, is a detailed understanding of the practices that are actually found to be at work in observed cases [Investigation 16].

According to ethnomethodology, communities necessarily use shared practices or member methods (Garfinkel & Sacks, 1970). Otherwise, intersubjective sense making would not be possible—any more than communication would without a common language. Therefore, the practices that one observes in a single case may be representative of widely used practices. Researchers familiar with a domain—such as experienced math educators—can often tell what seems like a typical group behavior within that educational arena.

LS and CSCL have made significant progress in recent decades, as documented by (Evans, Packer & Sawyer, 2016; Sawyer, 2014). However, it may be timely to pursue a new research paradigm explicitly—one in which CSCL plays a foundational role. For the CSCL and LS research communities to make the major paradigm shift advocated here will involve significant re-tooling and adoption of new methods. It will also require increased collaboration with colleagues in social science who are more familiar with analyzing interaction and language and with formulating rigorous descriptive accounts of group-interactional processes. Fortunately, the requisite technological recording capabilities are available and the evocative research questions are at hand.

The settings studied by LS and CSCL today are complex. Many diverse studies can contribute to an understanding of the learning taking place. Such studies can pose a broad spectrum of research questions, each with its own theoretical framing and methodological approach. Certainly, traditional quantitative and qualitative analyses at the individual unit of analysis can provide important parts of the picture, as can considerations of social practices and community participation. However, it is also necessary to consider the temporal processes of group interaction, through which the individual and the community are often mediated and through which learning takes place as a process, not just as an outcome.

Too often, CSCL is treated as a secondary niche within LS. This is probably because commonsensical attitudes assume that individual learning is the primary goal, and that collaborative learning is just one of many possible approaches. However, if one understands that individual learning is actually secondary and dependent upon group cognition, then CSCL should be viewed as foundational to human learning.

In his reconsideration of the CSCL paradigm, Koschmann (2001, p. 21) concluded that “we have yet to develop a consensus within the CSCL community with regard to what it means to learn and how to study the process.” I have argued here that a paradigm-shaping research question for LS would *treat learning as essentially an intersubjective, interactional process*, and would *study it by investigating the dynamic developmental processes through which individual, small-group and community cognitive practices emerge*.

Within CSCL, the seminal analysis by Teasley and Roschelle (1993) pursued a specific version of this question by asking how dyads of students created a joint problem space around a computer representation of velocity and acceleration. The VMT Project followed a different approach to the same question by exploring how students co-construct interactional, group-cognitive and mathematical practices in small online groups mediated by collaborative-dynamic-geometry tasks and tools (Stahl, 2013c; 2016). Taking approaches like these, research in a post-cognitive CSCL paradigm could lead research in LS by working out the interactional foundations of all learning through taking advantage of technologies, pedagogies and understandings afforded by CSCL.

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Investigation 16. Adopting Group Practices

Gerry Stahl

Abstract. The analysis of *group practices* can make visible the work of novices learning how to inquire in science or mathematics. These ubiquitous practices are invisibly taken for granted by adults but can be observed and rigorously studied in adequate traces of online collaborative learning. Such an approach contrasts with traditional pre/post comparisons that miss sequential interactional processes or that reduce group phenomena to individual or social factors. The analysis of the enactment of practices by small groups in CSCL contexts can systematically inform the design, testing and refinement of collaborative-learning software, curriculum, pedagogy and theory. The research of the Virtual Math Teams Project resulted in a new way of viewing collaborative learning. According to this view, CSCL can be re-conceptualized as the design of technology to foster the adoption of group practices by student teams.

Keywords. Group practice, CSCL theory, CSCL methodology, design-based research

A new method for CSCL

As a CSCL researcher, participant in all previous CSCL conferences and former Editor of *ijCSCL*, I have consistently observed that almost all published studies of collaborative learning reduce it either to individual mental representations or to cultural social practices; the small-group unit of analysis is under-researched (Stahl, 2006a). This may be partially because it is difficult to find data that adequately documents collaborative learning by reliably capturing all the discourse, gestures and artifacts that enter into *group* (i.e., small-group, collaborative) knowledge-building processes (Stahl, 2013a). Furthermore, the methods of traditional educational-psychology research are inadequate for investigating many core CSCL issues because they focus on individual cognition and assume that utterances can be categorized objectively (i.e., without interpretation based on understanding the sequential meaning-making) (Stahl, 2014). My conclusions are controversial within the CSCL research community because they methodologically eschew prevalent cognitivist and positivist assumptions associated with methodological individualism (Stahl, 2016b).

Throughout the history of CSCL as a research community, the methodological tension of the field has been informally described as an opposition of "quantitative" versus "qualitative" approaches (Jeong, Hmelo-Silver & Yu, 2014; Suthers, Dwyer, Medina & Vatrapu, 2010). Habermas refines this with the epistemological distinction between calculative and sense-making orientations, which pervades modern science (Hammond, 2015). Sfard (1998) saw the contrast as (individual) acquisition versus (cultural) participation. Viewed in terms of the unit of analysis, "socio-cognitive" psychology focuses on representations in individual minds and "socio-cultural" anthropology centers on socially defined practices. In other words, there has been no methodological focus on the small-group unit of analysis—precisely where one would naturally expect to observe collaborative learning in CSCL (Stahl, 2009). Some psychologists recognize that individual learning can be influenced by groups (Cress, 2008) and some sociologists show how social practices are enacted and maintained in group interaction (Garfinkel, 1967; Giddens, 1984). However, as noted by Schwarz and Baker (2017), even these studies rarely analyze empirical data of collaborative learning in ways that display processes of small groups building knowledge or acquiring practices.

Quantitative and qualitative methods are appropriate for measuring net changes due to hypothesized independent variables. However, CSCL needs ways to analyze the group processes that bring about such changes and that establish group practices—in order to guide iterative design-based research (DBR). To not only judge the statistical effectiveness of CSCL interventions in promoting collaborative learning, but also to identify specific problems and to suggest innovative functionality during DBR cycles, it is necessary to analyze, understand and theoretically conceptualize temporal group processes as such, in their sequential unfolding (Stahl, 2013b). Moreover—as anyone knows who has successfully implemented CSCL in classrooms or created effective technological support for it—many cycles of trial and refinement of approach as well as of technology design are necessary to develop effective CSCL pedagogy and tools: DBR as an integration of theory, design, research and practice.

Traditional methods provide evidence *that* change has taken place (between a pre and a post state), without describing *how* the change took place, beyond speculation based on assumptions from folk theories of cognition (Stahl, 2016b).

For instance, groundbreaking CSCL studies (Kapur & Kinzer, 2009; Scardamalia & Bereiter, 2014; Schwartz, 1995) indicated that important learning took place at the group level, without being able to show how it happened. A number of researchers have proposed that unique cognitive processes take place at the small-group level (Barron, 2003; Dillenbourg, Baker, Blaye & O'Malley, 1996; Hutchins, 1996; Rogoff, 1995), but they have not collected the required data for a systematic analysis at the group unit.

Sfard (2008) argued that learning math is a matter of acquiring many *practices* that are passed down in the culture of mathematics. Vygotsky laid the basis for collaborative-learning theory by proclaiming that practices are acquired socially (e.g., in dyads or small groups) first, and subsequently adopted by individuals. This notion of practices—when applied at the group unit of analysis—provides a way of conceptualizing regularities of group-cognitive processes. My colleagues and I set out to generate and analyze CSCL interactions in which we could observe such small-group practices emerging.

Studies of group practices

In 2002, we initiated the Virtual Math Teams (VMT) research project at the Math Forum. We gradually developed a prototypical CSCL DBR project, which investigated mathematical education with small teams of students in a custom online collaboration environment. The VMT software was instrumented to collect comprehensive interaction data and to provide it to researchers in useful formats. Later, we developed and incorporated a multi-user version of GeoGebra to provide computer support more specifically for dynamic-geometry math content.

My published books draw successive lessons from the phases of this research: *Group Cognition* (Stahl, 2006a) proposes analyzing knowledge-building phenomenon at the small-group unit of analysis. *Studying VMT* (Stahl, 2009) describes our scientific approaches to supporting and analyzing small-group problem solving in the VMT context. *Translating Euclid* (Stahl, 2013b) discusses the many facets of DBR for supporting constructionist CSCL using VMT.

Finally, *Constructing Dynamic Triangles Together* (Stahl, 2016a) follows utterance-by-utterance a team of students developing mathematical understanding through an eight-hour longitudinal case study, as the group progressively masters collaborative online dynamic geometry. It identifies about sixty "group practices" that the team explicitly, observably enacts. We found that these practices successively contribute to various core aspects of the group's abilities: to collaborate online; to drag, construct and transform dynamic-geometry figures; to use GeoGebra's software tools; to identify and construct geometric dependencies; and to engage in mathematical discourse about their accomplishments.

The notion of *group practices*, as it emerged in this research, provides a foundation for a new way of viewing, analyzing, theorizing and supporting CSCL. Group practices mediate between individual cognition and community culture (Stahl, 2006a, p. 16; 2013b, ch. 8). They can be observed and analyzed in small-group interactions. Thereby, the theory of group practice provides a research-based solution to the obstinate issues of meaning making, intersubjectivity, structuration and connecting levels of learning (Giddens, 1984; Stahl, 2012b; 2016a), while focusing analysis on the small-group unit, as central to collaborative learning. Intersubjective meaning making and knowledge sharing take place via group practices like turn taking, pointing, questioning and drawing. Individuals can transform the group practices into personal skills and mental abilities. Practices can also pass back and forth between small-group and classroom or cultural levels.

The group practices identified in VMT studies are all based on captured interaction data. These practices arose in observable breakdowns or interactional difficulties and were each enacted explicitly in student discourse. Ethnomethodologically speaking, the practices are observably issues for the participants themselves (Stahl, 2012a). They can be identified through close analysis of discourse and other forms of interaction, such as geometric sketching or pointing within the online VMT environment.

The identification of group practices has substantial implications for the design of CSCL software, curriculum, scripting, pedagogy and experimental interventions. In DBR, one develops an initial prototype environment and tries it out with groups of students. Based on observation of problems, the prototype is iteratively re-designed and refined. By observing breakdowns in group interaction and the gradual enactment of new group practices in response to the breakdowns, a designer can identify problem areas and constructive processes that need additional support. The analysis of group practices provides a systematic analysis method for driving CSCL design—something that has long been lacking in CSCL (Tim Koschmann, Marlene Scardamalia, personal communications).

Although DBR is a popular approach to the development of educational software, especially in CSCL, there is little agreement on how to evaluate trials in a way that contributes systematically to re-design. The theory of group cognition proposed that one could make collaborative learning—or group cognition—visible (Stahl, 2006a, Ch. 18), based on the principles of ethnomethodological description (Garfinkel, 1967). This is because meaning making is an intersubjective or small-group process, requiring group members to make their contributions visible to each other, and therefore also to researchers (Stahl, 2006a, Ch. 16). As the editor's introduction to an important book on ethnomethodology (Garfinkel, 2002) explains, “the sounds and movements that comprise social action are meaningful creations that get their meaning from the shared social contexts of expectation within which they are enacted…. Intended meanings, however, can only be shared if they can be successfully displayed before others in expected ways” (p.57).

This Investigation’s analyses of the meaning-making process focus on the sequential response structure (or “adjacency pairs”) of utterances, which build on previous utterances and elicit further possible, anticipated or expected responses—in keeping with the approach of conversation analysis (Schegloff, 2007). The analysis re-constructs the web of situated semantic references: “The meaning of the interaction is co-constructed through the building of a web of contributions and consists in the implicit network of references” (Stahl, 2009, p. 523). The analysis of this discourse data makes visible how small groups negotiate and adopt group practices.

The structure of group discourse

Note that meaning is constructed by more than one individual through the elicitation-response pair. That is why interaction analysis is considered to take place at the small-group unit of analysis. If one attributed the meaning of a single utterance to the mental state of the individual making the utterance, then that would be an analysis at the individual unit—and would imply some form of access to the individual’s mental state. Single utterances can rarely be adequately interpreted in isolation; they typically include indexical elements that reference prior utterances and other elements of the interactional situation (Zemel & Koschmann, 2013) [Investigation 5]. Therefore, they must be analyzed in terms of their sequential position with respect to utterances of other people.

Most published sequential analyses of conversation are limited to brief excerpts; this Investigation’s analysis of hour-long sessions—especially considered within the larger context of the VMT Project—goes beyond the analysis of even so-called “longer sequences” (Stahl, 2011) [Investigation 23]. Analysis of longer sequences is more important in studying geometry instruction than in most conversation analysis. While ethnomethodologically informed conversation analysis (Garfinkel, 1967; Goodwin & Heritage, 1990; Sacks, 1965/1995; Schegloff, 2007) is interested in how meaning is socially constructed in the momentary interaction, we go beyond the ethnomethodological focus. We are here concerned with both (a) longer chains of meaning making and (b) how the meaning-making process itself changes as the group learns to collaborate and to engage in mathematical discourse.

(a) Perhaps geometry's greatest contribution to the development of human cognition was to systematize the building of *chains of reasoning*—presented as deductive proofs or specially structured constructions of graphical figures (Latour, 2008; Netz, 1999). Euclid's proofs could extend to over forty steps, each specified in a prescribed technical language and accompanied by a diagram representing a correspondingly complicated construction. The cognitive capacity to follow—let alone to invent—such a sequence of deduction or construction required the development of meta-cognitive planning and agentic regulation skills (Charles & Shumar, 2009; Emirbayer & Mische, 1998; Stahl, 2005). These skills have since the time of the early Greek geometers become ubiquitous in literate modern society (Ong, 1998). They underlie our scientific worldview and technological lifestyle. Sophisticated planning skills have become second nature (Adorno & Horkheimer, 1945) to us and we now assume that people are born with rational skills of planning and arguing. It has taken seminal studies of philosophy (Heidegger, 1927/1996) and psychology (Suchman, 2007) to dispel the common rationalist assumption (Dreyfus, 1992) that our actions are the result of previous mental planning, rather than that reasoning is generally posterior rationalization (Stahl, 2013b, Ch. 3), and that we must learn how to make up these explanations after our actions as little retroactive stories (Bruner, 1990), in order to understand and justify them. We would like to see how young, novice teams could develop such sequential reasoning skills, guided by experiences involving geometric construction, analysis and planning. We hypothesize that studying geometry can be an occasion during which significant steps of learning about deductive reasoning can take place. We look at transcripts of discourse and interaction by virtual math teams for the adoption of group practices that involve group agency of sequences of task steps.

(b) Following a chain of development of group agency over time involves the longitudinal analysis of longer sequences of interaction or comparison of excerpts at different points in a temporally extended learning process. Analysis of a single moment can reveal how participants take their activity as instructional or can display signs of having learned something new (Koschmann & Zemel, 2006; Zemel, Çakir, Stahl & Zhou, 2009). However, it can be more informative to compare and contrast interactions at different times to reveal how groups and their participants have taken up previous experiences in current interaction (Sarmiento & Stahl, 2008) and *how that makes a difference to their current meaning making*.

We would like to observe the evolution of group practices and individual skills or understandings over time. Our analytic goal can be called a "learning trajectory." Such a learning trajectory has been characterized as follows:

> A researcher-conjectured, empirically supported description of the ordered network of constructs a student encounters through instruction (i.e., activities, tasks, tools, forms of interaction and methods of

> evaluation), in order to move from informal ideas, through successive refinements of representation, articulation and reflection, towards increasingly complex concepts over time. (Confrey et al., 2009, p. 346)

Note the central role of instruction here. Instruction is conceived here as the provision of a carefully designed learning environment. As Lehrer and Schauble (2012) put it, “The benchmarks of learning tend not to emerge unless someone carefully engineers and sustains the conditions that support them” (p. 705). Particularly in our study of the Cereal Team (see case studies 3 and 4, below) we analyze how the VMT environment guides the student team’s learning trajectory as they adopt group practices that enable them to refine their representations, articulations and reflections over time.

Theory of group practices

Our focus on *group practices* as foundational to collaborative learning is in keeping with the “practice turn” in contemporary social theory and epistemology (Schatzki, Knorr Cetina & Savigny, 2001). According to Reckwitz (2002), a practice is “a routinized type of behavior which consists of several elements, interconnected to one another: forms of bodily activities, forms of mental activities, ‘things’ and their use, a background knowledge in the form of understanding, know-how, states of emotion and motivational knowledge” (p. 249). Social practices form our background, tacit knowledge as proposed by twenty-first-century philosophy (Stahl, 2016b) as an alternative to eighteenth-century rationalist (Descartes, 1633/1999) and cognitivist (Kant, 1787/1999) philosophies [Investigation 15].

Practice theory was propounded by Bourdieu (1972/1995). He uses the term “habitus” for our systems of durable, transposable dispositions—or organization of conventionalized, routinized, objectified and embodied habits. As with other concepts, I construe practices primarily at the small-group unit of analysis, rather than as habits of individual bodies or cultural conventions of whole communities—in contrast to Bourdieu and his followers. Group practices are what make collaboration possible: “The homogeneity of habitus is what—within the limits of the group of agents possessing the schemes (of production and interpretation) implied in their production—causes practices and works to be immediately intelligible and foreseeable, and hence taken for granted” (Bourdieu, 1972/1995, p. 80).

Only because group members share the ability to use the same group practices, can the members understand each other’s actions and their references to those actions. The intersubjectivity of the group is based on this shared meaning [Investigation 18]. The sharing of meaning is a product of the group interaction that adopts the practice; it is produced in the interaction as the members construct the meaning together (Stahl, 2015b). Group practices are proposed—whether verbally or in action—and then discussed, negotiated, accepted, put into regularized practice, generalized across instances of practice and incorporated into the group's habitus. Then we may say that the group—and often its members—has learned. The analysis of group practices provides a vital key to theorizing, supporting, analyzing and facilitating computer-supported collaborative learning.

Collaborative learning certainly involves individual cognition and socio-cultural influences. Resources from the individual and cultural levels are necessarily introduced into the group interaction, made sense of, negotiated, shared and adopted through small-group processes. The adoption of group practices *mediates* the multiple levels involved in learning (Stahl, 2013a; 2013b). The analysis of group practices provides a powerful new method to study CSCL. By automatically capturing the complete interaction within inquiring online groups of students, CSCL research has the potential to observe and analyze the subtle development and use of group practices for the first time.

Enacting group practices

In analyses of VMT interactions, group practices largely account for the group's teamwork and for its ability to construct knowledge or problem solve as a group. The enacting of cultural or community practices as their own group practices—facilitated by teachers, texts, scripts, interactional resources and knowledge artifacts—is how small groups acquire skills from their social context and how the group participants exchange and appropriate each other's perspectives and skills as individual learning (Stahl, 2013b, ch. 8). *The answer to the question of how the group learns is that it successively adopts various practices and incorporates them in its on-going interaction.* As Vygotsky (1930/1978) proposed, such small-group learning generally precedes learning by "isolated individuals" (still surrounded by texts, motivations, and objectives from family members, workplace colleagues, classroom friends and other small groups).

As observed in our studies, the adoption process often follows a general pattern (Stahl, 2016a):

- First, the group encounters a "breakdown" situation in which they do not know what to do.

- Then someone makes a proposal for action. There may have been a preceding series of proposals, some ignored or failed (see Stahl, 2006a, ch. 21) and others rejected by the group.
- The proposal may be followed by a negotiation process as group members question, refine or amend the original proposal through secondary proposals.
- Finally, there is often an explicit round of agreement.
- Perhaps most importantly, the new practice is put to work in overcoming the breakdown situation.
- In the future, the practice may be simply applied without discussion. Of course, there could also be instances of backsliding, in which the group fails to apply a previously adopted practice where it could help.

This general pattern is not a rational model of mental decision-making. Rather, it involves tacit behavior, where a breakdown leads to explicit knowledge, followed by negotiation and eventually a return to tacit practices (Stahl, 1993, ch. 4). The adoption process is driven by interpersonal interaction engaged in the world, not by logical deductions in individual minds.

The catalog of group practices compiled from analysis of VMT data agrees well with lists of social practices enumerated in the literature (Stahl, 2016a). For instance, we identified online versions of practices defined by face-to-face conversation analysis: sequential organization (response structure), turn taking, repair, opening and closing topics, indexicality, deixis, linguistic reference and recipient design (Zemel, Çakir & Stahl, 2009). Other group practices correspond to practices CSCL has previously investigated: joint problem spaces, shared understanding, persistent co-attention, representational practices, longer sequences and questioning [Investigation 17]. Within both our work and other CSCL reports, practices in mathematics education include: mathematical discourse and technical terminology; pivotal moments in problem solving and the integration of visual/graphical reasoning; numeric/symbolic expression and narrative [Investigation 12].

The idea of centering CSCL analysis on group practices emerged from study of VMT data. Publications that present that data discuss the theory, methodology and implications of the focus on group practices extensively. To ground this Investigation in that data, we point to four case studies that analyze group practices:

Study 1: A group practice of referencing

Pointing, referencing, or deixis forms a ubiquitous class of gestures essential for maintaining collaboration, including online (Stahl, 2006b). In Log 1 from Team C in the VMT SpringFest 2006, three students use a whiteboard integrated with VMT's chat tool to explore arrays of hexagons. In line 709, Jason halts discussion until he can "see" what student 137 has proposed. Qwertyuiop has drawn an array of lines to check his understanding of 137's post (Figure 1). As analyzed in (Stahl, 2013b, Sec. 8.1) , this leads to work by the group to establish practices for making focal geometric figures visible to each other by coloring lines that outline or divide up the figures [Investigation 17].

line	time	student	chat post
705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance
714	19:18:53	137	How do you color lines?
715	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:12	Jason	it's the fifth one from the right
717	19:19:20	137	Thanks.
718	19:19:21	Jason	there ya go :-)
719	19:19:48	137	Er... That hexagon.

Log 1. Excerpt from Study 1.

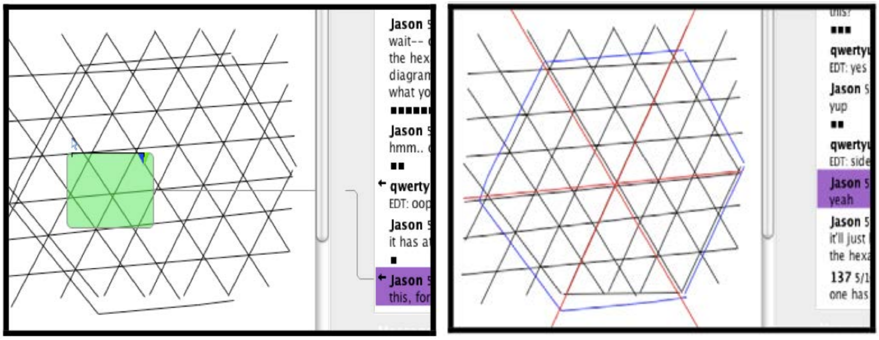

Figure 1. A large hexagon outlined with extra black lines and pointed to by a green square (left). The large hexagon divided in six sectors by red lines (right).

In the minute from his interruption of the mathematical talk at Line 709 to his resumption in Line 713, Jason demonstrates that he sees the hexagon that has been outlined by 137 in black lines (Figure 1, left), by making a new mathematical proposal and pointing from his chat posting to a small hexagon using the VMT pointing tool. Soon thereafter, the group divides the larger hexagon into six triangles using a practice involving colored lines (Figure 1, right). We also see the sharing of tool-use practices as Jason guides 137 in coloring lines in the VMT whiteboard, after which 137 colors his outline blue. Later, the group makes more complicated relationships within the array easily visible with colored lines, allowing the group to derive formulas working collaboratively step-by-step.

Study 2: Group practices over time and across individuals

In a longitudinal analysis of another VMT team—Team B in SpringFest 2006—Medina, Suthers and Vatrapu (2009) identify several group practices and show how they are enacted and repeatedly used across all four of their VMT sessions (see Figure 2).

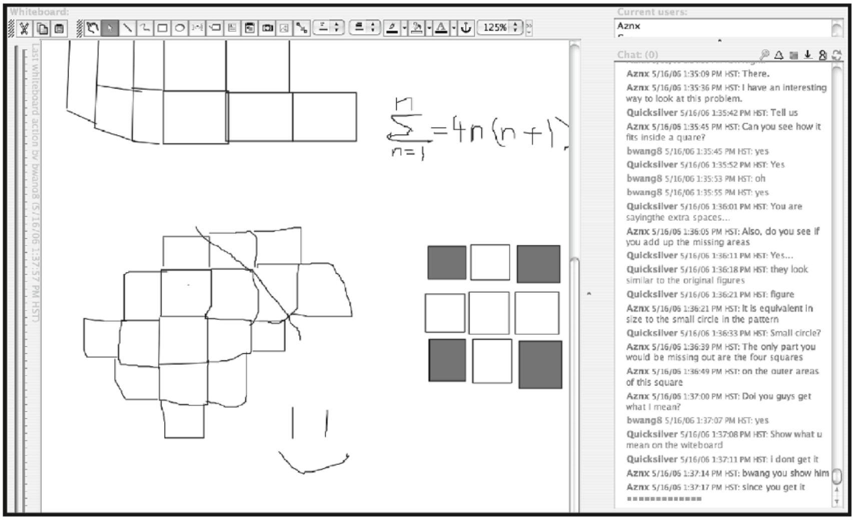

Figure 2. A growing pattern of squares forming a diamond shape, with corners filled in with colored squares to simplify calculations.

In each session, a different participant initiates interaction by first producing a whiteboard drawing that the other two subsequently orient to through chat. In their Sessions 2 and 3, the practice of *inscribe-first-solve-second* is iteratively enacted and composed with two additional practices—*modulate-perspective* and *visualize-decomposition*. In Session 2, Quicksilver's use of color and perspective emerges in the joint work in support of both representational and problem-solving practices. In Session 3, Bwang appropriates color to draw out the particular decomposition previously articulated by Aznx (Figure 2). This demonstrates both shared understanding and individual adoption of the shared group practices.

Study 3: A group practice supporting collaboration

It is particularly informative to observe novices confronting a completely new challenge. In the start of WinterFest 2013, teams of middle-school students faced VMT's multi-faceted software interface and a new form of mathematics, dynamic geometry. Here is the opening interaction of a group of three fourteen-year-old girls we call the Cereal Team (Figure 3; Log 2), analyzed in (Stahl, 2016a, Session 1).

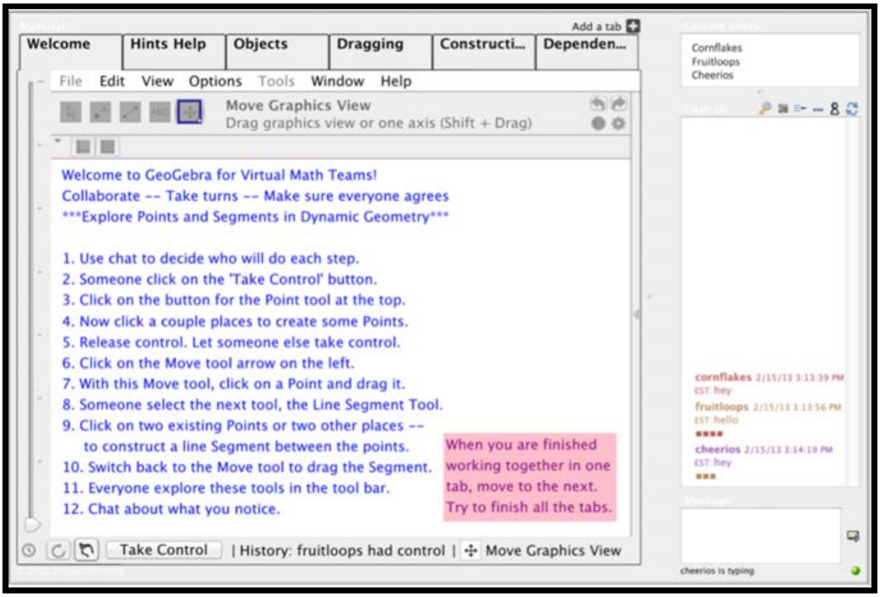

Figure 3. Students greet each other at the start of their use of VMT.

Line	Time	User	Message
3	13:39.4	cornflakes	hey
4	13:57.0	fruitloops	hello
6	14:19.1	cheerios	hey
7	14:45.6	cheerios	whose froot loops
8	14:53.9	cornflakes	xxxxxxxx [name *removed from log for privacy*]
9	15:10.8	cheerios	whose takimg control
10	15:20.1	cheerios	taking*
...			
21	16:18.4	cheerios	so whoses doing what
22	16:44.4	fruitloops	who wants to take control?
23	17:30.6	cheerios	xxxxxxxx do you want to [name removed for privacy]
24	17:52.2	fruitloops	no... cornflakes you take controll.....
25	18:01.7	fruitloops	who wants to do what steps?
26	18:02.9	cheerios	cornflakes take control
27	18:03.6	cornflakes	no cheerios you can
28	18:14.6	cheerios	cornflakes
29	18:25.4	fruitloops	cornflakes
30	18:33.6	cornflakes	NO
31	18:40.0	cheerios	why not
32	18:52.3	fruitloops	i just took control. lets takes turns
33	19:01.9	cheerios	alright
34	19:03.0	cornflakes	ok

Log 2. Excerpt from Study 3.

Note that the group carries over practices of greeting (lines 3, 4, 6) and correcting typos (line 10) from talking and texting to VMT. However, the group has no idea how to start computer-supported collaboration by taking control of the software and responding to the instructions. Each student strenuously resists leading the online group work. Finally, Fruitloops suggests, “Let’s take turns” (Line 32). Although suggested in the instructions, this suggestion has to be stated explicitly and agreed upon by all to become an effective group practice; thereafter, each session begins by a student taking her turn, and the group work proceeds smoothly.

Study 4: A group practice of mathematical problem solving

We analyzed the Cereal Team's adoption of many group practices during their eight hour-long sessions. One of their most impressive mathematical accomplishments is analyzed in (Stahl, 2016a, Session 6). The group explores a given dynamic figure of one square inscribed in another, and then constructs its own figure with the same geometric dependencies (Figure 4).

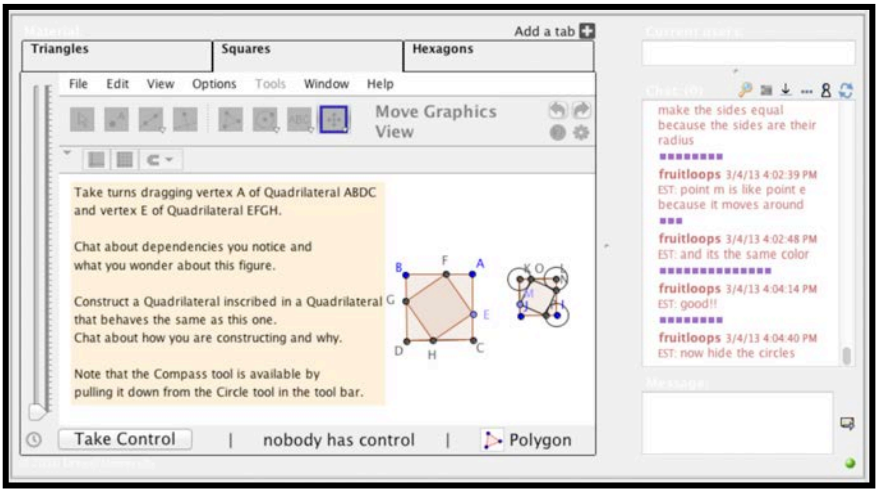

Figure 4. The team was given example square ABDC with inscribed square EFGH and the team constructs square IJKL with inscribed square MONP.

The analysis of this accomplishment and the group's discourse about it demonstrates the team's effective adoption of many mathematical, tool-usage and collaboration group practices. In particular, it makes visible how well the team members each learned the practices enacted by the group in previous sessions (Log 3, esp. Line 146).

135	39:20.3	cornflakes	olets start by cinstructing a regular square
136	39:48.0	fruitloops	i think we should make perpendicular lines somehow
137	39:58.8	cheerios	use the perpindicular line tool
138	43:21.9	fruitloops	the first line segment would be like ab
139	43:27.7	cornflakes	yes
142	51:24.7	cheerios	how do u know ji is straight
143	55:40.6	fruitloops	i dont know what to do because the points arent the same color
144	56:38.2	fruitloops	now after you make the perpendicular lines try to make the circles\
145	57:48.7	fruitloops	i think you need to know use the polygon tool and make the square
146	59:10.6	fruitloops	now we need to use the compass tool lilke we did in the triangles tab
147	59:57.5	fruitloops	because af is equal to ec and dh and bc
148	00:42.4	cheerios	i made a line segment which was if than i used the perpendicular line tool and made 2 lines on each side then used the compass tool and clicked on each point and then the center vertex was i and then made a another circle except the center vertex is j and connected all the points

Log 3. Excerpt from Study 4.

The analysis of this excerpt requires observing the shared geometric manipulations, noting the reuse of previously acquired group practices and carefully studying the text chat. The data for this is comprehensively preserved by the VMT system.

Designing for group practices

We have just summarized very briefly four case studies that were examined in considerable detail in other reports on VMT sessions. From these summaries, we can glimpse several lessons for the design of CSCL pedagogy, curriculum and technology.

Study 1 shows the importance of pointing practices (deictic reference) for establishing common ground. The VMT software provides tools for pointing from a chat posting to a region in the drawing area, for drawing lines around a region, and for coloring lines to highlight a region [Investigation 22]. The students use these to focus each other's visual attention on a referenced region. In an online environment, creating shared focus is a precondition of productive discourse. In the study's data, we can see how students discover the reference tools and how they inform other group members about them, adopting group practices of using these tools. If other groups failed to find these tools when they were needed or failed to use them appropriately, this could suggest to technology and pedagogy designers to make these tools more visible and to guide students to find and use these tools. Analysis of the group practices in this study suggests retaining the reference tools in future versions and designing more activities that explore and exploit them.

Analysis of group practices in *Study 2* contributes to the theory of CSCL, group cognition and group practice. For instance, it shows how abilities of one student become shared group practices, and how these then become abilities of the other students. Each member of the group in this study contributes a practice that may have been an individual skill or may have been brought in from the larger socio-cultural context. These become shared group practices, which then interact with other group practices, leading to innovations in collaborative mathematical cognition. Analysis focused on the creation, adoption and application of group practices can provide detailed views of computer support, collaborative knowledge building and the interplay of processes at different levels of description.

Study 3 is informative because it practically shows the creation of collaboration practices *tabula rasa*. Of course, the students are already teens with developed communication skills, but they are initially very reluctant to work together in the VMT environment (although they immediately start to explore it and work in it as individuals). The first thing that the screen of instructions (their script) says is, "Collaborate–take turns–make sure everyone agrees." Eventually the group adopts this advice and uses turn taking as a visible group practice. The students talk about whose turn it is and who will take the next turn. However, the group has to go through an adoption sequence before it can enact this group practice to overcome the team's breakdown in action. The design of the wording of the instructions resulted from the observation by the designers of previous studies, which indicated the need for group turn-taking practices. Researchers can now debate whether the instructions need further revision based on Study 3 and parallel studies with the same or re-worded instructions.

Study 4 is taken from another session by the team from Study 3, but now (five sessions later), this team of middle-school students is already achieving a geometry accomplishment that is challenging for most college-educated adults working individually. The chat excerpt reflects both geometry-construction actions and mathematical reflections by the team. An analysis of the group practices here reveals the importance of the compass tool (technology), of creating equal-length line segments (mathematics), and of explaining what one does so everyone can agree (collaboration). The group is successful because the technology, instructions and successive activities had been carefully designed to scaffold the adoption of the necessary group practices, based upon previous iterations of VMT trials.

Through analysis of the enactment of group practices while engaging in collaborative dynamic geometry, we determined that a central practice of dynamic geometry is the construction of dependencies. A *dependency* is a constraint on geometric objects that ensures invariance. For instance, in the exemplary Euclidean construction of an equilateral triangle, the sides of the triangle are constrained to be dependent on the radii of circles with equal radii, ensuring that the three sides of the triangle are of equal length to each other (Euclid's first proof). In dynamic geometry, one can drag a vertex of a triangle to make the triangle a different size or orientation; however, if the triangle has been constructed with the correct dependencies, the three sides will stay equal to each other, all getting longer or shorter together.

Geometric dependency is a very abstract notion, challenging for middle-school students to master, as can be seen in the extended analysis of the Cereal Team's group practices (Stahl, 2016). For instance, Öner (2016) [Investigation 9] specifically traces the team's struggle in their third session to move from a naïve view of geometry in terms of visual appearances to one of underlying constructed dependencies. Understanding the notion of dependencies in dynamic geometry can be operationalized in terms of identifying specific group practices of construction (which establish dependencies) and discourse (which references and reflects on the dependencies). This can then guide the researchers' analysis and design.

To learn more about effectively scaffolding group practices related to constructing and discussing dependencies, we designed activities and analyzed interaction data from trials. Our final curriculum (technology, teacher training, embedded instructions, geometry challenges, etc.) is all oriented toward fostering and supporting group practices of constructing dependencies and of discussing dependency (Stahl, 2015a). While aligned with Common Core introductory-geometry curriculum, the sequence of activities is designed to foster the successive adoption of group practices that build on each other to facilitate increasingly advanced collaboration, mathematics and argumentation. As designers, we configure activities to be used in ways we intend. However, we need to study how student teams structure their group practices in our designed environment to know how they enact our artifacts [Investigation 6]. What counts in CSCL is the actual student interaction—structured by group practices—which is always quite different from what the designers envisioned.

Just as we can see in Study 3 the team's difficulty in taking collaborative action before it has adopted the turn-taking group practice, we can repeatedly observe breakdowns in action in later sessions. Especially during periods of geometric construction, the Cereal team seems to flounder excessively. These are indications that additional group practices should be scaffolded and encouraged. For instance, there are many small tricks to doing constructions in dynamic geometry, and perhaps the curriculum should introduce some of these more explicitly for adoption as group practices. In addition, students tend to avoid discussing in chat what they are doing in the construction area. It might, for instance, be helpful to model effective geometric-construction techniques and collaborative-discussion patterns in classroom periods before small-group sessions, depending on the educational context. These are new design decisions to be made for future iterations, based on analysis of recent interaction data suggesting which possible group practices are important to support.

Analyzing group practices

Group practices are often derived from social practices (of the classroom, school mathematics, the general culture, etc.), but must be enacted or adopted and used repeatedly by the group to become effective. Groups do this in different, unpredictable ways as a result of their massively over-determined interactions. Every instance of collaborative learning is unique—it cannot be replicated or generalized. However, within a domain like collaborative online dynamic geometry, certain group practices typically recur regularly. Group practices can constitute central structural elements of group knowledge-building interactions. They structure the interaction. They also structure the domain—as practices related to dependency structure dynamic geometry. The cataloguing of group practices identified in the analysis of a corpus of interaction data from CSCL interventions can contribute to research that is directly applicable to CSCL design to support team interactions in target domains.

Traditional experimental methods aim to contribute incremental additions to a body of scientific findings. However, they are typically summative evaluations that judge the adequacy of supposedly well-defined situations, rather than formative explorations of situations under development and evolution. Summative evaluation is appropriate for studying unchanging natural phenomena. However, CSCL is a design science involving complex human interaction within social contexts in flux. We do not assume that the current design of technology, crafting of pedagogy, preparation of students, or orchestration of collaboration are finalized and perfect—ready for summative evaluation. Rather, we are interested in discovering whether we are making progress along those intermingling dimensions and how we should tweak things for our next design iteration.

In an insightful and comprehensive new review of theory and research in the domain of argumentation in education, Schwarz & Baker (2017) summarize the results of published studies on a variety of aspects of their topic. Invariably, they have to conclude that "more research on argumentation in diverse learning contexts" is needed in each aspect (p. 239). The research they are reviewing consists primarily of attempts to contribute incremental additions correlating variables and effects. What would this even mean in a field where the technology, education theory and socio-cultural context are fluid and successive studies cannot really be comparable? Furthermore, the real point in DBR research is not to evaluate the effectiveness of collaborative learning under current conditions, but to discover avenues to pursue in re-design of our tools, scripts and theories to create improved future conditions.

For a DBR science, case studies are generally more appropriate than summative evaluations (Yin, 2009). That does not mean that quantitative pre-post studies cannot be helpful in generating or checking hypotheses and suggesting phenomena to look for in detailed interaction analysis. In the VMT project, we often used quantitative and qualitative methods to pursue specific questions. However, we found that the deepest theoretical insights and the best design suggestions derived from detailed interaction analysis of interesting case studies. The examination of group practices can provide a methodological focus for such investigations in the CSCL research context.

Group practices in CSCL

The four case studies excerpted above illustrate the adoption and use of group practices for communicating (pointing), problem solving (problem decomposition), collaborating (turn taking), software usage (compass tool, perpendicular line tool), and geometric construction (of perpendiculars, squares, equal-length lines). By studying the group interactions involved in these practices, we can see in detail just what practices are needed for collaborative mathematics, how groups adopt them, and how well they are supported in our successive prototypes. This is much more useful information for CSCL theory and DBR design than just confirming that groups get a certain result more frequently under certain broad conditions.

A methodological focus on group practices can suggest the design of technologies, curriculum and pedagogy to support the adoption of key group skills. The analysis of the adoption and reuse of group practices in interaction data can pinpoint how concrete groups learn and achieve (or fail to achieve) group-cognitive accomplishments. The scaffolding of suggested group practices through the orchestration of teacher presentations before group sessions, help videos during sessions and classroom reflections after sessions can guide and enrich the collaborative-learning experience.

This new view of CSCL research suggests that CSCL should be reconceived or redefined as the design, analysis and orchestration of technology, curriculum and pedagogy to foster the adoption of productive group practices by student teams.

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Investigation 17. Co-experiencing a Shared World

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Abstract: The ability of people to understand each other and to work together face-to-face is grounded in their sharing of our meaningful natural and cultural world. CSCL groups—such as virtual math teams—have to co-construct their online, shared world with extra effort. A case study of building shared understanding online illustrates several aspects of co-experiencing a digital world. Asking each other questions is one common way of aligning perceptions. Literally looking at the same aspect of something as someone else helps us to see what each other means. The co-constructed shared world has social and temporal as well as objective dimensions. This world grounds communicative, interpersonal and task-related activities for online groups, making possible group cognition that exceeds the limits of the individual cognition of the group members.

Keywords. Questioning, bridging, seeing, world.

The Shared World of Meaning

We all find others and ourselves within one world: reality. We learn about and experience the many dimensions of this world together, as we mature as social beings. Infants learn to navigate physical nature in the arms of caregivers; toddlers acquire their mother tongue by speaking with others; adolescents are socialized into their cultures; and adults master the artifacts of the built environment designed by others. The world is rich with socially endowed meaning, and we come to perceive and experience it as immediately meaningful.

Because we share this meaningful world, we can understand each other and can work together on concerns in common. Our activities around our common objects of concern provide a shared structuring of our world in terms of implicit goals, interpersonal relations and temporal dimensions. These structural elements are reflected in our language: in references to artifacts, in social positioning, and in use of tenses. All of this is understood the same by us unproblematically based on our lived experience of the shared world and communal language. Of course, there are occasional misunderstandings—particularly across community boundaries—but these are exceptions, which prove the rule of shared understanding in general.

The "problem" of establishing intersubjectivity is a pseudo-problem in most cases [Investigation 18]. Human existence is fundamentally intersubjective from the start [Investigation 15]. We understand the world as a shared world and we even understand ourselves through the eyes of others and in comparison with others (Mead, 1934/1962). Rationalist philosophy—from Descartes to cognitive science—has made this into a problem by focusing on the mind of the individual as if it were isolated from the world and from other people. That raised the pseudo-problem of epistemology: how can the (encapsulated, solipsistic) individual mind know about states of the (physical, as opposed to mental) world and about states of other minds? Rationalist philosophy (as described by Dreyfus, 1992) culminated in an information-processing view of human cognition, modeled on computer architecture: understanding is viewed as primarily consisting of a collection of mental representations (or propositions) of facts stored in a searchable memory (Newell & Simon, 1972), implemented in brain neurons.

Critiques of the rationalist approach (e.g., Dreyfus, 1992; Schön, 1983; Suchman, 1987; Winograd & Flores, 1986) have adopted a phenomenological (Heidegger, 1927/1996; Husserl, 1936/1989; Merleau-Ponty, 1945/2002), hermeneutical (Gadamer, 1960/1988), or ethnomethodological (Garfinkel, 1967) approach, in which understanding is grounded in being-in-the-world-together, in the life-world (Lebenswelt), in cultural-historical traditions, and in tacit social practices. This led to post-cognitive theories, with a focus on artifacts, communities-of-practice, situated cognition, distributed cognition, group cognition, activity, and mediations by actor-networks. Human cognition is now recognized to be a social product (Hegel, Marx, Vygotsky) of interaction among people, over time, within a shared world. Knowledge is no longer viewed as primarily mental representations of individuals, but includes tacit procedural knowledge (Polanyi, 1966), designed artifacts (Hutchins, 1996), representational inscriptions (Latour, 1992), small-group processes (Stahl, 2006), embodied habits (Bourdieu, 1972/1995), linguistic meanings (Foucault, 2002), activity structures (Engeström, Miettinen & Punamäki, 1999), community practices (Lave, 1991), and social institutions (Giddens, 1984). The critique of human thought as purely mental and individual is now well established for embodied reality. But what happens in virtual worlds, where the physical world no longer grounds action and reflection? That is the question for this Investigation.

Constructing a Shared Virtual World

The problem of shared understanding rises again—and this time legitimately—within the context of computer-supported collaborative learning (CSCL). That is because when students gather in a CSCL online environment, they enter a virtual world, which is distinct from (although embedded within) the world of physical co-presence. They leave the world of nature, of physical embodiment, of face-to-face perception. They enter a world that they have not all grown into together. However, this does not mean that "shared understanding" is just a matter of overlapping opinions of mental models for online groups either.

In the Virtual Math Teams (VMT) Project, we have been studying how students interact in a particular CSCL environment designed to support online discourse about mathematics. In this Investigation, we will illustrate some of our findings about how interaction in the VMT environment addresses the challenge of constructing a shared virtual world, in which small groups of students can productively engage in collaborative mathematics.

We will present a case study of Session 3 of Team C in the VMT Spring Fest 2006. Here, students aged 12-15 from different schools in the US met online for four hour-long sessions. Neither the students nor the researchers knew anything about the students other than their login user names and their behavior in the sessions. A researcher joined the students in their group sessions, but did not engage with them in the mathematics. Between sessions, the researchers posted feedback in the shared whiteboard of the environment. The VMT Project is described and discussed in (Stahl, 2009); its theoretical motivation is presented in (Stahl, 2006). The VMT environment is shown in Figure 1. The complete chat log of Session 3 of Team C is given in the Appendix to this Investigation.

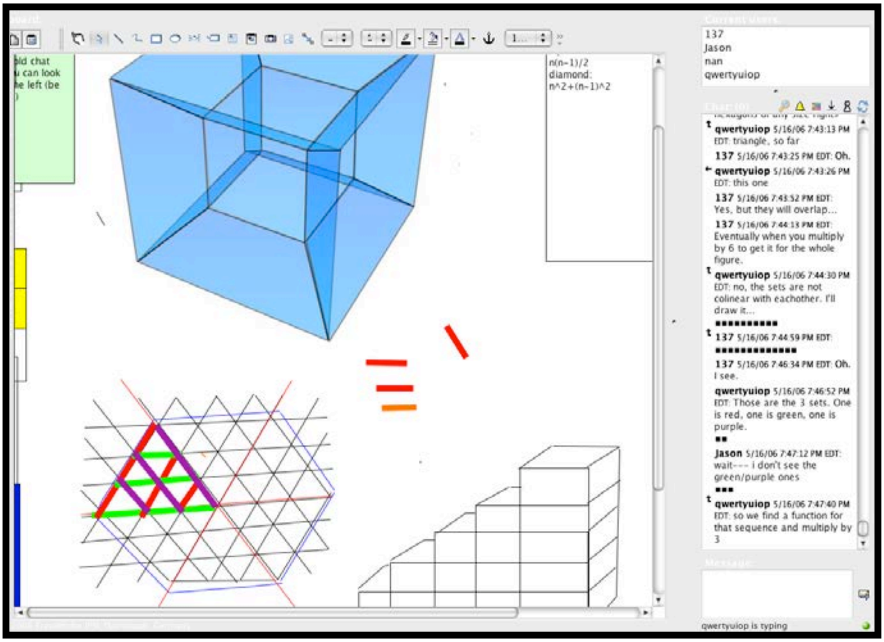

Figure 1. The VMT environment during Session 3 of Team C.

In the next four sections, we illustrate the following aspects of building shared understanding: (a) Asking each other questions is one common way of resolving or avoiding troubles of understanding and aligning perceptions. (b) Literally looking at the same aspect of something as someone else helps us to see what each other means. (c) The co-constructed shared world has social and temporal as well as objective dimensions. (d) This world grounds communicative, interpersonal and task-related activities for online groups.

Questioning to Share Understanding

We have analyzed how questions posed in the VMT environment often work to initiate interactions that resolve troubles of understanding and deepen shared understanding (Zhou, 2009; 2010; Zhou, Zemel & Stahl, 2008). This is in contrast to the rationalist assumption that questions are requests for propositional information. We will here review a number of questions from Session 3 of Group C and indicate how they lead to shared understanding. Unfortunately, due to space limitations, we will not be able to provide the full context for these questions or a detailed conversation analysis.

The question by Qwertyuiop in Log 1 (line 685) serves a coordination function, making sure that all the students have read the feedback to Session 2 before any work begins in the new Session. This is an effort, taking the form of a question, to maintain a shared experience by having everyone take this first step together.

Log 1. Question by Qwertyuiop.

Chat Index	Time of Posting	Author	Content
685	19:06:34	qwertyuiop	has everyone read the green text box?
686	19:06:44	Jason	one sec
687	19:06:45	137	Yes...
688	19:07:01	Jason	alright im done

Log 2 is part of a complicated and subtle process of co-constructing shared understanding. It is analyzed in detail in Investigation 12. The student named 137 has attempted to construct a grid of triangles in the whiteboard (similar to those in the lower left corner of Figure 1). He or she has failed (as expressed by the ironic "Great") and has erased the attempt and solicited help by posing a question. Qwertyuiop requests clarification with another question and then proceeds to draw a grid of triangles by locating and then tweaking three series of parallel lines, following much the same procedures as 137 did. Qwertyuiop's understanding of 137's request is based not only on the "Yeah…" response to his/her "just a grid?" question, but also on the details of the sequentially unfolding visual presentation of 137's failed drawing attempt.

Log 2. Question by 137.

694	19:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?
695	19:11:51	qwertyuiop	just a grid?
696	19:12:07	137	Yeah...
697	19:12:17	qwertyuiop	ok...

In Log 3, the moderator, Nan, asks a question to make visible in the chat what members of the group are doing. Qwertyuiop is busy constructing the requested grid in the whiteboard and the others are presumably watching that drawing activity and waiting for its conclusion. The students do not express any indication that there is a problem in their understanding of each other's activities. However, due to the nature of the virtual environment—in which the attentiveness of participants is only visible through their chat and drawing actions—Nan cannot know if everyone is engaged during this period of chat inaction. Her question and the responses to it make visible to her and to the students the fact that everyone is still engaged. The questioning may come as a minor interference in their group interaction, since Nan's questioning positions her as someone outside the group (not part of "everyone"), exerting authority by asking for an accounting, although it is intended to increase group shared understanding ("everyone know what other people are doing").

Log 3. Question by Nan.

698	19:14:09	nan	so what's up now? does everyone know what other people are doing?
699	19:14:25	137	Yes?
700	19:14:25	qwertyuiop	no-just making triangles
701	19:14:33	137	I think...
702	19:14:34	Jason	yeah
703	19:14:46	nan	good:-)
704	19:14:51	qwertyuiop	triangles are done

See What I Mean

Studies of the use of interactive whiteboards in face-to-face classrooms have shown that they can open up a "shared dynamic dialogical space" (Kershner, Mercer, Warwick & Staarman, 2010) as a focal point for collective reasoning and co-construction of knowledge. Similarly, in architectural design studios, presentation technologies mediate shared ways of seeing from different perspectives (Lymer, Ivarsson & Lindwall, 2009) in order to establish shared understanding among design students, their peers and their critics. Clearly, a physical whiteboard that people can gather around and gesture toward while discussing and interpreting visual and symbolic representations is different from a virtual shared whiteboard in an environment like VMT.

We have analyzed in some detail the intimate coordination of visual, narrative and symbolic activity involving the shared whiteboard in VMT sessions (Çakir, 2009; Çakir, Stahl & Zemel, 2010; Çakir, Zemel & Stahl, 2009) [Investigation 12]. Here, we want to bring out the importance of literally looking at some mathematical object together in order to share the visual experience and to relate to—intend or "be at"—the object together. People often use the expression "I do not see what you mean" in the metaphorical sense of not understanding what someone else is saying. In this case study, we often encounter the expression used literally for not being able to visually perceive a graphical object, at least not being able to see it in the way that the speaker apparently sees it.

While empiricist philosophy refers to people taking in uninterpreted sense data much like arrays of computer pixels, post-cognitive philosophy emphasizes the phenomenon of “seeing as.” Wittgenstein notes that one sees a wireframe drawing of a cube not as a set of lines, but as a cube oriented either one way or another (Wittgenstein, 1953, sec. 177). For Heidegger, seeing things as already meaningful is not the result of cognitive interpretation, but the precondition of being able to explicate that meaning further in interpretation (Heidegger, 1927/1996, pp. 139f). For collaborative interpretation and mathematical deduction, it is clearly important that the participants see the visual mathematical objects as the same, in the same way. This seems to be an issue repeatedly in the online session we are analyzing as well.

In line 705 of Log 4, student 137 proposes a mathematical task for the group. This is the first time that the term “hexagonal array” is used. Coined for the first time in this posting, the term will become a mathematical object for the group as the discourse continues. However, at this point, it is problematic for both Qwertyuiop and Jason because it is new to them. In line 706, Qwertyuiop poses a question for clarification and receives an affirmative, but minimal response. Jason, unsatisfied with the response, escalates the clarification request by asking for help in seeing the diagram in the whiteboard *as* a “hexagonal array,” so he can see it *as* 137 sees it. Between Jason’s request in line 709 and acceptance in line 710, Qwertyuiop and 137 work together to add lines outlining a large hexagon in the triangular array. Demonstrating his ability to now see hexagons, Jason thereupon proceeds with the mathematical work, which he had halted in the beginning of line 709 in order to keep the group aligned. Jason tentatively proposes that every hexagon “has at least 6 triangles” and he makes this visible to everyone by pointing to an illustrative small hexagon from the chat posting, using the VMT graphical pointing tool.

Log 4. Proposal by 137.

705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay

711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance

In Log 5, 137 asks the team to share its knowledge about how to color lines in the VMT whiteboard. Jason gives instructions for 137 to visually locate the appropriate icon in the VMT interface. Demonstrating this new knowledge, 137 changes the colors of the six lines outlining the large hexagon, from black to blue, making the outline stand out visually (see Figure 1). 137 thereby finally clarifies how to look at the array of lines *as* a large hexagon, a task that is more difficult than looking at the small hexagon that Jason pointed to. In this excerpt, the group shares their working knowledge of their virtual world (including the software functionality embedded in it), incidentally to carrying out their task-oriented discourse within that world.

Log 5. Request by 137.

714	19:18:53	137	How do you color lines?
715	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:12	Jason	it's the fifth one from the right
717	19:19:20	137	Thanks.
718	19:19:21	Jason	there ya go :-)
719	19:19:48	137	Er... That hexagon.

In Log 6, Jason proposes a specific mathematical task for the group to undertake, producing a formula for the number of triangles in a hexagonal array of any given side length. (As we shall see below, the group uses the term "side length" as the measure of a geometric pattern at stage n.) Qwertyuiop responds to this proposal with the suggestion to "see" the hexagon (of any size) as a configuration of six triangular areas. (To see what Qwertyuiop is suggesting, look at Figure 1; one of the six triangular areas of the large hexagonal array has its "sticks" colored with thick lines. Looking at this one triangular area, you can see in rows successively further from the center of the hexagon a sequence of 1 small triangle, then 3 small triangles, then 5 small triangles.)

Log 6. Proposal by Jason.

720	19:20:02	Jason	so... should we try to find a formula i guess
721	19:20:22	Jason	input: side length; output: # triangles
722	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.
723	19:20:48	137	Like this?

724	19:21:02	qwertyuiop	yes
725	19:21:03	Jason	yup
726	19:21:29	qwertyuiop	side length is the same...
727	19:22:06	Jason	yeah
728	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
729	19:22:19	137	Each one has 1+3+5 triangles.

In line 723, 137 seeks confirmation that he is sharing Qwertyuiop's understanding of the suggestion. After posting, "Like this?" with a reference back to Qwertyuiop's line 722, 137 draws three red lines through the center of the large hexagon, dividing it visually into six triangular areas. Upon seeing the hexagon divided up by 137's lines, Qwertyuiop and Jason both confirm the shared understanding. Now that they are confident that they are all seeing the mathematical situation the same, namely *as* a set of six triangular sub-objects, the group can continue its mathematical work. Jason draws the consequence from Qwertyuiop's suggestion that the formula for the number of small triangles in a hexagon will simply be six times the number in one of the triangular areas of that hexagon, thereby subdividing the problem. 137 then notes that each of those triangular areas has 1+3+5 small triangles, at least for the example hexagonal array that they are looking at. The fact that the three members of the group take turns making the consecutive steps of the mathematical deduction is significant; it demonstrates that they share a common understanding of the path of deduction and are building their shared knowledge collaboratively.

The observation, "Each one has 1+3+5 triangles," is a key move in deducing the sought equation. Note that 137 did not simply say that each triangular area had nine small triangles. The posting used the symbolic visual representation, "1+3+5." This shows a pattern of the addition of consecutive odd numbers, starting with 1. This pattern is visible in the posting. It indicates that 137 is seeing the nine triangles *as* a pattern of consecutive odd numbers—and thereby suggests that the reader also see the nine triangles *as* such a pattern. This is largely a visual accomplishment of the human visual system. People automatically see collections of small numbers of objects as sets of their specific size (Lakoff & Núñez, 2000). For somewhat larger sets, young children readily learn to count the number of objects. The team has constructed a graphical representation in which all the members of the team can immediately see features of their mathematical object that are helpful to their mathematical task. The team is collaborating within a shared virtual world in which they have co-constructed visual, narrative and symbolic objects in the chat and whiteboard areas. The team has achieved this shared vision by enacting (within and for their group) practices specific to mathematics as a profession for shaping witnessed events, such as invoking related math terms and drawing each other's attention to relevant objects in the scene (Goodwin, 1994). They have learned and taught each other how to work, discuss and perceive as a group in this shared virtual world, via the adoption of group practices [Investigation 16].

Dimensions of a Virtual World

There has not been much written about the constitution of the intersubjective world as the background of shared understanding, particularly in the CSCL online context. This is largely the result of the dominance of the cognitive perspective, which is primarily concerned with mental models and representations of the world; this rationalist view reduces the shared world to possible similarities of individual mental representations. Within the VMT Project, we have analyzed the dimensions of domain content, social interaction and temporal sequencing in the co-construction of a virtual math team's world or joint problem space (Sarmiento & Stahl, 2008; Sarmiento-Klapper, 2009a; Sarmiento-Klapper, 2009b). In this work, we have found the following conceptualizations to be suggestive and helpful: the "joint problem space" (Teasley & Roschelle, 1993) and the "indexical ground of reference" of domain content (Hanks, 1992); the social "positioning" of team members in discourse (Harré & Gillet, 1999) and their self-coordination (Barron, 2000); and the temporal sequentiality of discourse (Schegloff, 1977) and the bridging of temporal discontinuities.

In its previous sessions, Team C had tried to derive formulae for the number of two-dimensional objects (small squares or small triangles) in a growing pattern of these objects, as well as the number of one-dimensional sides, edges or "sticks" needed to construct these objects. A major concern in counting the number of sides is the issue of "overlap." In a stair-step two-dimensional pattern (like the 2-D version of the stair-step pyramid in the lower right section of Figure 1), one cannot simply multiply the number of squares by 4 to get the number of sides because many of the sides are common to two squares. In Session 1, Team C had seen that in moving from one stage to the next stage of the stair-step pattern most new squares only required two new sides.

In Log 7, Qwertyuiop moves on from the derivation of the number of triangles to that of the number of sides. He "bridges" back to the group's earlier insight that the addition of "each polygon corresponds to [an additional] 2 sides." In bridging to past sessions, we found, it is necessary for a group to re-situate a previous idea in the current context. In line 731, Qwertyuiop is reporting that for their hexagon formula, such situating does not work—i.e., that the current problem cannot be solved with the same method as the previous problems. The team then returns to the formula for the number of triangles and efficiently solves it by summing the sequence of consecutive odd numbers using Gauss' technique—the sum of n consecutive odd integers is n(2n/2)—which they had used in previous sessions.

Log 7. Bridging by Qwertyuiop.

731	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles
732	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?
733	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...
734	19:24:49	137	And there are n terms so... n(2n/2)
735	19:25:07	137	or n^2
736	19:25:17	Jason	yeah
737	19:25:21	Jason	then multiply by 6
738	19:25:31	137	To get 6n^2

In Log 8, Qwertyuiop makes a particularly complicated proposal, based on a way of viewing the sides in the large hexagon drawing. He tries to describe his view in chat, talking about sets of collinear sides. Jason does not respond to this proposal and 137 draws some lines to see if he is visualizing what Qwertyuiop has proposed, but he has not. Qwertyuiop has to spend a lot of time drawing a color-coded analysis of the sides as he sees them.

Log 8. Proposal by Qwertyuiop.

742	19:25:48	qwertyuiop	an idea: Find the number of a certain set of colinear sides (there are 3 sets) and multiply the result by 3
746	19:26:36	137	As in those?
747	19:27:05	qwertyuiop	no-in one triangle. I'll draw it...
748	19:28:10	qwertyuiop	those
749	19:28:28	qwertyuiop	find those, and then multiply by 3
750	19:28:50	137	The rows?
751	19:30:01	qwertyuiop	The green lines are all colinear. There are 3 identical sets of colinear lines in that triangle. Find the number of sides in one set, then multiply by 3 for all the other sets.
752	19:30:23	137	Ah. I see.

He has decomposed the set of sides of one triangular area into three subsets, going in the three directions of the array's original parallel lines. He can then see that each of these subsets consists of 1+2+3 sides. There are 3 subsets in each of the 6 triangular areas. Based on this and generalizing to a growing hexagonal array, which will have sums of consecutive integers in each subset, the team can derive a formula using past techniques.

At some point, they will have to subtract a small number of sides that overlap between adjacent triangular areas. Qwertyuiop has proposed a decomposition of the hexagonal array into symmetric sets, whose constituent parts are easily visible. Thus, his approach bridges back to previous group practices, which are part of the shared world of the group—see the analysis of a similar accomplishment by Group B in (Medina, Suthers & Vatrapu, 2009). The hexagonal pattern, which Team C came up with on its own, turns out to be considerably more difficult to decompose into simple patterns that the original problem given in Session 1. It strained the shared understanding of the group, requiring the use of all the major analytic tools they had co-constructed (decomposing, color-coding, visually identifying sub-patterns, summing series, eliminating overlaps, etc.).

In Log 9, the group work is interrupted by an interesting case of bridging across teams. At the end of each session, the teams had posted their findings to a wiki shared by all the participants in the VMT Spring Fest 2006. During their Session 3, Team B had looked at Team C's work on a pattern they had invented: a diamond variation on the stair-step pattern. In their wiki posting, Team C had used their term, "side length." Because members of Team B did not share Teams C's understanding of this term, they were confused by the equation and discussion that Team C posted to the wiki. Team B's question sought to establish shared understanding across the teams, to build a community-wide shared world. As it turned out, Team C had never completed work on the formula for the number of sides in a diamond pattern and Team B eventually discovered and reported the error in Team C's wiki posting, demonstrating the importance of community-wide shared understanding.

Log 9. Bridging by Nan.

804	19:48:49	nan	(we got a question for you from another team, which was posted in the lobby:
805	19:48:53	nan	Quicksilver 7:44:50 PM EDT: Hey anyone from team c, our team needs to know what n was in your equations last week
806	19:49:04	Jason	oh
807	19:49:15	137	The length of a side.

808	19:49:16	qwertyuiop	was n side length?
809	19:49:33	Jason	are you talking about the original problem with the squares
810	19:49:48	137	I think nan is.
811	19:49:58	qwertyuiop	i think it's squares and diamonds
812	19:49:58	Jason	oh
813	19:50:12	Jason	then if you look in the topic description, theres a column for N;
814	19:50:14	Jason	thats what it is
815	19:50:17	nan	ok, quicksilver said they got it
816	19:50:25	Jason	so yes it is # sides
817	19:50:26	nan	thanks guys

Grounding Group Cognition

CSCL is about meaning making by groups (Stahl, Koschmann & Suthers, 2006). At its theoretical core are questions about how groups of students collaborating online co-construct and understand meaning. In this essay, we conceptualize this issue in terms of online groups, such as virtual math teams, building a shared meaningful world in which to view and work on mathematical objects.

Log 10 illustrates a limit of shared understanding, closely related to the notion of a "zone of proximal development" (Vygotsky, 1930/1978, pp. 84-91). The original stair-step pattern consisted of one-dimensional sides and two-dimensional squares. In their Session 2, Team C had generalized this pattern into a three-dimensional pyramid consisting of cubes. Now Qwertyuiop proposes to further generalize into a mathematical fourth dimension and derive formulae for patterns of one, two, three and four-dimensional objects. He had previously imported a representation of a four-dimensional hyper-cube (see the upper area of Figure 1) into the whiteboard for everyone to see.

Log 10. Generalization by Qwertyuiop.

20:12:22	qwertyuiop	what about the hypercube?
20:12:33	137	Er...
20:12:39	137	That thing confuses me.
20:13:00	137	The blue diagram, right?
20:13:13	qwertyuiop	can you imagine extending it 4 dimensions, and a square extends into a grid?
20:13:17	qwertyuiop	yes

20:13:30	137	I didn't get that?
20:13:32	qwertyuiop	I'm having trouble doing that.
20:13:45	qwertyuiop	didn't get this?
20:13:50	137	Ya.
20:15:02	qwertyuiop	If you have a square, it extends to make a grid that fills a plane. A cube fills a space. A simaller pattern of hypercubes fills a "hyperspace".
20:15:19	137	The heck?
20:15:29	137	That's kinda confusing.
20:15:43	qwertyuiop	So, how many planes in a hyper cube latice of space n?
20:16:05	137	Er...
20:16:07	qwertyuiop	instead of "how many lines in a grid of length n"
20:16:17	qwertyuiop	does that make any sense?
20:16:30	137	No. No offense, of course.

At this point late in Session 3, Jason had left the VMT environment. Qwertyuiop was unable to guide 137 to see the drawing in the whiteboard as a four-dimensional object. Apparently, Qwertyuiop had been exposed to the mathematical idea of a fourth dimension and was eager to explore it. However, 137 had not been so exposed. They did not share the necessary background for working on Qwertyuiop's proposal. The only resource available for scaffolding joint meaning making on this topic was the graphic that Qwertyuiop imported—and that was apparently not enough by itself.

This shows that tasks for student groups, even tasks they set for themselves, need to be within a shared group zone of proximal development or be adequately supported by the collaboration environment. The stair-step problem was in their zone—whether or not they could solve it themselves individually, they were able to solve it collectively, with enough shared understanding and background knowledge that they could successfully work together. Their three-dimensional pyramid turned out to be quite difficult for them to visualize in a shared way. Their diamond pattern seemed to be easy for them, although they forgot to work on some of it and posted an erroneous formula. The hexagonal array required them to develop their skills in a number of areas, but they eventually solved it nicely. However, the hyper-cube exceeded at least 137's ability (or desire) to participate in investigating it.

Rationalist philosophy reduces the complexity of social human existence to a logical, immaterial mind that thinks about things by representing them internally. It confuses the mind with the brain and conflates the two. It assumes that someone thinking about a hexagon or working on a math problem involving a hexagon must primarily be representing the hexagon in some kind of mental model. But one of the major discoveries of phenomenology (Husserl, 1936/1989) was that (mental) intentionality is always the intentionality of some (non-mental) object and that cognition takes place as a "being-with" that object, not as a mental act of some transcendental ego [see Investigation 18].

As an example, we have seen that the members of Team C are focused on the graphical image of the hexagon in their virtual world on their computer screens. They reference this image and transform it with additional lines, colors and pointers. They chat about this image, not about some personal mental representations. They work to get each other to see that image in the same way that they see it. This "seeing" is to be taken quite literally. Their eyes directly perceive the image. They perceive the image in a particular way (which may change and which they may have to learn to see).

"Seeing" is not a metaphor to describe some kind of subjective mental process that is inaccessible to others, but a form of contact with the object in the world. Accordingly, we may say that shared understanding is a matter of the group members being-there-together at the graphical image in the whiteboard.

Being-there-together is a possible mode of existence of the online group. The "there" where they are is a multi-dimensional virtual world. This world was partially already there when they first logged in. It included the computer hardware and software. It included the VMT Spring Fest as an organized social institution. As they started to interact, the students fleshed out the world, building social relationships, enacting the available technology, interpreting the task instructions and proposing steps to take together. Over time, they constructed a rich world, furnished with mathematical objects largely of their own making and supporting group practices that they had introduced individually but which they had experienced and adopted as a group [Investigation 16].

Being-there-together in their virtual world with their shared understanding of many of this world's features, the group was able to accomplish mathematical feats that none of them could have done alone. Each individual in the group shared an understanding of their group work at least enough to make productive contributions that reflected a grasp of what the group was doing. Their group accomplishments were achieved through group processes of visualization, discourse and deduction. They were accomplishments of group cognition, which does not refer to anything mystical, but to the achievements of group interaction. The group cognition was possible because of, and only based upon, the shared understanding of the common virtual world. Shared understanding is not a matter of similar mental models, but of experiencing a shared world.

Of course, there are limits to group cognition, just as there are limits to individual cognition. We saw that Team C could not understand Qwertyuiop's ideas about the fourth dimension. Without shared understanding about this, the group could not engage in discourse on that topic. Group cognition can exceed the limits of the individual cognition of the group members, but only by a certain amount. The individuals must be able to stretch their own existing understanding under the guidance of their peers, with the aid of physical representations, tools, concepts, scaffolds and similar artifacts, whose use is within their grasp—within their zone of proximal development (Vygotsky, 1930/1978). We have seen that Team C was able to solve a complex mathematical problem that they set for themselves involving a hexagonal array by building up gradually, systematically and in close coordination a meaningful virtual world.

An analysis of the log of the interaction in our case study has demonstrated much about the team's group cognition. Their group work proceeded by contributions from different individuals, with everyone contributing in important ways. Their questions showed that their individual cognition was initially inadequate to many steps in the work; but their questions also served to expand the shared understanding and to ensure that each member shared an understanding of each step. Because the students demonstrated an understanding of the group work through their successive contributions, we can see not only that individual learning took place, but we can analyze the interactional processes of group learning or group cognition through which it took place by detailed analysis of the chat and drawing actions.

As Vygotsky argued, not only does group cognition lead individual cognition by several years, but individual cognition itself develops originally as a spin-off of group cognition. Individuals can learn on their own, but the cognitive and practical skills that they use to do so are generally learned through interaction with others and in small groups.

This is a powerful argument for the use of CSCL in education. It is incumbent upon CSCL research to further analyze the processes by which this takes place in the co-construction of shared understanding within co-experienced virtual worlds. As we have seen, participants in CSCL virtual environments co-construct worlds to ground their interactions. These virtual worlds exploit intersubjective meaning-making, perceptual joint attention and referential practices learned in the physical social world and adapted to the co-experienced online world.

Appendix

Following is the complete chat log of Session 3 of Group C of VMT Spring Fest 2006. A Replayer file of the entire Group C interaction, including whiteboard and chat is available on request from the author.

663		17:20:42	nan	joins the room
665		19:01:25	Jason	joins the room
666		19:02:22	137	joins the room
667	19:02:30	19:02:37	nan	hi Jason and 137, welcome back
668	19:02:49	19:02:49	Jason	hi
669	19:03:05	19:03:06	137	Hi.
670	19:02:56	19:03:09	nan	i'll be your facilitator tonight
671	19:02:51	19:03:13	Jason	it looks like ssjnish is having connection problems again, even after i pointed him to an email on how to clear his Java cache
672		19:03:45	qwertyuiop	joins the room
673	19:04:07	19:04:13	nan	hi qwertyuiop
674	19:04:23	19:04:23	qwertyuiop	hi
675	19:04:24	19:04:36	nan	do any of you know if david is coming?
676	19:04:33	19:04:41	137	So we do what we did last time again?
677	19:04:46	19:04:47	nan	yes
678	19:04:42	19:04:52	137	I forgot to ask David at school.
679	19:04:59	19:05:04	137	I don't think he'd remember.
680	19:04:48	19:05:11	nan	first take a few minutes to read the feedback posted on the whiteboard
681	19:05:19	19:05:21	nan	no problem
682	19:05:23	19:05:27	nan	i guess we can start
683	19:05:38	19:05:48	nan	david can join later when he comes
684	19:05:53	19:05:54	137	Right.
685	19:06:19	19:06:34	qwertyuiop	has everyone read the green text box?
686	19:06:43	19:06:44	Jason	one sec
687	19:06:43	19:06:45	137	Yes...
688	19:07:00	19:07:01	Jason	alright im done
689	19:06:27	19:07:02	nan	did you see some little squares aftermessage? i haven't seen thoese before, interesting
690	19:07:10	19:07:11	qwertyuiop	yes
691	19:07:07	19:07:12	Jason	yeah, they just indicate whiteboard activity
692	19:07:31	19:07:32	137	Oh.
693	19:07:22	19:07:40	nan	i see. i was on a leave for two weeks and this version is the latest
694	19:11:02	19:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?
695	19:11:47	19:11:51	qwertyuiop	just a grid?
696	19:12:04	19:12:07	137	Yeah...
697	19:12:14	19:12:17	qwertyuiop	ok...
698	19:13:40	19:14:09	nan	so what's up now? does everyone know what other people are doing?
699	19:14:23	19:14:25	137	Yes?
700	19:14:18	19:14:25	qwertyuiop	no-just making triangles
701	19:14:31	19:14:33	137	I think...
702	19:14:32	19:14:34	Jason	yeah
703	19:14:44	19:14:46	nan	good:-)
704	19:14:45	19:14:51	qwertyuiop	triangles are done
705	19:14:46	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:22	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?

707	19:16:00	19:16:02	137	Ya.
708	19:16:13	19:16:15	qwertyuiop	ok...
709	19:16:20	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:28	19:17:30	Jason	hmm.. okay
711	19:17:42	19:17:43	qwertyuiop	oops
712	19:17:35	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:55	19:17:58	Jason	in this, for instance
714	19:18:48	19:18:53	137	How do you color lines?
715	19:18:58	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:06	19:19:12	Jason	it's the fifth one from the right
717	19:19:19	19:19:20	137	Thanks.
718	19:19:18	19:19:21	Jason	there ya go :-)
719	19:19:44	19:19:48	137	Er... That hexagon.
720	19:19:52	19:20:02	Jason	so... should we try to find a formula i guess
721	19:20:13	19:20:22	Jason	input: side length; output: # triangles
722	19:20:12	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.
723	19:20:44	19:20:48	137	Like this?
724	19:21:01	19:21:02	qwertyuiop	yes
725	19:21:00	19:21:03	Jason	yup
726	19:21:23	19:21:29	qwertyuiop	side length is the same...
727	19:22:05	19:22:06	Jason	yeah
728	19:22:06	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
729	19:22:04	19:22:19	137	Each one has 1+3+5 triangles.
730	19:22:17	19:22:23	Jason	but then we're assuming just regular hexagons
731	19:21:53	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles
732	19:22:43	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?
733	19:23:43	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...
734	19:24:22	19:24:49	137	And there are n terms so... n(2n/2)
735	19:25:01	19:25:07	137	or n^2
736	19:25:17	19:25:17	Jason	yeah
737	19:25:18	19:25:21	Jason	then multiply by 6
738	19:25:26	19:25:31	137	To get 6n^2
739	19:25:21	19:25:39	Jason	but this is only with regular hexagons... is it possible to have one definite formula for irregular hexagons as well
740	19:24:19	19:25:46	nan	(sorry to interrupt) jason, do you think you can ask ssjnish to check the email to see the instructions sent by VMT team, which might help?
741	19:25:42	19:25:48	Jason	i'm not sure if its possible tho
742	19:24:39	19:25:48	qwertyuiop	an idea: Find the number of a certain set of colinear sides (there are 3 sets) and multiply the result by 3
743	19:25:57	19:26:03	Jason	i did--apparently it didn't work for him
744	19:26:05	19:26:13	Jason	or his internet could be down, as he's not even on IM right now
745	19:26:10	19:26:13	nan	i see. thanks!
746	19:26:20	19:26:36	137	As in those?
747	19:26:46	19:27:05	qwertyuiop	no-in one triangle. I'll draw it...
748	19:28:09	19:28:10	qwertyuiop	those
749	19:28:18	19:28:28	qwertyuiop	find those, and then multiply by 3
750	19:28:48	19:28:50	137	The rows?

751	19:29:01	19:30:01	qwertyuiop	The green lines are all colinear. There are 3 identical sets of colinear lines in that triangle. Find the number of sides in one set, then multiply by 3 for all the other sets.
752	19:30:20	19:30:23	137	Ah. I see.
753	19:31:00	19:31:07	137	Wait. Wouldn't that not work for that one?
754	19:31:11	19:31:12	Jason	yeah
755	19:31:12	19:31:15	Jason	beacuse that's irregular
756	19:31:09	19:31:17	137	Or are we still only talking regular ones?
757	19:31:20	19:31:22	137	About
758	19:30:38	19:31:24	qwertyuiop	side length 1 = 1, side length 2 = 3, side length 3 = 6...
759	19:32:32	19:32:50	137	Shouldn't side length 2 be fore?
760	19:32:52	19:32:53	137	*four
761	19:33:06	19:33:10	qwertyuiop	I count 3.
762	19:33:20	19:33:25	137	Oh. Sry.
763	19:33:24	19:33:30	qwertyuiop	It's this triangle.
764	19:33:44	19:33:45	137	We
765	19:33:47	19:33:54	qwertyuiop	I don't see the pattern yet...
766	19:33:50	19:34:01	137	We're ignoring the bottom one?
767	19:34:11	19:34:29	qwertyuiop	no, 3 is only for side length 2.
768	19:34:36	19:34:52	137	And I think the'y;re all triangular numbers.
769	19:35:06	19:35:17	qwertyuiop	"triangular numbers"?
770	19:35:28	19:35:37	Jason	you mean like 1, 3, 7, ...
771	19:35:39	19:35:39	Jason	?
772	19:35:48	19:35:59	137	Like 1,3,6,10,15,21,28.
773	19:35:51	19:36:02	qwertyuiop	the sequence is 1, 3, 6...
774	19:36:02	19:36:30	137	Numbers that can be expressed as n(n+1)/2, where n is an integer.
775	19:36:44	19:36:45	qwertyuiop	ah
776	19:37:09	19:37:18	137	So are we ignoring the bottom orange line for now?
777	19:37:32	19:37:36	qwertyuiop	"green"?
778	19:37:44	19:37:48	137	THe short orange segment.
779	19:37:49	19:38:05	137	PArallel to the blue lines.
780	19:37:58	19:38:05	qwertyuiop	I don't think so...
781	19:38:20	19:38:26	137	Wait, we are counting sticks right now, right?
782	19:38:35	19:38:48	qwertyuiop	yes-one of the colinear ets of sticks
783	19:38:55	19:39:08	qwertyuiop	oops-"sets" not " ets"
784	19:39:22	19:39:42	137	So we are trying to find the total number of sticks in a given regular hexagon?
785	19:39:50	19:40:18	qwertyuiop	not yet-we are finding one of the three sets, then multiplying by 3
786	19:40:25	19:40:40	qwertyuiop	that will give the number in the whol triangle
787	19:40:34	19:40:51	137	Then shouldn't we also count the bottom line?
788	19:40:52	19:41:01	Jason	are you taking into account the fact that some of the sticks will overlap
789	19:41:25	19:41:41	137	Then number of sticks needed for the hexagon, right?
790	19:41:16	19:42:22	qwertyuiop	Yes. The blue and green/orange lines make up on of the three colinear sets of sides in the triangle. Each set is identical and doesn't overlap with the other sets.
791	19:42:50	19:42:50	Jason	ok
792	19:43:03	19:43:11	Jason	this would be true for hexagons of any size right>
793	19:43:09	19:43:13	qwertyuiop	triangle, so far
794	19:43:25	19:43:25	137	Oh.

795	19:43:25	19:43:26	qwertyuiop	this one
796	19:43:42	19:43:52	137	Yes, but they will overlap...
797	19:43:59	19:44:13	137	Eventually when you multiply by 6 to get it for the whole figure.
798	19:44:01	19:44:30	qwertyuiop	no, the sets are not colinear with eachother. I'll draw it...
799		19:44:59	137	
800	19:46:22	19:46:34	137	Oh. I see.
801	19:46:22	19:46:52	qwertyuiop	Those are the 3 sets. One is red, one is green, one is purple.
802	19:47:04	19:47:12	Jason	wait--- i don't see the green/purple ones
803	19:47:18	19:47:40	qwertyuiop	so we find a function for that sequence and multiply by 3
804	19:48:25	19:48:49	nan	(we got a question for you from another team, which was posted in the lobby:
805	19:48:52	19:48:53	nan	Quicksilver 7:44:50 PM EDT: Hey anyone from team c, our team needs to know what n was in your equations last week
806	19:49:04	19:49:04	Jason	oh
807	19:49:12	19:49:15	137	The length of a side.
808	19:49:10	19:49:16	qwertyuiop	was n side length?
809	19:49:26	19:49:33	Jason	are you talking about the original problem with the squares
810	19:49:44	19:49:48	137	I think nan is.
811	19:49:43	19:49:58	qwertyuiop	i think it's squares and diamonds
812	19:49:58	19:49:58	Jason	oh
813	19:49:59	19:50:12	Jason	then if you look in the topic description, theres a column for N;
814	19:50:12	19:50:14	Jason	thats what it is
815	19:50:09	19:50:17	nan	ok, quicksilver said they got it
816	19:50:22	19:50:25	Jason	so yes it is # sides
817	19:50:21	19:50:26	nan	thanks guys
818	19:51:11	19:52:19	qwertyuiop	what about: f(n)=2n-1 where n is side length
819	19:52:55	19:53:03	137	I don't think that works.
820	19:53:07	19:53:18	137	Howbout just n(n+1)/2
821	19:53:37	19:53:41	Jason	for # sticks?
822	19:53:38	19:53:48	qwertyuiop	that's number of sides for one set
823	19:53:50	19:53:51	qwertyuiop	?
824	19:53:57	19:53:59	Jason	oh ok nvm
825	19:54:26	19:54:29	137	Ya.
826	19:54:36	19:54:58	qwertyuiop	then x3 is 3(n(n+1)/2)
827	19:55:04	19:55:07	qwertyuiop	simplified to...
828	19:55:11	19:55:37	qwertyuiop	(n(n+1)1.5
829	19:55:34	19:55:44	137	On second thought, shouldn't we use n(n-1) for these:
830	19:55:31	19:55:55	nan	just a kind reminder: Jason mentioned that he needs to leave at 7p central time sharp
831	19:56:05	19:56:19	nan	rest of you can continue if you like
832	19:56:19	19:56:25	137	Is that 5 pm PST?
833	19:56:27	19:56:31	137	or 4pm?
834	19:56:32	19:56:32	nan	yes
835	19:56:41	19:56:42	137	Ah.
836	19:56:42	19:56:56	nan	which is a couple of min from now, right, Jason?
837	19:57:15	19:57:16	qwertyuiop	Jason?
838	19:57:30	19:57:33	137	I think he left?

839	19:57:43	19:57:52	Jason	sorry i was away for a couple minutes
840	19:57:58	19:58:02	Jason	yeah i'll need to go pretty soon
841	19:58:23	19:58:25	qwertyuiop	back to this?
842	19:58:32	19:58:34	137	Ya
843	19:58:39	19:58:49	qwertyuiop	why not n(n-1)?
844	19:58:39	19:58:50	Jason	you guys pretty much have the formula for this hexagon problem...
845	19:58:57	19:59:28	qwertyuiop	We almost have it for the triangle. I don't know about the hexagon.
846	19:59:35	19:59:50	Jason	well that's just multiplied by a certain number for a hexagon, provided that it is regular
847	19:59:58	20:00:14	qwertyuiop	but the sides of the triangles making up the hexagon overlap
848	19:59:52	20:00:18	Jason	well i have to leave now; sorry for not participating as much as i wanted to, it's a pretty busy night for me with school and extracurricular stuff
849	20:00:31	20:00:35	Jason	see you guys Thursday!
850	20:00:44	20:00:48	nan	thanks for participating
851	20:00:53	20:00:57	nan	see you Thursday
852	20:00:57	20:01:00	137	Cya/
853		20:01:07	Jason	leaves the room
854	20:01:19	20:01:31	137	Anyways, if we multiply the orange by 3, we get the:
855	20:01:14	20:01:34	nan	do two of you want to continue working for a bit or stop here?
856	20:01:40	20:01:44	nan	i guess that's the answer
857	20:01:47	20:01:48	nan	go ahead
858	20:01:57	20:02:14	137	So then we add 12n for:
859	20:01:28	20:02:15	qwertyuiop	actually, this doesn't complicate it that much. The overlaps can be accounted for with "-6n"
860	20:02:54	20:02:55	137	Oh.
861	20:02:56	20:03:07	137	I like addition more than subtraction.
862	20:03:11	20:03:16	qwertyuiop	do you see why that works
863	20:03:18	20:03:18	qwertyuiop	?
864	20:03:12	20:03:29	137	So: 9n(n+1)-6n.
865	20:03:41	20:03:45	qwertyuiop	9, not 3?
866	20:04:13	20:04:14	137	?
867	20:04:18	20:04:35	qwertyuiop	you have "9n(n..."
868	20:04:37	20:04:47	qwertyuiop	not "3n(n..."?
869	20:04:51	20:05:00	137	But we need to multiply by 6 then divide by 2
870	20:05:10	20:05:22	qwertyuiop	x6 and /2 for what?
871	20:05:44	20:05:47	137	FOr each triangle
872	20:05:48	20:06:02	137	and /2 because it's part of the equation.
873	20:06:03	20:06:06	137	of n(n+1)/2
874	20:05:36	20:06:20	qwertyuiop	it's x3 for the 3 colinear sets, then x6 for 6 triangles in a hexagon... where's the 9 and 2?
875	20:06:28	20:06:28	qwertyuiop	oh
876	20:06:35	20:06:38	137	So 18/2.
877	20:06:42	20:06:50	137	A.K.A. 9
878	20:06:48	20:07:08	qwertyuiop	(n(n+1)/2)x3x6
879	20:07:14	20:07:15	137	Yeah.
880	20:07:20	20:07:27	qwertyuiop	which can be simplified...
881	20:07:42	20:07:46	137	To 9n(n+1)

882	20:08:01	20:08:04	qwertyuiop	that's it?
883	20:08:10	20:08:12	137	-6n.
884	20:08:17	20:08:24	137	So 9n(n+1)-6n
885	20:08:20	20:08:34	qwertyuiop	i'll put it with the other formulas...
886	20:09:39	20:09:47	qwertyuiop	number of triangles is...
887	20:10:27	20:10:28	137	That.
888	20:10:37	20:10:43	137	6n^2
889	20:11:25	20:11:26	qwertyuiop	oops
890	20:12:12	20:12:22	qwertyuiop	what about the hypercube?
891	20:12:29	20:12:33	137	Er...
892	20:12:36	20:12:39	137	That thing confuses me.
893	20:12:56	20:13:00	137	The blue diagram, right?
894	20:12:37	20:13:13	qwertyuiop	can you imagine extending it 4 dimensions, and a square extends into a grid?
895	20:13:16	20:13:17	qwertyuiop	yes
896	20:13:26	20:13:30	137	I didn't get that?
897	20:13:21	20:13:32	qwertyuiop	I'm having trouble doing that.
898	20:13:41	20:13:45	qwertyuiop	didn't get this?
899	20:13:49	20:13:50	137	Ya.
900	20:13:57	20:15:02	qwertyuiop	If you have a square, it extends to make a grid that fills a plane. A cube fills a space. A smaller pattern of hypercubes fills a "hyperspace".
901	20:15:17	20:15:19	137	The heck?
902	20:15:25	20:15:29	137	That's kinda confusing.
903	20:15:16	20:15:43	qwertyuiop	So, how many planes in a hyper cube latice of space n?
904	20:16:04	20:16:05	137	Er...
905	20:15:48	20:16:07	qwertyuiop	instead of "how many lines in a grid of length n"
906	20:16:11	20:16:17	qwertyuiop	does that make any sense?
907	20:16:23	20:16:30	137	No. No offense, of course.
908	20:16:35	20:16:43	qwertyuiop	ok... let me think...
909	20:16:58	20:17:19	qwertyuiop	Imagine our first problem with a grid of squares.
910	20:17:29	20:17:31	137	Right.
911	20:17:23	20:18:07	qwertyuiop	The squares are 2 dimensional and they can be arranged in a grid to tessalate over a plane. The plane is also 2 dimensional.
912	20:18:39	20:18:41	137	Right.
913	20:18:12	20:18:54	qwertyuiop	If you use 3 dimensional cubes, they can be arranged to fill a 3 dimensional space.
914	20:19:08	20:19:17	137	And that structure's 4 dimensional?
915	20:18:56	20:19:25	qwertyuiop	If you have hypercubes, they can be arranged to fill a 4 dimensional "hyperspace"
916	20:19:32	20:19:36	qwertyuiop	what's 4D?
917	20:19:45	20:19:46	137	?
918	20:19:43	20:20:04	nan	you may want to make your ideas available on the wiki before you go
919	20:20:06	20:20:09	nan	which may take some time
920	20:20:15	20:20:24	137	Actually, I only have around 10 minutes left.
921	20:19:51	20:20:29	qwertyuiop	you say "and that structure's 4 dimensional?" -what's "that"
922	20:20:33	20:20:34	nan	oh
923		20:20:35	137	
924	20:20:36	20:20:43	137	The hypercube

925	20:20:50	20:20:50	qwertyuiop	yes
926	20:21:00	20:21:02	137	Um...
927	20:20:58	20:21:09	qwertyuiop	I have homework to do, too...
928	20:21:03	20:21:33	137	So how the heck are we supposed to calculate the number of four-dimentional figures?
929	20:21:32	20:21:42	nan	do you want to stop here and start putting ideas on wiki?
930	20:21:47	20:21:47	qwertyuiop	ok
931	20:21:51	20:21:52	137	Sure.
932	20:22:02	20:22:09	qwertyuiop	resume from here next time?
933	20:22:16	20:22:17	nan	sure
934	20:22:18	20:22:19	137	Ya.
935	20:22:32	20:22:48	qwertyuiop	We have the 2 hexagon equations to put on the wiki.
936	20:23:04	20:23:04	137	Right.
937	20:23:11	20:23:18	qwertyuiop	Where's the wiki again?
938	20:23:27	20:23:30	nan	open "view topic"
939	20:23:23	20:23:31	137	Somewhere in the View topic button
940	20:23:39	20:23:41	nan	there's link
941	20:23:53	20:23:54	qwertyuiop	I see it.
942		20:24:28	137	leaves the room
943	20:24:57	20:25:02	qwertyuiop	i'll write it.
944		20:25:05	qwertyuiop	leaves the room
945		20:25:19	nan	leaves the room

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Investigation 18. From Intersubjectivity to Group Cognition

Gerry Stahl

Abstract. The term "intersubjectivity" is ambiguous. It can refer to the problem of how two or more minds can inter-relate: understand each other and work together from their individual cognitive positions. It can also refer to a form of joint cognition that is shared by a group and transcends, unifies or even founds the cognition of the participating individuals. This Investigation traces an historical evolution in philosophy from the former view to the later, considering in turn Plato, Descartes, Kant, Husserl, Schutz, Heidegger, Merleau-Ponty, Tomasello and Vygotsky. It proposes a view of intersubjectivity as group cognition, appropriate to CSCW and CSCL, illustrated with a CSCW example of paired programming and a CSCL example from online collaborative geometry.

Keywords. Intersubjectivity, shared understanding, joint intentionality, we-awareness, group cognition, group agency, joint participation, perspectival individuality, joint attention, shared meaning making, being-there-with-others, shared world.

The Issue of Intersubjectivity

The question of how it is possible for people to understand each other has been a controversial theme throughout the history of philosophy. It is a foundational issue for the social sciences, in which researchers try to understand the behaviors and statements of other people. It is of particular relevance to Computer-Supported Cooperative Work (CSCW) and Computer-Supported Collaborative Learning (CSCL), where participants have to understand, work with and learn with each other. Philosophers have posed the issue of how an individual can understand another and how a small group or community can have a joint understanding, shared intentionality or we-awareness. Studies of CSCW not only adopt insights from the philosophy of intersubjectivity to ground their methodology, but they also contribute to the analysis of how intersubjectivity is established in concrete settings, including in virtual environments. Similarly, CSCL research can investigate how groups of people learn to construct intersubjective understandings in both traditional and technologically enhanced interactions. While classical phenomenology of intersubjectivity started from the cognitions of a solitary mind, the notion of intersubjectivity has subsequently shifted to a more social view. Recent studies of intersubjectivity suggest a structure of group cognition, which can provide a foundation for collaboration in work and learning that incorporates but transcends individual cognition.

While "intersubjectivity" is a modern term, it points to an issue that is both as old as philosophy and as current as the lead article in a CSCW journal special issue on the topic (Tenenberg, Roth & Socha, 2015). Intersubjectivity is what makes we-awareness possible. By referencing a realm between or encompassing multiple people, intersubjectivity raises the question of whether knowing, thinking or being aware are at base matters of individual consciousnesses or of collectivities. The following historical review of the philosophy of intersubjectivity will trace a shift from a foundation in solitary minds to one in human groups or communities. It will consider the central statements concerning intersubjectivity from: Husserl, Schutz, Heidegger, Merleau-Ponty, Hegel, Marx, Vygotsky and Tomasello. Implications of the philosophic conception of intersubjectivity for CSCW and CSCL methodology—in which the analytic foundation in individual or group cognition is currently highly contentious—will then be suggested and be related to research in these fields.

The issue of intersubjectivity is paramount to our times. The major geo-political issues of the day concern how people around the shrinking globe can understand each other and relate in unity to their shared world. How can the rich and the poor see eye to eye on global ecology; how can former colonial powers and former colonies work together for peace and mutual benefit; how can populations with incompatible politics, ideology, religion and economic interests co-exist? We do not adequately grasp how people understand each other even in dyads, let alone in international communities. Researchers in CSCW and CSCL could contribute to such a comprehension, but they tend to get distracted with methodological concerns based on outmoded philosophies and approaches misappropriated from the natural sciences.

The problematic of intersubjectivity emerged in response to the growth of the social sciences a century ago. The first explicit systematic discussion was in the phenomenology of Husserl, grappling with issues in traditional philosophy.

Although the historical movement from intersubjectivity to group cognition followed multiple intertwined paths, this Investigation will present a single conceptual thread. It will review the core discussions of intersubjectivity in the primary philosophic texts that defined the concept. As we will see, the term "intersubjectivity" is ambiguous. It can refer to the problem of how two or more minds can inter-relate: understand each other and work together from their individual cognitive positions. It can also refer to a form of joint cognition that is shared by a group and transcends, unifies or even founds the cognition of the participating individuals. This Investigation will trace an evolution in philosophy from the former view to the later and will propose a view of intersubjectivity as group cognition, appropriate to CSCW and CSCL.

The Philosophy of Subjectivity: Plato, Descartes, Kant

Socrates was the poster child for the self-reflective individual, who radically examined his own life and thought. However, in the end he submitted to Athenian society as the collectivity to which he fundamentally belonged. Perhaps horrified by the consequence of Socrates' refusal to break with his corrupt, irrational and unheeding community, Plato (340 BCE) metaphorically left his fellow citizens behind in the dark cave of their traditions and illusions to emerge into true knowledge as an isolated individual. Thenceforth, truth, knowledge and learning were no longer matters founded in traditional society, but concerned eternal ideas discoverable through individual critical reflection.

The focus on individual thought found its ultimate formulation in Descartes (1633). In his argument—popularly formulated as "I think, therefore I am"—Descartes claimed that as much as he tried to doubt the reality of everything, he could not doubt that he was thinking, because his doubt was itself an instance of him thinking. If he was thinking, then there must be a subject (namely him) who was doing the thinking. Descartes thereby established as a foundation for philosophy and all knowing that an individual thinking subject existed. This raised subsequent problems, which were much harder for Descartes and his successors to address: how can this radically doubting individual subject be certain about knowledge of any object in the physical world (the problem of epistemology) and how can this isolated individual subject be certain about knowledge of other people's minds (the problem of intersubjectivity). How can one even know that a world or that other people exist external to the individual thinking subject (the problem of solipsism)?

There were many attempts to address the problems left in Descartes' wake. These produced philosophies of empiricism, rationalism, materialism, idealism, etc. Some principles from these classical philosophies were adopted as foundations of scientific method and are still assumed in many contemporary research methodologies. Kant (1787) came up with a synthesis of the major philosophic approaches of his time, still focusing on the individual human mind as the seat of pure reason. He argued that the only access we have to the world is to versions of objects that we have constructed ourselves from our sense perceptions. We structure what we sense from the world that is external to our individual minds. We do so in terms of categories of time, space and causality, which we impose in constructing the world as meaningful and knowable. That provides us with a view of the world that makes sense to us, with persistent, meaningful objects. Kant's solution to the problem of epistemology provides a form of constructivism that makes impossible "objective" knowledge (other than logical deductions) in a naïve sense. Kant demonstrated that there are many questions that are meaningless to pose—often because they presume to peek behind the constructions that our understanding of the world unavoidably erects.

The Phenomenology of Intersubjectivity: Husserl

While philosophy has always been concerned with the nature of subjectivity, the first major discussion of inter-subjectivity was by Husserl. He devoted his popular introduction to phenomenology to the problem of intersubjectivity. His *Cartesian Meditations* (Husserl, 1929) was presented at the Sorbonne in 1929. (Merleau-Ponty was in the audience as a student.) This was a couple years after Husserl's student and assistant, Heidegger, had published *Being and Time*, but Husserl's presentation was as yet unaffected by that. Husserl was concerned with the crisis of the philosophical foundations of the sciences. Dilthey and others had differentiated the human sciences from the natural sciences. Einstein and quantum theory were shaking the physical sciences with the idea that observation was relative to the observer. The foundations of logic and mathematics were in dispute. Weber and others were formulating social sciences (linguistics, anthropology, as well as sociology) in terms of meaning and interpretation, hard to objectify.

Husserl began from Descartes' argument. It starts with the solitary subject ("I") doubting everything except its own existence. In five chapters or "meditations," Husserl builds toward the central problem, intersubjectivity: How can I know another person—that he[2] exists or what he means when he speaks? For a social science today, such as CSCW or CSCL, this asks: How does one person relate to co-workers or fellow students as equally human, how does one understand the meaningful actions and statements of others? Also: How does a researcher analyze the meaning created in the discourse and in the work products of cooperating workers or collaborating students?

After introducing Descartes' position in his first meditation, Husserl shows how minds construct meaningful objects. At first, cognition is intentional, that is, directed toward some phenomenon.[3] For instance, if my consciousness is directed toward a six-sided die, I perceive at any instant only evidence of certain sides. However, over time my consciousness can synthesize the die as having six sides, perceptible from different perspectives. Then the die is intended by my consciousness as "given" with more than the immediate evidence. The meaningful die is temporally constituted by a series of perceptions and synthesizing acts in my stream of consciousness. I understand the perceived view of the die as having a horizon of possibilities, anticipations or potential remembrances that is given with the immediate perception as belonging to the meaning of the phenomenon of the observed view of the die. Husserl's third and fourth meditations outline his extensive phenomenological analyses of how the solitary subject constitutes his world and his lived temporality, starting from elementary cognitive experiences.

For Husserl, we construct or constitute our experiences of things, including other people, through sequences of cognitive acts, which are generally not conscious, but pre-reflective. Our knowledge of another person is constituted through our own processes of constructing our experience of them. We can, for instance, construct an understanding of someone else's behavior as the behavior of a person who is human like us, has a stream of consciousness like ours and has understandings like ours. We can assemble evidence for our understanding of the other person from experiences we have had—both our experiences of the other person and our own experiences that are similar or relevant. For instance, we observe our own bodies and those of others—and we see that the other is like us.

In his concluding fifth meditation, Husserl reaches the goal of his presentation and gives us a summary of the first major extended analysis of intersubjectivity. He departs from Descartes and argues that we can experience other people as also

[2] The masculine pronoun is used here to refer to people of all genders, in keeping with traditional English grammar and philosophic usage.

[3] The notion of intentionality was first developed by Husserl's teacher, Brentano (1874). Intentionality means that consciousness is always consciousness *of* something, always directed at something. Consciousness is not a purely mental phenomenon but extends into the "external" world.

sentient beings who experience the world as we do. In fact, this makes the world a shared, intersubjective one. We experience the socially shared world from our own perspective, and we see other people as also experiencing this same world from their positions:

> I experience others in shifting experiential manifolds. On the one hand, as objects in the world. Not just as mere natural things, but also experienced as psychically active in the natural bodies to which they each belong. On the other hand, I experience them simultaneously as subjects of this world, as experiencing this world—this same world that I experience myself. They are experiencing it with me, as I experience it and as within it I experience them.
>
> Even within my purely cognitive life, I experience the world including other people and the associated meaning not as a so-to-say private construction of synthesis, but as other than mine, as *intersubjective*, as existing for everyone, as having its objects accessible to everyone….
>
> To the character of the world and particularly of nature as objective, there belongs its being there for everyone, as constituted by us whenever we speak of objective reality. To this belongs the objects of the experienced world having mental characteristics, which refer to human subjects by their origins and meaning—and in general refer to other subjects and their actively constituting intentionality. This includes all cultural objects (books, tools and all kinds of works, etc.), which also carry with them the experiential meaning of being there for everyone. That is, for everyone of the corresponding cultural community, such as the European or more specifically the French. (Husserl, 1929, §43, my translation)

Husserl overcomes the solipsism of Descartes by showing that I experience others as fellow subjects in a shared world. However, this all takes place in my own consciousness and experiences. So, it is not meaningful to ask if my understanding of the other person's behavior is identical to the other person's understanding of their own behavior. The gulf of intersubjectivity is spanned by Husserl in that we can construct an understanding of the other person as a person, having their own understandings. Nevertheless, we cannot erase the gulf and obtain direct knowledge of their understanding. Any two people construct their own understanding of the shared world (including themselves and each other) from the perspective of their own subjectivity (stream of consciousness, personality, personal history, body position, etc.).

The Social Science of Intersubjectivity: Schutz

Schutz explicitly applied Husserl's approach to the social sciences, specifically to Weber's sociology. In 1932, he published a detailed and relatively clear book on the meaning-full construction of the social world (Schutz, 1932), centered around a chapter on "Foundations of a theory of intersubjective understanding." While occasionally referencing Heidegger, Schutz remained true to Husserl's phenomenology, starting from the cognitions of an individual consciousness and constructing the intersubjective world upon that basis. This was also consistent with the methodological individualism of Simmel and Weber, which held that "all concrete social phenomena should be traced back to the modes of individual behavior" (Schutz, 1932/1967, p. 4).

Schutz starts from Husserl's conclusion of the intersubjectivity of the world, namely that people take for granted the existence of other people as having the same kinds of temporal streams of consciousness and as sharing the same social world. However, since people constitute the world from their own perspective (in terms of their own bodily location, personal history, ingrained habits, action goals and subjective experiences), "the concept of the other person's intended meaning remains at best a limiting concept" (p. 98). We can only approach an understanding of another's cognition to a degree and without certainty.

To understand another person takes a reflective act. The other person typically does not understand his own action in this way: he is simply acting, not reflecting on his action. Thus, it does not even make sense to ask if a researcher's understanding of a subject's action corresponds to the subject's own understanding, since the subject probably does not have that kind of reflective understanding. If a researcher tries to triangulate his interpretation by asking a subject questions (in a test, a questionnaire, an interview, a focus group), then the subject may start to reflect on the relevant prior actions, but his newly constructed understanding or response was not something present at the time of the action, let alone motivating it in advance or causing it. Nor is the subject's retroactive self-understanding qualitatively superior to an observer's understanding of the subject, except that the subject may have access to a richer array of information about himself and his past. Like the researcher's analysis, the subject's self-understanding is also a speculative reconstruction from a series of perceived experiences.

Schutz provides analyses of meaning making, sign systems and artifacts, as they enter into our understanding of other people and of their communications, actions and interactions. He also describes concepts of "in-order-to motives" and "because motives," which can be used for understanding statements and actions, without attributing explicit knowledge to the actor. These feed into Schutz' interesting discussions of (a) the thou-orientation, (b) the we-relationship, (c) face-to-face situations and (d) direct social observation.

(a) The *thou-orientation* is a pre-reflective awareness of another person as a fellow human, who has consciousness and experiences similar in kind to my own. It thus embodies the intersubjectivity in which others are recognized as indubitable, aware, thoughtful and human. To understand another in this way is to attribute meanings, desires and plans to him. It is the first stage of intersubjectivity as a relationship between two individual subjects.

(b) When the thou-orientation becomes reciprocal, it forms a *we-relationship*, in which another and I experience the world together as a shared world. Schutz provides this example: "Perhaps while I was following the bird's flight, I noticed out of the corner of my eye that your head was moving in the same direction as mine. I could then say that the two of us, that *we*, had watched the bird's flight" (p. 165). Although we have experienced something together, that does not mean that we had the same subjective experience. For me to think about your experience, I have to step back from our we-relationship and reflect on evidence about your experience that are available to me. This is a second stage of intersubjectivity including reciprocity: I am aware that you are experiencing the same world as I am, and we are doing it together.

(c) When two people are engaged *face-to-face*, they participate together in an ongoing series of acts of meaning-establishment and meaning-interpretation (such as elicitation/response pairs of discourse utterances, in which I say something and you respond, thereby establishing the meaning of my utterance through its implicit interpretation by your response). In orienting to objects of joint attention, the participants experience the objects as common to both their experiences. They are simultaneously aware of what each other experiences as being experienced together. The shared intersubjective world is constituted by this experience in the face-to-face situation. Over time, I understand my partner in terms of his motives (personality, habits). Furthermore, I can check my understanding of the other by asking him questions (e.g., to jointly create meaning and to avoid or repair potential misunderstandings). This all takes place within the merged experiential streams of the face-to-face situation. Although Schutz does not discuss the face-to-face mode of intersubjectivity in any detail, he hints here at an intersubjectivity that is more than the sum of its parts, the two individual subjectivities. Meanings are created through the interaction between the participants; there are group processes like repair of understandings; and the experience of the world is partially shared, not completely subjective. Schutz' face-to-face intersubjectivity provides a brief foretaste of group cognition.

(d) Schutz then contrasts the face-to-face situation (e.g., of participants collaborating) with *direct social observation* (e.g., by a social-science researcher).

Direct social observation is very different from the face-to-face situation. The observer is not engaged in the same undertaking as his subject, nor is he engaged with the subject in a shared context of action. Furthermore, the observer does not have the same kinds of access through interaction to check on and repair his understandings of the subject's subjective experiences, motivations or attempts. The close mutuality and reciprocal mirroring of the face-to-face situation is missing in a context of objective observation. Schutz specifies three possible indirect approaches for scientific observation of a subject's motives: An observer can interpret the subject's behavior in terms of what he imagines he himself might have done under the circumstances. Alternatively, he can take into account the customary behavior of that kind of person (e.g., applying Weber's ideal types). Finally, he can interpret the observed behavior "in terms of the effect which it actually has and assume that the effect is what was intended" (p. 175). These modes of understanding other people and of intersubjectivity appear in various methodologies of CSCW and CSCL research.

The Being of Intersubjectivity: Heidegger

By the time Husserl's and Schutz' analyses of intersubjectivity were published, Heidegger's implicit repudiation of these theories was already widely read. Although Heidegger emerged from the Husserlian school of phenomenology and was deeply steeped in traditional philosophy, his *Being and Time* presented a radical rejection of the starting point of individual consciousness. In this sense, he left behind not only the constructivism of Kantian pure reason, but also the cognitivism of any methodological individualism. Heidegger's analysis of human existence began with the unity of being-in-the-world, where people exist through their essential involvement in the world. This involvement includes being-there-together in the shared world with other people.

Heidegger's analysis of being-there-with-others (Heidegger, 1927, §§25-27) is laced with barbs against the positions of Husserl and Schutz. Heidegger refers to the enterprise of seeking a transition from the isolated individual to the other as a "mis-understanding" and explicitly rejects the conception of the unity of the self "as the identity of the I maintaining itself in the multiplicity of its 'experiences'" (p. 122).

Human being—as our openness to the world—is defined according to Heidegger, first and foremost, by the collectivity of other people, with whom we are concerned and with whom we share a joint world, filled with meaningful artifacts and natural objects that we deal with together. However, this collectivity is described abstractly by Heidegger—not in terms of our family, friends, colleagues, neighbors, community or society. In fact, it is portrayed in rather dark tones, as an oppressive or at least obscuring view of the world through the outlook of an unenlightened mass culture.

Heidegger argues that because we are caught up in this distracting and obscuring culture and are constantly busily distracted by other people, with the objects in the world of our concern and in our projects involving them, we cannot see our own true nature as being-there-with-others. Rather, we see things—including other people and even ourselves—in terms of an ontology of physical objects and mental ideas (*à la* Plato, Descartes and the common sense of the collective). Unfortunately, after his brief but central and pivotal analysis of being-there-with-others, Heidegger shifts from the social basis of human existence, which he had finally uncovered, to a focus on the individual self as a secondary ontological mode, which supposedly provides greater understanding of human being than the collective view. He values this derived mode as more "authentic," although ironically it is close to the individualistic reflective mode of Husserl. Heidegger, thus, retreats from the social foundation he briefly established. By not elaborating this more concretely through contact with the other mainstream of German philosophy developed by Hegel and Marx, Heidegger remains at the level of politically conservative cultural criticism (Adorno, 1964/1973) and heads toward his fateful political error (Stahl, 1975).

The Corporeality of Intersubjectivity: Merleau-Ponty

Merleau-Ponty studied both Husserl and Heidegger carefully, including especially their responses to Descartes' problem of intersubjectivity [4]. Merleau-Ponty (1945/2002) fleshed out their analyses with an in-depth analysis of the role of the body and of embodied perception in human being and thinking. His chapter on other people and the human world comes as the culmination of his phenomenological description of human existence. He argues that the experience of another person—such as my sense of the other's grief or anger—is given immediately in my perception of his bodily contact and expression, not mediated through some form of my reflection on what his inner experiences must be like based on remembrances of similar experiences of my own (p. 356). We thus strive

[4] When Merleau-Ponty died, he was found with his head literally in a book by Descartes, perhaps struggling to the end with the question of intersubjectivity.

to project a shared world, in which we can communicate, for instance about our grief or anger. We each do so from our own bodies, as corporeal actors.

Intersubjectivity is given with our being embodied in a shared world and forms a basis for our subjectivity. Intersubjectivity could not be "constituted" subsequently by isolated individual consciousnesses. As Merleau-Ponty says, "My greatest attempt at impartiality would never enable me to prevail over my subjectivity (as Descartes so well expresses it by the hypothesis of the malignant demon), if I had not, underlying my judgments, the primordial certainty of being in contact with being itself, if, before any voluntary adoption of a position, I were not already situated in an intersubjective world" (p. 355). Merleau-Ponty adopts Heidegger's view of being-there-with-others as fundamental to the human condition. However, he does so more concretely and persistently. He refers to the perception of the other's body as material, meaningful and expressive. He cites evidence from child development that infants exist in a shared world without even differentiating themselves from others—so that subjectivity is seen to be a derived and learned phenomenon, not an absolute Cartesian starting point.

In addition, Merleau-Ponty looks at the role of language in the perception of other people. Language is essentially social; it transcends the individual and it merges the perspectives of multiple speakers. He describes eloquently how dialogue can establish a shared thinking in the verbal interaction of two people:

> My thought and his are interwoven into a single fabric, my words and those of my interlocutor are called forth by the state of the discussion, and they are inserted into a shared operation of which neither of us is the creator. We have here a dual being, where the other is for me no longer a mere bit of behavior in my transcendental field, nor I in his; we are collaborators for each other in consummate reciprocity. Our perspectives merge into each other, and we co-exist through a common world. In the present dialogue, I am freed from myself, for the other person's thoughts are certainly his; they are not of my making, though I do grasp them the moment they come into being, or even anticipate them. And indeed, the objection which my interlocutor raises to what I say draws from me thoughts which I had no idea I possessed, so that at the same time that I lend him thoughts, he reciprocates by making me think too. It is only retrospectively, when I have withdrawn from the dialogue and am recalling it that I am able to reintegrate it into my life and make of it an episode in my private history. (p. 354)

Through elicitation and response, the utterances of people in dialog produce a cognitive stream that is not attributable to either speaker individually, but is a group process that only makes sense as such. This is a description of collaboration as an intersubjective form of cognition. There is a common world, in which the two personal perspectives are integrated in a single process of intersubjective meaning making—a “shared fabric.” The view of an individual’s contribution to the dialog is a retroactive view, the result of subsequent reflection and appropriation into one’s (linguistic) self-narrative.

Merleau-Ponty’s description of the intersubjective source of my own creativity is particularly striking. The other draws from me thoughts “which I had no idea I possessed.” Of course, I did not “possess” such thoughts ahead of time—they emerged from the discourse. Nevertheless, they were understood by everyone as being my thoughts, from my perspective and due to my agency. Here we get a glimpse of the power of intersubjective collaboration—and of how it is systematically covered over by commonsense views, interpretations and retrospective accounts.

This model of intersubjectivity goes beyond Husserl’s and Schutz’ analyses of the individual’s “transcendental field.” It also escapes Heidegger’s version of intersubjectivity as an obfuscating mass culture. Merleau-Ponty agrees that one can step back from intersubjective engagement to reflect on one’s personal life, but now with positive insights about one’s own thinking that would not otherwise have occurred. Finally, we have a conception of intersubjectivity that values the potential of collaboration and of our concrete joint life in a shared world. Here, intersubjectivity can be a primordial experience, which provides a foundation for individual consciousness as derivative.

In recent decades, followers of phenomenology have adopted the shift of starting point from the individual to the shared world, pioneered in Heidegger’s being-there-with-others, the later Husserl’s lifeworld and Merleau-Ponty’s intersubjectivity. For instance, Schegloff (1991, p. 168) writes, “In Western tradition, it is the single, embodied, minded individual who constitutes the autonomous reality.” He then contrasts the view of phenomenologically inspired ethnomethodology and conversation analysis to this earlier dominant cognitivist tradition: “Interaction and talk-in-interaction are structured environments for action and cognition, and they shape both the constitution of the actions and utterances needing to be ‘cognized’ and the contingencies for solving them.” As their names suggest, ethnomethodology describes the pervasive methods that people (ethno) use for creating social order during their interactions, and conversation analysis describes the patterns of talk that people use to support intersubjective understanding of the public meaning that is thereby created in the shared world. This approach details the rich and orderly variety of mechanisms that are used in human interaction to constitute and maintain intersubjectivity.

In addition to his phenomenological roots, Merleau-Ponty appreciated the other major philosophic tradition in twentieth-century European thought, that of Hegel and Marx, to which we turn next.

The Dialectic of Intersubjectivity: Hegel and Marx

When the movement of social history became conspicuous with the American and French revolutions, the march of Napoleon and the early stirrings of the industrial working class, Hegel captured the nature of his dynamic times in his philosophy. His early lectures in particular defined a break with Kantian methodological individualism and described the social nature of man (Habermas, 1967; Hegel, 1806). This led to a philosophic approach to subjectivity contrasting to that of Husserlian phenomenology, which had remained neo-Kantian.

Until Hegel, human nature and human cognition were conceived as based in the individual person, as fully determined from birth ahistorically or universally—not dependent on one's biography or social context. The theories that minds develop, that social relations transform or that humanity evolves all came after Hegel—in process-oriented sciences inspired by his philosophy. For Freud, Marx and Darwin, to understand a psyche, a social formation or a species requires understanding the history of its development, complete with conflicts and resolutions.

Hegel elaborated a dynamic view, in which mind develops all the way from primitive sense perception to sophisticated self-consciousness and cultural worldview. In the methodological Preface to his most influential presentation of the development of mind, Hegel (1807) wrote that one must analyze a phenomenon by looking at its unity as the result of its clashing temporal appearances:

> The bud, the blossom and the fruit's fluid nature make them into moments of an organic unity within which … one is equally as necessary as the other…. The subject matter is not exhausted in its ends; rather, it is exhaustively treated while it is worked out. Nor is the result that is reached the actual whole itself; rather, *the whole is the result together with the way the result comes to be*…. What is the most difficult of all is to grasp both what unites the process and the result, and to give a full exposition of what that is. (§2 & 3, my translation and italics)

Let us see how Hegel treated interaction between two people in his famous master/slave dialectic. A person first becomes aware of himself as a particular individual at this developmental stage within Hegel's system. The analysis focuses on the interaction of people and involves them working with objects in the world. The cognitive effect (self-consciousness) is a result of the whole dynamic of the interaction, not a pre-existing causal agent within the interaction. The prototypical interaction is here that of a worker creating an artifact; the worker recognizes himself as reflected in the product that he created to meet the needs of another person:

> Work *gives form* to its object. The worker's transforming relationship toward the object is transformed into the object's form and becomes something *persisting*, because for the worker the object gains self-sufficiency. This transforming mediation—the *activity* of forming—is also the *individuality* of consciousness or the pure being-for-itself of consciousness, which in the work process now steps out of consciousness and takes on the character of persistence. The consciousness of the worker thereby arrives at a perception of the self-sufficient artifact as a perception *of his self*. (Hegel, 1807, p. 238, my translation)

Hegel shows how human consciousness emerges through productive activity in the intersubjective and physical world. The worker and the master (for whom the object is produced) are formed as such (i.e., as self-conscious individual actors) through the interaction with each other and with artifacts (tools and products of work) in the world. Hegel describes the emergence of self-consciousness from within the process of mutual recognition of self, world and other. In particular, it is the worker, who produces an artifact in the physical world at the bidding of an other, who is then able to perceive his labor as externalized and made persistent in the artifact. The worker's self-consciousness emerges through his activity in the shared world, where he comes to see himself as objectified in his artifacts and through the eyes of others.

Marx (1867) builds on this analysis of social interaction. He situates Hegel's idealist analysis in the historical context of early capitalism. The artifact that is produced by the worker's labor and that externalizes his self within its social relations to other people is specified within settings of capitalist production into a commodity (an artifact produced for sale on the open market). The worker's self-consciousness is reified, alienated and fetishized because the commodity that reflects his identity is no longer his (but the capitalist's, who sells it) and because his social relations to potential users of the artifact are transformed into the abstract monetary value of the commodity. The meaning of the labor that went into forming the product's use-value undergoes multiple complex social transformations: it is externalized into an artifact, the artifact enters commodity relations and the commodity is reflected back to the worker as monetary exchange-value belonging to his boss. For Marx, individuals in capitalist society are analyzed as results of their interactions as wage laborers, owners of the means of production or consumers of commodities. He critiques the traditional notion of the abstract individual consciousness as an ideology of individualism that obscures concrete, historically specific human reality.

In his methodological *Grundrisse*, Marx (1858) identifies the interaction in which the worker exchanges his labor time for the capitalist's wages as the "cell form" for analysis. His analysis in *Capital* (1867) starts out from the simple dyadic interaction of a worker exchanging the product of his labor with another person. As his inquiry into social production in the capitalist era develops, this elemental intersubjective relation of production is mediated by its dialectical relationship to technology as the social means of production (e.g., the factory system, machinery and automation in their historical development).

Intersubjectivity in this approach of Hegel and Marx is a concrete social and historical product of human labor with material artifacts. The subjectivity of individuals is a subsequent by-product of their interactions within the shared social world. The Kantian view of the individual mind producing the world is stood on its head. Mind is seen to be a social product, and individualism is characterized as an ideology serving competitive capitalism.

In a contemporary extension of this tradition, Habermas (1971) has argued for viewing communicative action as the basis for intersubjectivity and social theory. He starts by explicitly rejecting the individualism of Kant and Husserl, which do not allow escaping from monadic subjectivity. Incorporating the linguistic turn of Wittgenstein (1953), Habermas reconstructs the possibility of moral behavior and social science from the interpersonal relationship between people engaged in communicative action. The dialectical tradition takes as its starting point the social interaction among people in place of Descartes' isolated subject. It focuses on the dynamic and conflictual mediations of this interaction within the concrete, historical world.

The Mediation of Intersubjectivity: Vygotsky

Vygotsky provides a psychology of human cognition appropriate to Marx's methodology of social science. He adopts Marx' analytic cell form: the interaction among people mediated by artifacts. For Vygotsky, the notion of artifact encompasses both tools and language. Artifacts are both physically present in the world and meaningful to people. Their meaning is not projected from individual minds, but is intersubjectively emergent from social interactions, as in the dialectical presentations of Hegel and Marx.

As discussed in the section of Investigation 15 on a post-cognitive educational paradigm, Vygotsky analyzed the genesis of the pointing gesture as an artifact whose form and meaning emerge from the interaction between multiple actors, such as an infant and its mother. The actors form an intersubjective group, whose joint attention is directed toward some intended object by the co-constructed pointing gesture. This view of intentionality as emergent in the shared world stands in sharp contrast to the rationalist assumption that individuals "have" personal mental aims which they then express in speech or action and which others may notice and interpret in their own minds. Marx, Wittgenstein and Heidegger (the primary founders of the major approaches in twentieth-century theory)—and their followers—soundly reject the cognitive picture of agency (see, e.g., Dennett, 1991; Dourish, 2001; Dreyfus, 1992; Ehn, 1988; Suchman, 2007). The rationalist, mentalist view persisted in the theories of intersubjectivity of Husserl and even Schutz, as we have seen.

Although Vygotsky was a trained psychologist and even a behaviorist during part of his career, his research agenda points toward a vision of group cognition. For instance, Vygotsky's analysis of learners' "zones of proximal development" stresses the origin of higher forms of human cognition in developmental processes involving intersubjective meaning making. In a formulation evoking Hegel, he writes of the need to analyze such developmental processes, not just learning outcomes: "The zone of proximal development defines those functions that have not yet matured but are in the process of maturation, functions that will mature tomorrow but are currently in an embryonic state. These functions could be termed the 'buds' or 'flowers' of development rather than the 'fruits' of development." He then cites a study in which children "could do only under guidance, in collaboration and in groups at the age of three-to-five years what they could do independently when they reached the age of five-to-seven years" (Vygotsky, 1930/1978, pp. 86, 87). In Vygotsky's approach, cognitive development is founded on the intersubjectivity of collaborative learning. The results of collaborative learning may later appear as individual learning through subtle processes of "internalization."

In his less quoted section on "Problems of Method," Vygotsky (1930/1978, pp. 58-75) called for a new paradigm of educational research almost a century ago. Arguing that one cannot simply look at visible post-test results of an experiment, he approvingly quoted Marx: "if the essence of objects coincided with the form of their outer manifestations, then every science would be superfluous." He then emphasized the Hegelian approach: "*To study something historically means to study it in the process of change*; that is the dialectical method's basic demand. To encompass in research the process of a given thing's development in all its phases and changes—from birth to death—fundamentally means to discover its nature, its essence." In Vygotsky's analysis based on his experiments with young children, the skills and understanding of individuals is traced back to their long-forgotten origins in intersubjective meaning making.

Vygotsky (1930/1978) outlines an intersubjective conception of the development of human cognition and collaborative learning, which treats the interaction, development and learning of small groups with artifacts in the shared world as foundational. We shall see a concrete example of this approach toward the end of this Investigation. One's understanding of oneself, of artifacts (including representations, gestures, signs, symbols, language) and of the meaningful world is constructed primarily and originally intersubjectively, socially and culturally. The individual is a result of subsequent processes of internalization, including the transformation by young children of speech as intersubjective communication into self-talk and then into silent verbal rehearsal or thinking.

The Evolution of Intersubjectivity: Tomasello

Tomasello (2014) complements Vygotsky's dialectical psychology with a corresponding evolutionary anthropology. He offers us a theory of intersubjective intentionality based on an analysis of human evolution and how human intentionality diverged from that of other primates throughout pre-history. He complements Vygotsky's historical, developmental analysis of a child's learning with a similarly historical analysis of the development of human cognition by the species.

Under environmental pressures, humans developed increasingly complex forms of cooperative sociality (see also Seddon, 2014). Tomasello describes a two-step evolutionary sequence: *joint intentionality* followed by *collective intentionality*. At both of these transitions, a similar process took place. "A change of ecology led to some new forms of collaboration, which required for their coordination some new forms of cooperative communication, and then together these created the possibility that, during ontogeny, individuals could construct through their social interactions with others some new forms of cognitive representation, inference, and self-monitoring for use in their thinking" (p. 31).

Perhaps the first step took place in the context of collaborative foraging. Early human individuals— in response to a changing feeding ecology—began to join other individuals in pairs in pursuit of shared goals, and they jointly attended to situations relevant to their common goals.[5] "Each participant in the collaboration had her own individual role and her own individual perspective on the situation as part of the interactive unit" (p. 78). Tomasello highlights this dual-level structure—*simultaneous joint participation and perspectival individuality*—as a defining structure of what he calls joint intentionality. For him, it is foundational for all

[5] Evolutionary development of mirror neurons and increased brain structure on the biological level may have accompanied and facilitated this increased sense of mutuality on the cultural level as a competitive advantage (Gallese & Lakoff, 2005), but see also (Hickok, 2014).

subsequent manifestations of human shared intentionality. Of course, early humans had always lived in family units and small tribes (like other primates), but now they began to carry out tasks like strategic hunting in small teams as an "interactive unit."

The second step took place more recently, as agriculture and domestication of animals led to the founding of the first great civilizations. Modern humans became predominantly cultural beings by identifying with their specific cultural group and collectively creating various kinds of cultural conventions, norms and institutions (p. 80). They thus became thoroughly group-minded individuals. Tomasello argues that the development of joint and collective intentionality provided a necessary foundation for the development of human language and culture, which allowed for the escalating evolutionary emergence of modern human cognition and thinking (p. 128). This rapid form of evolution took place through historically transmitted culture (Donald, 1991; 2001), rather than as biological adaptation. Increasingly, our individual cognition became mediated by and derivative of group, collective, cultural and now even global cognition.

Intersubjectivity—as the recognition of other people as having the same kinds of comprehension capabilities as we do (so-called "theory of mind")—involves perspective taking, being able to view from the other person's position. For instance, to understand what someone says to me, I have to be able to understand the utterance as coming from the other person, as she might have understood it in articulating it. I also have to understand it as having been designed for me to understand it ("recipient design"). So, I have to recognize the speaker as someone who understands meaning and can create it, as well as someone who knows how I might understand what she says. This mutual or reciprocal recognition is a precondition for distinctively human communication (e.g., as evolved beyond animal vocal signaling). Intersubjectivity is a foundation for—a condition for the possibility of—modern human interaction (Duranti, 2010).

Of course, our understanding of each other is only tentative and partial. There is no possibility of absolute knowledge of other minds or of identity of mental contents, as Husserl and Schutz argued. Shared understanding is, rather, taken-for-granted, not objective. Furthermore, the sharing is generally developed only to the point necessary to maintain communication (Linell, 2014). In general, understanding is always partial and pragmatic; I only understand even my own thoughts enough to continue engaging in the current activity that involves those thoughts.

As Heidegger put it, understanding is an aspect of our being-in-the-world, of situated activity rather than of abstractly mental cognition. We understand something *as* something to the extent necessary for our dealings with it. Accordingly, our shared understanding with other people should be seen as an aspect of our being-there-with-others in the same world [Investigation 17]. We share understandings because we share one world; and we do so to the extent necessary for our care for things in the world and our concern for other people as part of our existence in the social world (with our background, our plans, our situation).

The discussion of intersubjectivity in twentieth-century philosophy and social-science theory has moved decisively away from the rationalism of Descartes and its focus on the reasoning of an individual mind. We are embodied in a shared world, and we understand ourselves, each other and our world through social interaction, gradual cognitive development and cultural transmission. Intersubjectivity can be more than just the confrontation of independent individuals. It can include the collaborative production of joint meaning in a shared world, where the interaction can result in a unity that is more than a simple aggregate of the inputs of the individuals.

The refined conceptions of shared understanding in our intersubjective world that emerge from the preceding review are suggestive for research in CSCW and CSCL. We turn now to examples of empirical studies from these fields.

Intersubjectivity in CSCW

The lead article in the CSCW journal special issue on intersubjectivity, by Tenenberg, Roth & Socha (2015), documents an instance of intersubjectivity in which there is joint attention and mutual recognition. Many of the characterizations of forms of intersubjectivity summarized above can be related to the recorded actions of software programmers Hank and Danny in that case study and to the analysis of the data by the article authors.

All of the theory sources considered in this Investigation have discussed the importance of one person seeing the other and being able to observe that they were attending to the same objects. This was a central theme in the lead article as well. The pair-programming work environment being studied was carefully structured so that the participants could see each other and could track each other's general gaze. This environment was an interesting hybrid of face-to-face and computer mediated. In fully online alternative systems mentioned in the article, the awareness of joint attention was either supported with specific functionalities or seen to be problematic.

The article explicitly focuses on the initial alignment phase of Hank and Danny working together. Consequently, we do not get to observe much of how they subsequently proceed in accomplishing their shared work in a fully intersubjective mode. The data presented gives a glimpse into a very narrow—but critical—slice of the intersubjective experience. As the authors note, Hank and Danny are very much at home in their specific work world and only need to align around the particular task at hand. These programmers are experienced at working together in this paired manner. The physical and technical environment has been carefully set up to support their closely coupled cooperative work, and they move around within it skillfully, without displaying explicitly much of the understanding or practices that contribute to such being-there-with-others.

Paired programming—like intersubjectivity itself—can be viewed in two ways. In one (like Husserl's), there is cooperation between two subjectivities, who coordinate their actions and reciprocal understandings of each other in two parallel streams of individual cognition. Excerpts 1 and 2 in the article include division of labor, for instance where Danny will write a list on paper while Hank operates the computer. In this view, one programmer may bring in resources (knowledge, skills, processes, artifacts) that the first does not have, or the second programmer can provide an immediate check on the work accomplished by the first.

In the alternative view (like Tomasello's), the pair collaborates in a single cognitive process of jointly accomplishing the programming task. For instance, the interaction presented in Log 1 can be seen as the pair narrowing in on a relevant object together through their joint attention to a list on the screen and their interactive construction of an increasingly narrow focus within that list.

3.1	Danny:	((Just before he starts talking, Danny moves his left hand that is holding a pen so that the pen points to a specific item on a dropdown menu on the left monitor)) I bet you if
3.2		((at apex of point, with pen tapped on screen))
3.3		((Hank selects item on list that is four items below Danny's point, which is highlighted on the display))
3.4	Danny:	you (go?) ((starts to withdraw hand))
3.5	Hank:	((Hank uses mouse to move cursor two elements higher on the list))
3.6	Danny:	bidoni
3.7	Hank:	((Hank moves up two additional elements on list, stays there))
3.8	Danny:	m-t-m black

Log 1. Interaction by pair-programmers.

The authors first describe the actions of the programmers: "Danny uses physical gestures and speech that complement and complete one another to direct Hank to a specific location. Hank uses the mouse for placing the cursor preparatory to acting with it, which, in its visibility to Danny takes a role in the 'conversation' that the two are having concerning the specific location of the next operation." Then the authors summarize the interaction as follows: "They thus combine a variety of semiotic resources to give this fragment its orderly, sequential character." What they call the programmers' "conversation" (including words, cursor movements, pointing gestures and mutual bodily visibility) is in fact a single, well-ordered achievement. It is irrelevant which programmer introduced which resource. All the resources received their meaning from the unfolding joint process of locating the cursor on a particular font name so that the team could work on that object. The actions of the two programmers form a single orderly sequence.

In the analysis of this work as a collaboration, the two programmers are seen to be checking—or grounding (Clark & Brennan, 1991)—their understanding of each other through their utterances, repairs, gestures and gazes. This reciprocal testing of interlocutors' understandings corresponds to the mutual reciprocity of knowledge in some of the theories of intersubjectivity reviewed above. Certainly, Husserl and Schutz, with their orientation to individual consciousnesses, relied heavily on one subject's knowledge that the other knows that the first knows that…. Even Tomasello focuses on the recursive recognition of other minds as sentient and perspectival. While Tomasello is persuasive that the evolution of this capability of recursive recognition to arbitrary levels was a necessary evolutionary precondition for modern human cognition and collaboration, that does not mean that we must always engage in some sort of mental recognition that you understand that I understand, etc. There may be occasions when this is indeed necessary, but only then does it actually have to be carried out. Furthermore, we have the ability to respond to questioning by making retroactive statements of mutual recognition to arbitrary levels of recursion. However, this need not enter into most activities of joint understanding. Such mutual recognition is already implicit in the fact of joint understanding. It is taken for granted in Heidegger's being-there-with-others, in which we care for each other as human, or in Merleau-Ponty's gaze, in which we see the body of the other as another human perspective on our shared world.

In his recommendations for social-science analysis, Garfinkel (1967) noted that common ground is established by the methodical ways in which things are said, not by a process of verifying agreement of the sets of presumed mental contents stored in the heads of the speaker and of the hearer:

> For the conduct of their everyday affairs, persons take for granted that what is said will be made out according to methods that the parties use to make out what they are saying for its clear, consistent, coherent, understandable, or planful character, i.e., as subject to some rule's jurisdiction—in a word as rational. To see the "sense" of what is said is to accord to what was said its character "as a rule." "Shared agreement"

> refers to various social methods for accomplishing the member's recognition that something was said-according-to-a-rule and not the demonstrable matching of substantive matters. The appropriate image of a common understanding is therefore an operation rather than a common intersection of overlapping sets. (p. 30)

The authors of the lead article have gone to pains to avoid mentalist explanations. They formulate their discussion of aligning visual fields in terms of the methodical ways of establishing joint attention to a shared object rather than as checking that one subject knows that the other is looking at the object, and the other knows that the first knows that, and so on. The establishment of joint attention—so necessary for collaboration—entails that the people involved are looking at the same object *together*. They do not just happen to be both individually oriented to the object, but are oriented toward it in a coordinated way. They do not have to be separately aware of the assumed recursive mutuality of this relationship—unless there is some kind of breakdown that needs to be repaired by checking verbally on the mutuality of gaze to some recursive depth. A contribution of the lead article analysis is to explicate the need to support the participants' operations of maintaining awareness of the mutuality of their joint attention and to describe their methods of doing so in their hybrid environment.

Just as there is an ambiguity to the method of paired programming between cooperation (with division of labor) and collaboration (working together on each step, although possibly from different perspectives or with different resources), so there is an ambiguity to the transcript in Log 1. While we have viewed the interaction there as a single, coherent, meaningful achievement, it could also be viewed in terms of the distinct actions of two individual subjects. One could speculate that Danny had himself identified the item in the list on the computer screen from the start by tapping on it with his pen. Then Hank followed Danny's guiding gestures to eventually recognize the same item by highlighting it with his cursor. This is a pervasive ambiguity in the analysis of CSCW data. To decide in favor of an analysis that treats the group as the primary agent or one that focuses on the contributions of individuals generally requires detailed interactional data, which is rarely available to researchers. For instance, if the transcript did not include Danny's bodily gestures and Hank's computer actions in addition to the spoken discourse, it would be impossible to analyze the identification of the font as a joint achievement.

The alignment phase involves a transition from individual cognition to intersubjective cognition. It therefore contains elements of each and can be analyzed at either the individual or group unit of analysis. At the individual level, it appears that subjects are monitoring each other's gaze or focus of attention. Here is where the reciprocal and recursive recognition come in and the conception of communicative signals being exchanged. Especially in the case of dyads, it is tempting to analyze individual intentionality and agency in a traditional, individualistic way; in somewhat larger groups, the interaction is often harder to attribute to individuals as the discussion builds on individual utterances in complex ways and takes twists that no one participant could have planned. At the group level of description, the group is beginning to act as a unity, creating social order and joint meaning in a shared world—not through independent acts of the individual participants, but through the interaction of the group.

The ambiguity is important. The point is not so much to always opt for an individual or a group focus, but to recognize their intertwining: that the individual is a social product, but also that the intersubjective has the individual at its poles. Sometimes one unit of analysis is more useful than the other. Efforts at alignment, in particular, involve a transition from multiple individual cognitions to unified group cognition. Philosophies of dialogicality have long tried to maintain this balance of what Tomasello calls joint intentionality with individual perspectives, which is not well supported by our inherited conceptualizations (Rommetveit, 2003; Wertsch, 1991). Interaction analysis—as carried out in the lead article—has shown us how to analyze the displayed utterances of individuals as part of intersubjective processes of group meaning making and social-order construction, without hypothesizing hidden mental phenomena (Schegloff, 1991).

To understand we-awareness or intersubjectivity once a team has come into alignment and is working smoothly together, it would be useful to analyze excerpts of interaction in later phases with the same kind of detail provided for the alignment phase in the lead article. Fuller examples of completely online group work would also be relevant to CSCW. The authors note a paucity of appropriate, detailed data about computer-mediated CSCW interactions on work like paired programming using different mediating technologies. In addition, we might add, there is little data reported about how people first learn to interact skillfully within such contexts. For a suggestion of how intersubjectivity might be analyzed and supported in more contexts, we turn to CSCL.

Intersubjectivity in CSCL

The relation of CSCW to CSCL has not been widely noted or clearly articulated. Both involve computer support for people interacting. While CSCW has the advantage of studying people who are expert at their work and experienced at working together, CSCL has the advantage of observing how such expertise and such interaction between people is originally constituted and learned. CSCL education can prepare students for careers in CSCW workplaces, and CSCW can display domain-related practices for adoption in CSCL curricula. The two fields share an interest in how individual and intersubjective cognition complement each other and how computer-support artifacts or environments can mediate between them.

We will now review a specific research agenda that explores the nature of intersubjectivity in a variety of small-group math-education settings. We will present examples of intersubjective knowledge building under several diverse, but typical learning conditions, involving computer mediation.

Based on research in CSCW and CSCL, Stahl (2006) proposes a form of intersubjectivity called *group cognition*. Group cognition can be thought of as a form of intersubjectivity that goes beyond the mutual recognition of individual minds in Husserl and the recursive thou-relationship of Schutz to a being-there-with-others that Heidegger and Merleau-Ponty briefly hint at. Its analysis is based on the social-historical-cultural approach of Hegel, Marx and Vygotsky. It is a developed form of Tomasello’s joint intentionality with individual perspectives. Group cognition is a vision of intersubjectivity for CSCW and CSCL, which exceeds the accomplishments of individual cognition within group efforts, especially in online settings.

In group cognition, multiple people participate in coherent interactions that achieve cognitive accomplishments which are best analyzed at least in part at the group unit, rather than attributing contributions and agency entirely to individual minds. When a number of people are involved in group-cognitive processes or activities, their individual utterances or actions are taken as merged in a single cognitive system, which is distributed across the people and the artifacts that are involved (Hutchins, 1996). Ideas, practices, habits and traditions from the larger culture are also brought in, so that the group cognition mediates between individual and community levels of description (Stahl, 2013, Ch. 8).

The original elaboration of the notion of group cognition emerged from a series of studies of software environments to support perspectives, negotiation and group formation in specific workplace and school settings (Stahl, 2006, e.g., Ch. 3, 6, 8). It provided, for instance, a detailed example of group cognition, in which a face-to-face student group co-constructed the applicability of a scientific representational artifact in an educational computer simulation in 1998 (Ch. 12 & 13). However, the collection of studies also acknowledged that the vision of group cognition as an effective form of collaborative learning is rarely achieved in practice. Furthermore, it noted the difficulty of finding or collecting data that is adequate for establishing and analyzing group cognition, let alone for observing the mediation across units of analysis.

Later (from 2002-2015), the Virtual Math Teams (VMT) system was developed as a test ground for studying group cognition. VMT is a collaboration environment for mathematical problem solving by online small groups of students. Reports on pedagogical and methodological issues in VMT (Stahl, 2009) include analysis of a text chat in which several online students solved a challenging word problem collaboratively that none could solve individually (Ch. 5). The analysis argues that their chat could be viewed as a group-cognitive accomplishment, integrating a chain of interactive responses similar to a solution that could have been stated by one person, but here involving the whole group as the problem-solving agent. Another case study (Ch. 7) discusses how three students working online in VMT with a shared graphics whiteboard maintain joint attention to geometric details and organize their graphical, symbolic and narrative interactions to solve an intricate problem in combinatorics collaboratively.

More recently, the VMT environment was extended with a custom multi-user version of GeoGebra, an application for dynamic geometry. A stimulating problem often given to people once they become comfortable with dynamic geometry is that of constructing inscribed triangles that behave like a given pair of inscribed triangles. (See the instructions and inscribed triangles ABC/DEF in Figure 1.) This is a difficult task even for adults who enjoy mathematics. The VMT research team looked closely at the logs of a group of three 14-year-old girls who succeeded with this problem in less than an hour. None of the students had studied geometry before joining an after-school math club as part of the research project; they spent four hours working together on collaborative dynamic geometry before this session.

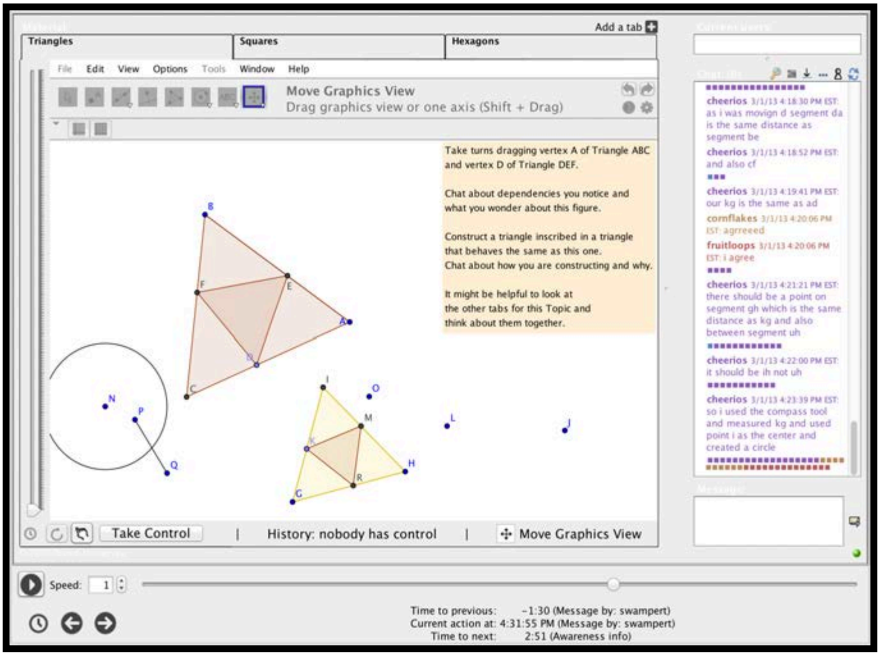

Figure 1. The state of the inscribed-triangles construction after Fruitloops finished triangle KMR inscribed in GHI.

The analysis of the team's work concludes that the students' success was an instance of group cognition (Stahl, 2013, Ch. 7.3). None of the students could construct the triangle configuration themselves, and the process of construction involved all three exploring, planning and carrying out the construction. Each of the three girls displays a different characteristic behavior pattern throughout their work in the eight hour-long sessions of our study. Yet, the team is impressively collaborative. This illustrates nicely the notion of individual perspectives within intersubjective group interaction.

What is particularly striking in the team's successful construction of the inscribed triangles is that on first appearance it seems like the team's insightful and skilled work is actually done primarily by the student who until then had seemed the least insightful and skilled. If one just looks at the chat postings (see panel in the right side of Figure 1), Cheerios does all the talking and Fruitloops (who is usually the most reflective and insightful) and Cornflakes (who explores the technology and often shows the others how to create geometric objects) simply register passive agreement. However, the actual GeoGebra construction actions tell a far more nuanced story. First, for most of the hour, each of the three students in the "Cereal Team" takes extended turns exploring the given example of inscribed triangles by dragging the vertices to discover dependencies in the construction that dynamically maintain the invariances of equilateral triangles. The dragging of figures is displayed simultaneously on each student's computer. Only one person at a time can create or drag geometric objects, in order to maintain joint attention by everyone to a single, shared sequence of actions.

Cheerios observes Fruitloops experimenting with the use of the GeoGebra compass tool just before Cheerios takes control and makes her discovery. Cheerios continues to manipulate Fruitloops' construction, involving a circle whose radius was constructed with the compass tool to be dependent on the length of a line segment. Then Cheerios very carefully drags points on the original inscribed-triangle figure to discover how segments BE and CF are dependent upon the length of segment AD, refining prior movements by the other students. The dynamic relationship between the side lengths becomes visually salient as she increases the size of the triangles or their orientation and as she drags point D along side AC.

Cheerios has a sense that the compass tool should be used to measure segment KG, but she does not quite understand how to make use of that tool. Following Fruitloops' example, Cheerios uses the compass to draw a circle around point I, whose radius equals length GK (see Figure 2, left). However, she is unable to further implement the plan she has already projected in chat.

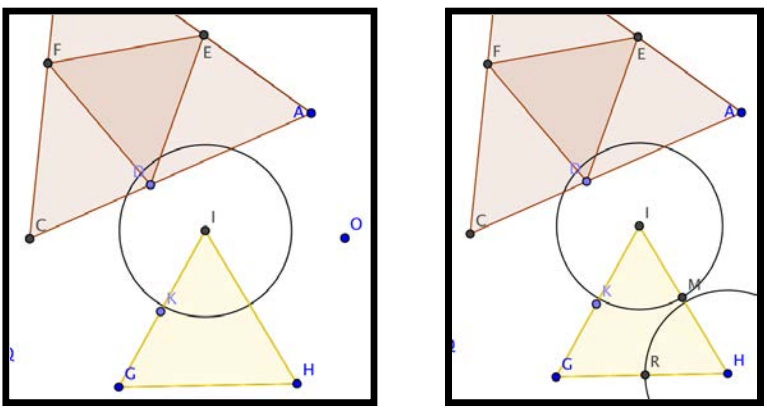

Figure 2. The state of the construction after Cheerios finished (left) and after Cornflakes finished (right).

Next, Cornflakes takes control of the construction, places a point, M, where Cheerios' circle intersects side HI and then repeats the process with the compass to construct another point, R, on the third side of the exterior triangle (see Figure 2, right).

Fruitloops then takes control and uses the polygon tool to construct a shaded interior triangle, KMR, connecting Cornflakes' three points on the sides of the exterior triangle (see Figure 2). She then conducts a drag test, dragging points on each of the new triangles to confirm that they remain equilateral and inscribed dynamically, just like the example figure. At that point, the students have been working in the room for over an hour and end their session, having succeeded as a team.

The VMT software is fully instrumented, so that researchers can obtain detailed logs and even replay the sessions (as shown in Figure 2, a screen image from the replayer) to see precisely what the students all saw on their screens. Of course, as Schutz pointed out, researchers have a reflective relationship to the interaction, which is quite different from the engagement of the students. The intersubjectivity of the students, when things are functioning optimally, can be that of group cognition, where they act as one subject, constructing shared meaning through their interaction. The intersubjectivity of the researchers with the students involves systematic (methodical, self-conscious, research-driven, theory-laden) efforts to understand the meanings previously created by the students, based on a culture and world partially shared by the students and researchers.

Intersubjectivity as Group Cognition

The kind of data generated by teams of students using VMT can support detailed research into the nature of interaction and intersubjectivity in CSCW and CSCL situations. For instance, the VMT research team has now analyzed all eight hours of the Cereal Team's interaction (Stahl, 2016). In particular, the analysis tracks their enactment and acquisition of various member methods or group practices. It documents how the students form into an effective team and how they align and develop joint attention. By adopting specific sequences of group practices, the team learns how to collaborate, to manipulate technological affordances, to engage in collaborative dynamic-geometry problem solving and to enter into mathematical discourse. One can see, displayed in the team interaction, group cognition in action as a specific form of intersubjectivity.

We see the potential productivity of collaboration in the way that the three students, participating from within their personal zones of proximal development, bring different resources to the interaction. Further, the interaction itself—including the reactions of the GeoGebra application to student attempts at geometric construction—elicits, as Merleau-Ponty put it, "thoughts which I had no idea I possessed." Ideas, skills and approaches from different sources mix and spontaneously generate new, shared knowledge through the interaction itself and its internal logic or implicit connotations. Collaborative learning may be guided through reflection by the participants and through feedback from the problem-solving process itself. For instance, observation of the results of various people's efforts at geometric manipulations and constructions may lead to the discovery of solutions that cannot be attributed to any one of the participant's minds or even to a simple aggregation of their individual contributions. The dynamic, over-determined behavior of their joint geometric-construction moves in their shared online world contributes to the unfolding of a solution path as well. The VMT environment incorporating its multi-user version of dynamic geometry provides visual feedback to construction or dragging actions, adding a computer-supported dimension to what Schön (1992) calls the back-talk of the materials of the (problem-solution) design situation.

In the Cereal Team's work, we see multiple instances of one student contributing a skill or insight from their individual perspective or developmental zone into the group work—usually in response to what another student did or tried to do. The other students learn from this—often from just one occurrence, where the contribution is discussed and consequently adopted by the team as a group practice. Subsequently, another student brings the newly learned skill into the group work, and it is accepted without comment. In this way, first, the group learns a skill or insight and through that, each of the other individuals learns it. For instance, in the session just described, it took each of the three students doing some of the necessary actions to construct the inscribed triangles. However, in their next session, all three students very clearly knew how to carry out all those actions when the group worked on a related challenge of constructing inscribed squares.

In the longitudinal developmental trajectory of the Cereal Team (followed in detail in Stahl, 2016), as the team first learns to collaborate online and to engage in dynamic geometry, we can observe the reciprocal interpenetration of individual and collective understanding in the group-cognition form of intersubjectivity. We see what our review of theories of intersubjectivity characterized as: simultaneous joint participation and perspectival individuality, as well as joint attention, shared meaning making, group agency and being-there-with-others in a shared world.

The Cereal Team took up in their discourse mathematical terms like "constraint" and "dependency," which were introduced in their session instructions. The choice of classical geometry problems and the wording of their presentation to the students guided the student exploration and discourse, mediating the interaction with resources from the mathematical community. By responding to the cues in the instructions and incorporating these technical terms in their discourse with each other, the students gradually developed new conceptions. At first not understanding the terms at all, they passed through everyday uses of them to more rigorous mathematical statements—in a process recalling Vygotsky (1934/1986). The transitions in individual and group understanding of the role of dependencies in dynamic geometry can be tracked in the logs of their interaction (Stahl, 2016).

While all the reviewed theories of intersubjectivity noted the important role of language, Vygotsky is especially clear about the mediation of language—both spoken and silently thought—in how we understand each other and our shared world. Heidegger's later work (e.g., 1959/1971) also emphasizes how language can be seen as a source of meaning making—most visibly in poetry. For him, "speech speaks" (through us) and we live in language as the "house of being." As Tomasello (2014) notes, the cultural richness of spoken language incorporates eons of human shared experiences. In the mixing pot of group discourse, phrases evoke each other and thereby generate creative ideas.

Of course, competent language users are needed to speak and understand the phrases. However, the source of the creative generation and the deductive flow can be analyzed in terms of the meanings sedimented in the phrases, rather than being attributed to rational motives in the minds of individual participants. Group cognition and its associated intersubjectivity can be conceived in primarily linguistic, rather than mental, terms (as recommended by Habermas, 1971/2001). Its intentionality is not that of some kind of group mind or even primarily of the minds of the individual participants, but of the intersubjectively shared discourse and the historically mediated, referred intentionality of a culture, expressed in its passed-down meanings. That is why a goal of math education is to involve students in math discourse and collaborative exploration.

Group cognition is a form of intersubjectivity, in which the words and actions of group members are aligned in a coherent unity, which can be analyzed as a semantic (meaning-making) or cognitive (symbol-manipulating) system in its own right. This vision of a potentially powerful form of group intersubjectivity can inspire and guide the design of supportive technology and pedagogy in CSCW and CSCL, as it has done in the VMT project (Stahl, 2006; 2009; 2013; 2016).

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Investigation 19. The Constitution of Group Cognition

Gerry Stahl

Abstract: Cognition is no longer confined to the solitary musings of an armchair philosopher, but takes place, for instance, in problem-solving efforts of teams of people distributed around the world and involving various artifacts. The study of such cognition can unfold at multiple units of analysis. In this Investigation, three cases of problem solving by virtual math teams demonstrate the mix of individual, group and social levels of cognition. They show how a resource like a mathematical topic can bridge the different levels. Focusing on the under-researched phenomena of group cognition, the presentation highlights three pre-conditions for the constitution of group cognition: longer sequences of responses, persistent co-attention and shared understanding. Together, these structure a virtual analog of physical embodiment: being-virtually-there-together, where what is virtually there is understood as co-experienced.

Keywords. Group cognition, longer sequences, response pairs, persistent co-attention, shared understanding, embodiment, co-experience.

Cognition at Multiple Levels

There is a venerable tradition in philosophy that cognition is a mysterious faculty of individual human beings. Increasingly since the late 19th century, it has become clear that even when thoughts appear to be expressed by an individual, they are the product of more complex factors. Cognitive abilities and perspectives develop over time through one's embeddedness in a physical, social, cultural and historical world. Thinking is closely related to speaking, a form of communication with others. Particularly in our technological world, thinking is mediated by a broad variety of artifacts and by other features of the context in which we are situated.

Rather than thinking about thinking, I try to explore cognition by generating data in which one can observe cognitive processes at work (Stahl, 2006; 2009; 2013; 2016). I do this by having small groups of students collaborate on mathematical problems in a computer-mediated setting, where their whole interaction can be captured. The motivation for this approach is the theory of Vygotsky (1930/1978), the socio-cultural psychologist who proposed that higher-level human mental abilities are acquired first in small-group interactions. In exploring such group cognition, I have found that there is a rich interplay of processes at individual, small-group and community levels of cognitive processing.

In the following, I will summarize three case studies of online collaborative problem solving in order to illustrate how cognitive processes at multiple levels can work together. (1) In the first case, two students solve a high-school math problem that has stumped them for some time. The problem-solving steps the dyad goes through as a team are typical for how proficient students solve problems individually. In the discourse captured in this case, one can see how the *group* integrates contributions from the two *individual* participants to accomplish a task in accordance with *community* standards of practice—illustrating the productive interplay between cognitive levels. The sequence of ten discourse moves by the group details their extended *sequential approach* to the problem. (2) In the second study, three students develop techniques for helping each other to see what they are seeing in the diagram they have drawn for a math problem. This *persistent co-attention* to a shared object of analysis allows the team to solve their problem as a group. (3) Similarly in the third example, the students are able to work together because they effectively manage their *shared understanding* of the problem.

I propose that it is often fruitful to analyze cognition on multiple levels and that the processes at the different levels work together. A variety of *interactional resources* are typically at work bridging the levels. In the three illustrative case studies, topics in high-school mathematics centrally figure as resources that bring together individual, small-group and community cognitive processes.

Virtual Math Teams

The study of group cognition requires careful review and analysis of all the interaction within a group during the achievement of a cognitively significant task, such as solving a challenging problem. I have arranged for this by designing an online software environment in which several people can meet and interact effectively to solve math problems. This Virtual Math Teams (VMT) environment supports synchronous text chat and a shared whiteboard for drawing figures (Stahl, 2009). During the project, it was expanded to incorporate a multi-user version of dynamic geometry, in which geometric figures can be interactively constructed and dynamically dragged (Stahl, 2013). The software is instrumented to capture all interaction and to allow it to be displayed, replayed and analyzed. This avoids the many problems of audio and video recording in classrooms. Students communicate online, avoiding the interpretational issues of eye gaze, bodily gesture and vocal intonation. When possible, groups are composed of students who do not know each other outside of the online setting, so that researchers reviewing a record of interaction can know everything about the participants and their background knowledge that the participants know about each other. Since group cognition is defined as consisting of those knowledge-building or problem-solving processes that take place in the group interaction (Stahl, 2006), the VMT environment can capture a complete history of group-cognitive events.

When a group enters the VMT environment, it is presented with a challenging math problem, designed to guide the group interaction in an academically productive direction. The problem acts as a resource for the group. The group must interpret the problem statement, elaborate the way in which it wants to conceive the problem and determine how to proceed. A math problem can serve as an effective interactional resource for bridging across cognitive levels. Typically, it introduces content—definitions, elements, procedures, principles, practices, proposals, theorems, questions—from the cultural traditions of mathematics and from school curriculum. In so doing, it recalls or stimulates individual cognitive responses—memories, skills, knowledge, calculations, deductions. It is then up to the group interaction to bring these together, to organize the individual contributions as they unfold in the on-going interaction in order to achieve the goals called for by the community, institutional, disciplinary and historical sources. In this way, the group interaction may play a central role in the multi-level cognition, interpreting, enacting and integrating elements from the other levels, producing a unified cognitive result and thereby providing a group experience that can subsequently lead to community practice or individual skill.

Constructing Diamonds

Cognition is neither a unitary phenomenon nor a temporally fixed one. Hegel described the logical stages he thought were involved in the development of cognition in his *Phenomenology of Mind* (1807/1967). Vygotsky explored the development of a person's cognition through psychological experiments reported in *Mind in Society* (1930/1978), emphasizing the priority of inter-subjective group cognition:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (*interpsychological*), and then *inside* the child (*intrapsychological*). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher [human mental] functions originate as actual relations between human individuals. (p. 57)

Research on computer-supported collaborative learning (CSCL) (Stahl, Koschmann & Suthers, 2013) can make visible the development and the unfolding of cognitive functions in small groups, shedding light on the less visible processes that can subsequently be carried out by people individually or "internally." A research method for undertaking such analysis is suggested by the field of conversation analysis (CA) (Sacks, 1965/1995). CA was inspired by ethnomethodology, a sociological approach focused on describing the "work" that people typically do in interactions with others to establish social order and to construct meaning (Garfinkel, 1967). CA applies this approach to analyzing everyday conversation. A central finding of CA is that the work of conversation is accomplished through the sequential construction of "adjacency pairs," short sequences in which one person's utterance elicits a response in the form of a following utterance by an interlocutor—for instance a question-answer pair. In looking at examples of mathematical problem solving by groups, we are more interested in "longer sequences," in which a series of adjacency pairs are constructed to accomplish the larger cognitive goal.

Longer sequences have only been suggested in CA (Sacks, 1965/1995, II p. 354; Schegloff, 2007, pp. 12, 213), not actually analyzed. In the final excerpt from a VMT interaction among three students, I analyzed their successful problem-solving effort as a longer sequence, consisting of ten discourse moves, each linguistically organized as an adjacency pair (Stahl, 2011) [Investigation 25]. I treated their four-hour-long online interaction in terms of a temporal hierarchy of: a group event, four scheduled sessions, several conversational topics, many discourse moves, adjacency pairs, textual utterances and indexical references [Investigation 24]. In the first session, the students had been asked to work on a topic in mathematical combinatorics, determining the number of squares and composite sticks needed to build a stair-step pattern at different stages of growth. By the fourth session, the students had set themselves the topic of analyzing a diamond pattern, illustrated by them at stages n=2 and n=3 in the screen image of the VMT software interface in Figure 1.

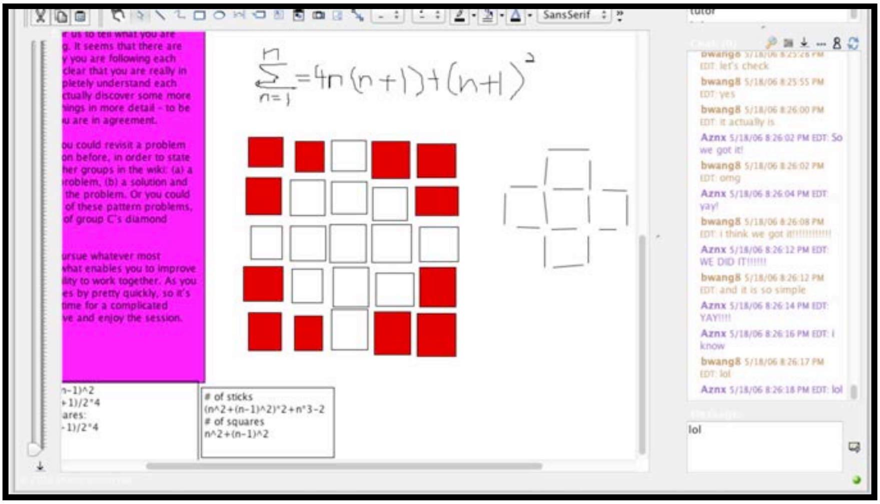

Figure 1. Discussion and drawings of diamond pattern.

In their final conversational topic, two students with login names of Bwang and Aznx decide to try again to solve this problem, despite not being able to do so for the past two hours and despite the fact that their scheduled online time is already over. In the course of ten minutes, 100 chat lines of text are posted. The analysis of these chat lines highlights ten adjacency pairs, which were central to this discourse. Each adjacency pair is listed in Log 1, under an added descriptive heading. This selection from the interaction should give a sense of the problem-solving process—see Investigation 25 for a more detailed analysis.

Move 1. Open the topic

Bwang: i think we are very close to solving the problem here
Aznx: We can solve on that topic.

Move 2. Decide to start

Bwang: well do you want to solve the problem
Aznx: Alright.

Move 3. Pick an approach

Aznx: How do you want to approach it?
Bwang: 1st level have 1*4 … 4th level have (1+3+5+7)*4

Move 4. Identify the pattern

Aznx: So it's a pattern of +2s?
Bwang: yes

Move 5. Seek the equation

Bwang: what is it

Aznx: n^2 … or (n/2)^2

Move 6. Negotiate the solution

Aznx: its n^2

Bwang: so that's wrong

Move 7. Check cases

Aznx: would be 4n^2

Bwang: it actually is

Move 8. Celebrate the solution

Bwang: i think we got it!!!!!!!!!!!!

Aznx: WE DID IT!!!!!!

Move 9. Present a formal solution

Aznx: So you're putting it in the wiki, right?

Bwang: yes

Move 10. Close

Aznx: we should keep in touch

Bwang: yeah

Log 1. Ten moves of the problem-solving topic.

There are several things to note here:

- Most importantly, the sequence of identified moves is strikingly similar to how an experienced math problem solver might approach the topic individually, as described at a particular granularity.
- The two students take turns contributing to the shared topic. The group direction is not set by either individual, but results from their interaction.
- Most opening utterances solicit a response, often in the explicit form of a question, and they always await a response.
- Each move is a situated response to the current state of the students' understanding of the topic as expressed in the discourse—rather than some kind of logical progression following a plan based on anything like a goal-subgoal hierarchy (Suchman, 2007).
- The focus of the group discourse moves is on the sharing, negotiation and agreement about their progress, rather than on details of mathematical facts or computations.

- The math content is handled by the individuals and contributed by them into the collaborative setting, for instance in move #3 or #5.
- The temporal structure of topics, moves and adjacency pairs is not imposed by the analyst but is projected in the remarks of the participants as integral to how they sense for themselves what they are doing and proceed.

If one follows the development of the students' understanding in their postings across the four sessions, one is struck by changing roles and confidence levels, as well as by their mastery of practices that one or the other introduced into the group. It is quite plausible that over time the lessons acquired in their collaborative interactions will become manifested in their individual cognitive skills. The longer sequences of argumentation or problem solving become "internalized" (as Vygotsky called it) or adopted as cognitive practices of individuals. The power of collaborative learning is partially to bring together multiple perspectives, which can be debated, negotiated, synthesized, contextualized, structured and refined. However, another advantage is to extend the cognitive effort into *longer sequences* of argumentation through the stimulation and enjoyment of productive social interaction, increasing the time-on-task as needed to solve challenging problems. Thus, groups can achieve cognitive accomplishments that their members cannot—and the members can learn from these achievements.

Visualizing Hexagons

Elsewhere, we have analyzed in some detail the intimate coordination of visual, narrative and symbolic activity involving the text-chat and shared whiteboard in VMT sessions [Investigation 12]. Here, we want to bring out the importance of literally looking at some mathematical object together in order to share the visual experience and to relate to—to "intend" or to "be at"—the entity together. People often use the expression "I do not see what you mean" in the metaphorical sense of not understanding what someone else is saying. In our second case study, we often encounter the expression used literally for not being able to visually perceive a graphical object, at least not being able to see it in the way that the speaker apparently sees it.

While empiricist philosophy refers to people taking in uninterpreted sense data much like arrays of computer pixels, post-cognitive philosophy emphasizes the phenomenon of "seeing as." Wittgenstein notes that one immediately sees a wire-frame drawing of a cube not as a set of lines, but as a cube oriented either one way or another (1953, §177). For Heidegger, seeing things as already meaningful is not the result of cognitive interpretation, but the precondition of being able to explicate that meaning further in understanding (1927/1996, pp. 139f). For collaborative problem solving and mathematical deduction, it is clearly important that the participants see the visual mathematical objects as the same, in the same way. This seems to be an issue repeatedly in the online session excerpted in Log 2, involving three high school students with login handles of Jason, Qwertyuiop and 137 [Investigation 17].

705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance

Log 2. Seeing a hexagonal array collaboratively.

Student 137 proposes a mathematical task for the group in line 705 of Log 2. This is the first time that the term, "hexagonal array," has been used. Coined in this posting, the term will become sedimented (Husserl, 1936/1989, p. 164) as a mathematical object for the group as the discourse continues. However, at this point it is problematic for both Qwertyuiop and Jason. In line 706, Qwertyuiop poses a question for clarification and receives an affirmative, but minimal response. Jason, unsatisfied with the response, escalates the clarification request by asking for help in seeing the diagram in the whiteboard *as* a "hexagonal array," so he can see it *as* 137 sees it. Between Jason's request in line 709 and acceptance in line 710, Qwertyuiop and 137 work together to add lines outlining a large hexagon in the triangular array. Demonstrating his ability to now see the hexagons, Jason thereupon proceeds with the mathematical work, which he had halted in the beginning of line 709 in order to keep the group aligned. Jason tentatively proposes that every hexagon "has at least 6 triangles" and he makes this visible to everyone by pointing to an illustrative small hexagon from the chat posting, using the VMT graphical pointing tool. Later, the students take turns using these group-defined methods of supporting shared vision and attention: using colored lines and the pointing tool, as seen in Figure 2.

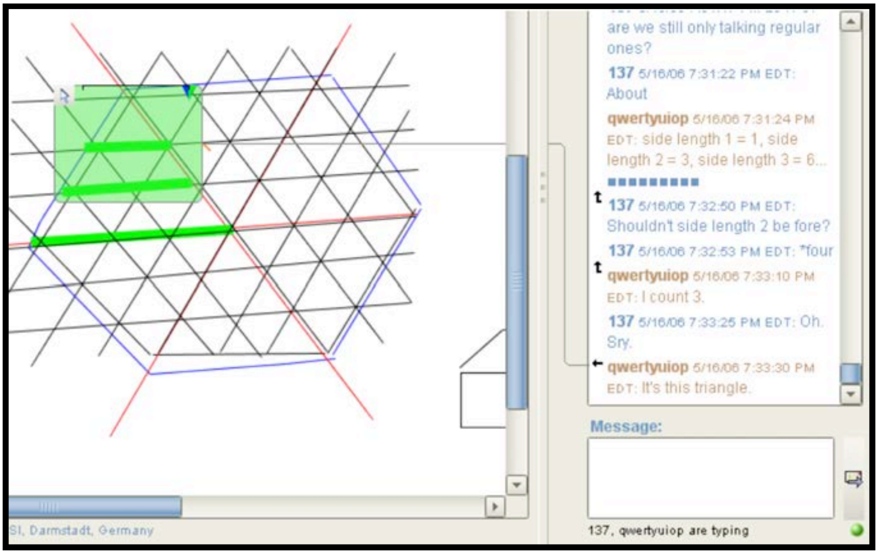

Figure 2. Discussion and drawing of hexagon grid.

Jason dramatically halted group work with his "wait." For him, it was impossible to continue until everyone could see the same thing in the way that 137 saw it. During this session, the students taught each other how to change the color and thickness of lines they constructed in the shared whiteboard. These were affordances of the VMT software, but the students had to learn how to use the features and they developed certain shared group practices of using colored lines to outline, highlight and draw attention to specific elements of the hexagonal grid. For instance, in Figure 2, blue lines outline a hexagon of side-length 3; red lines divide that hexagon into six symmetric triangles; thick green lines pick out the three horizontal lines of length 1, 2 and 3 in one of the triangles; and the VMT pointing tool focuses attention on that triangle.

There are many ways to count the number of unit sticks in the large hexagon. In order to count them as a group, everyone's attention much be focused on the same elements, such as the green horizontals. Then it is possible for each participant to count that subset visually: 1+2+3=6. Through similar shared attention to structural elements of the hexagon, all the group members know that there are three such arrays of lines like the green ones at different orientations in each of the six triangles. They can also see how this array of lines will increase as the hexagon itself progresses to successively longer side-lengths. The achievement of the necessary *persistent co-attention* to construct and to follow this complicated analysis was the result of subtle interactions and the development of shared practices within the group.

Inscribing Triangles

Our final case involves a group of three middle-school students given a topic in dynamic geometry (Stahl, 2013, Section 7.3). The students have not yet had a course in geometry, but have already spent four hours together in a version of VMT that incorporates interactive, multi-user support for dynamic geometry. In this topic, the students are given constructions of an equilateral triangle inscribed inside another equilateral triangle and a square inscribed inside another square (see Figure 3). In dynamic geometry, a student can drag one point of a figure like the inscribed triangles and all the other points and lines will move accordingly, maintaining the geometric relationships or dependencies that have been built into the construction of the figure. In previous sessions, the students had learned the dynamic-geometry equivalent of Euclid's first two propositions: the construction of an equilateral triangle (using software tools equivalent to a straight edge and compass) and the copying of a line-segment length.

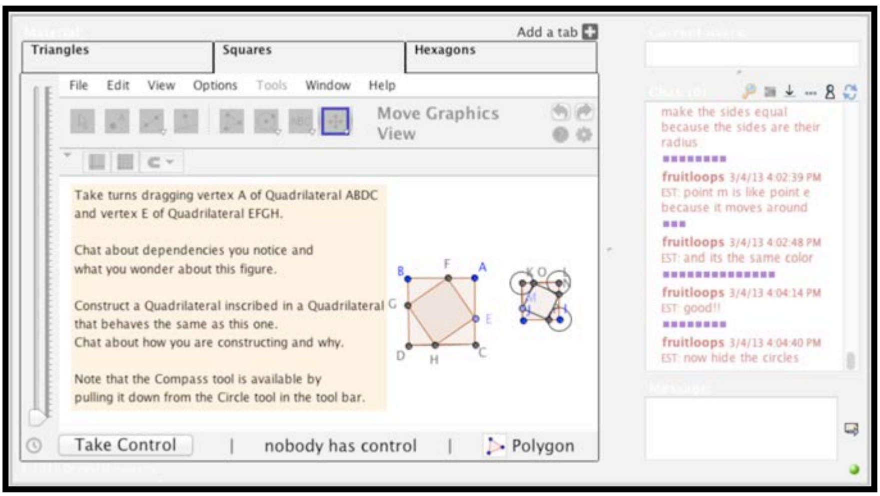

Figure 3. Discussion and constructions of inscribed squares.

In their fifth session, the three students took turns dragging points of the equilateral triangles and discussing the dependencies that were maintained. Then they tried to duplicate the given figure and to build in the relevant dependencies. For instance, the dependency defining the equilateral character of the outer triangle is that the lengths of the second and third sides must always be the same as the length of the base, even when the endpoints of the base segment are dragged, changing its length. Euclid's construction maintains this dependency because the lengths of all three sides are radii of circles of equal radius. Read today, Euclid's *Elements* (300 BCE/2002) in effect provides instructions for dynamic-geometry constructions. The “elements” of geometry are not so much the points, lines, circles, triangles and quadrilaterals, but the basic operations of constructing figures with important relationships, such as congruence or symmetry. Just as Euclidean geometry contributed significantly to the development of logical, deductive, apodictic cognition in Western thought and in the formative minds of many prospective mathematicians, so collaborative experiences with dynamic geometry may foster in students ways of thinking about dependencies in the world.

The students in the case study used Euclid’s method to construct the outside triangle, but soon realized that the same procedure could not be used to construct the inscribed triangle because of the additional constraint that its vertices all had to be on the sides of the inscribing triangle, which they had constructed. Considerable further dragging of points in the given figure and experimentation with various construction approaches were tried. Finally, the students noticed that when one point of the inner triangle was dragged along a side of the outer triangle, the other vertices of the inner triangle moved in a corresponding way, such that their positions along their sides of the outer triangle were the same as that of the dragged vertex on its side. Then they quickly decided to use the method they had learned for copying a line-segment length. They copied the length from one outer vertex of their new equilateral triangle to a point for an inner vertex. Then they placed this length along the other sides, starting at both of the other vertices. This determined the locations of the other inner vertices. When they connected the three points, they formed an inscribed equilateral triangle. When any point or line was dragged, both the inner and outer triangles remained equilateral and inscribed.

In their sixth session, the students tackled the topic of inscribed squares. All their previous work in dynamic geometry had involved triangles and they had not been exposed to a method of constructing a dynamic square. They spent most of the hour exploring possible construction methods, eventually inventing a method that was elegantly similar to that of the triangle construction. All three students then immediately saw how to construct the interior square by copying the length from a corner of the exterior square to a corner of the interior one along a side. In Figure 3, the circles used for copying the length are still visible. The clarity with which each of the students understood how to inscribe a square—once they were able to construct the exterior dynamic square—shows how well they had each individually mastered the technique from their prior collaborative experience involving the dynamic triangles.

Their collaborative solution of the inscribed-triangles topic is quite typical. We have observed a number of small groups working on this topic, including math teachers, researchers, graduate students and middle-school students. They all go through a similar process of dragging the original figure, experimenting with construction attempts, discovering the dependency of the distances between the interior and exterior vertices, then realizing how to copy that distance and finally checking that their construction has the same behavior as the given figure. While this topic poses a problem that is difficult for individuals, small groups often stick with it and solve it through collaborative effort within an hour or less. It takes a combination of many trials, observations and connections to accomplish the task. The collaborative approach allows individuals to contribute specific pieces of the puzzle, to build on each other's proposals and to discuss the implications.

The chat discourse is striking in how much the students make sure that everyone agrees with and understands each step that the group as a whole takes in constructing their figures. In addition to expressing agreement and affirming understanding, the students also demonstrate their *shared understanding* by fluidly building on each other's contributions. Successive steps are generally taken by different students, indicating that they are all following the logic of the collaborative effort.

Contributing to Group Cognition

The cognition in group cognition is not the same as individual cognition; it relies upon individual cognition to make essential contributions. However, one cannot say that all of the cognition should be analyzed at the individual unit, because the work of assembling the high-level argumentative structure occurs at the group unit of analysis. Surely, putting together longer sequences of problem-solving arguments must be considered a cognitive activity as much as the work that goes into making the detailed contributions to individual steps. In addition, the personal contributions of individual utterances are largely responses to what has gone before in the group interaction. Not only are these contributions expressions that would not have occurred without the preceding opening up for them and elicitation of them by the group process, but many of the contributions are largely reactions at the group level, which reference and inter-relate resources available in the discourse context more than they introduce new elements from the personal perspective and individual background of the actor. The important cognitive achievement of solving the problem is emergent at the group level, rather than a simple collection of expressions of individual cognitive accomplishments.

Coherent and impressive examples of group cognition—such as solving a math problem that the group members would not have been able to solve on their own—do not occur whenever a number of people come together in conversation. In fact, the research field of computer-supported collaborative learning has documented that desirable forms of collaborative knowledge building are hard to find. The three studies summarized above indicate some reasons for this. First, it is difficult to set up a group interaction where everything relevant to the cognition at the group level of analysis is captured in a form adequate for detailed analysis. It took years of research to develop and deploy the VMT environment to successfully generate adequate data for the analysis of group cognition. Secondly, the group interaction must be directed and guided to focus on an appropriate cognitive task. Certain challenging math problems, carefully presented, seem to provide effective resources for stimulating interesting episodes of group cognition.

Additionally—as the three studies summarized here have documented—the groups must work consistently to ensure the presence of certain preconditions of effective group cognition. They must persist in building *longer sequences* of responses to each other, they must maintain continuous *co-attention* to a shared focus of discussion, and they must build and sustain a *shared understanding* of the topic of conversation.

The Constitution of Group Cognition

The phenomenological tradition has always conceived of cognition as embodied in the world, rather than as a Cartesian mental process. Husserl (1929/1960, §14) emphasized that cognition is cognition *of* something; it is located at its object, not at some internal representation of that external object. Heidegger (1927/1996) therefore started from the experience of being-in-the-world instead of thinking-in-the-head. For him, cognition is a matter of being-with and caring-for things and people. The world is a shared world and the things we are there with are always already understood as meaningful. In Merleau-Ponty's (1945/2002) famous example of the blind man with the cane, the cane does not so much augment or extend the man's senses and mental awareness of external reality as it locates his cognition in the physical world at the tip of the cane; he senses the object at the tip directly, without focusing on the cane or the intervening distance.

If we look at the presented examples of group cognition, we see that the students are "there" in their group interaction with virtual mathematical objects, seen in specific ways. Aznx and Bwang have drawn the horizontal sticks and the vertical sticks separately (not shown in the summary above). They have noticed a four-way symmetry, which allows them to reduce the problem of counting the sticks to a tractable pattern. They are focused together on the diamond as that symmetric pattern of sticks. Similarly, Jason, Qwertyuiop and 137 have worked hard to view their hexagonal array as a symmetrical pattern of sticks forming lines within triangles that make up a hexagon. As these groups work out their algebraic solutions to the topic, they are present together in a shared virtual world at an object of interest, which they all see as structured in the same way. In the third case, after much work individually and collaboratively, and incorporating ideas from the ancient tradition of Euclidean geometry, the three students working on the inscribed squares all observe that when square EFGH is dragged within square ABCD the following segments along the outer square change but stay equal in length to each other: AE, CH, DG, BF. They then can all see that they have to construct square MONP within square IJKL so that segments IP, JM, KO, LN stay the same (see Figure 3). They collaborate in a shared world, manipulating a shared object physically (i.e., by moving a mouse), visually and imaginatively within a shared approach to their problem, the geometric objects, the dynamic dependencies, the representational figure and the software affordances.

Following the phenomenologists, ethnomethodologists showed that the shared social world is constituted continuously through group interaction (Garfinkel, 1967). In our VMT data, we can study precisely how that is accomplished. We see that it takes place over longer sequences of discourse moves, each centered on elicitation/response adjacency pairs. Carrying out these longer sequences requires maintaining persistent co-attention to a shared object; the being-there-together at the object provides a shared focus for the discourse. Accompanying this, there must be a shared understanding of the object and of the discourse context so that group members understand each other. If someone does not know what someone else means by a "hexagonal array" or by its "side-length," does not see the same elements of a symmetrical pattern or the same set of line segments moving together, then the collaborative problem solving cannot continue productively.

Kant (1787/1999) argued that the human mind constitutes meaningful reality through a process of creative discovery, in which structure is imposed by the mind to create and discover (constitute) objects in the world. In the preceding examples, we see how group interaction can constitute the character of objects in the shared world and we have suggested that the shared meaningful world is itself constituted through such interaction. The nature of reality—such as the symmetries of diamond patterns, hexagonal arrays and inscribed squares—is discovered through the creation of interpretive views of objects. Effective perspectives are constrained by reality, which is not knowable except through these views.

The creation of perspectives at the level of group cognition shifts the constitutive role from Kant's individual cognition to group and social cognition. Like the students in the virtual math teams, *we first learn to see things as others see them in group-cognitive processes* (which generally incorporate culturally sanctioned approaches). Subsequently—due to the power of language (e.g., naming, verbal description)—we can be there with those objects (diamonds, hexagons, squares) when we are not physically (or virtually) present with them in a shared group setting. We can even “internalize” (to use Vygotsky’s metaphor) our ability to be-there-with these meaningful objects in the internal speech of individual thought.

However, the fact that introspection of adults discovers (and assumes) the existence of many individual mental objects does not mean that those objects were not at some point in our development internalized from group-cognitive experiences in community contexts. An adequate analysis of cognition should recognize the constitutive roles of group cognition and their integration with phenomena of individual and social cognition.

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Investigation 20. Theories of Shared Understanding

Abstract: Recent research on instructional technology has focused increasingly on the potential of computer support to promote collaborative learning, shared understanding and collaborative knowledge building. Socio-cultural theories have been imported from cognate fields to suggest that cognition and learning take place at the level of groups and communities as well as individuals. Various positions on this issue have been proposed and a number of theoretical perspectives have been recommended. In particular, the concept of common ground has been developed to explain how meanings and understandings can be shared by multiple individuals. This Investigation takes a critical look at the concept of shared meaning as it is generally used and proposes an empirical study of how group cognition is constituted in practice.

Keywords: Shared meaning, knowledge building, shared understanding, common ground, group cognition, interpretation, perspectives, participation metaphor.

Among those researchers working on computer-assisted learning, a community has emerged in the past decade known as computer-supported collaborative learning, or CSCL (Crook, 1994; Dillenbourg, 1999; O'Malley, 1995). In an influential attempt to define this paradigm of research, Koschmann (1996) argues that previous forms of instructional technology research "approach learning and instruction as psychological matters (be they viewed behavioristically or cognitively) and, as such, are researchable by the traditional methods of psychological experimentation" (p. 10f). That is, they focus on the mind of the individual student as the unit of analysis when looking for instructional outcomes, learning, meaning making or cognition. By contrast, the paradigm of CSCL "is built upon the research traditions of those disciplines—anthropology, sociology, linguistics, communication science—that are devoted to understanding language, culture and other aspects of the social setting" (p. 11). This radical paradigm shift, focusing on "the social and cultural context as the object of study, produces an incommensurability in theory and practice relative to the paradigms that have come before" (p.13).

The incommensurability between CSCL and other paradigms of computer-assisted learning becomes clear if we phrase it this way: in the CSCL perspective, it is not so much the individual student who learns and thinks, as it is the collaborative group. Given that we have for millennia become used to taking learning and thinking as activities of individual minds, it is hard to conceive of them as primarily group activities. Of course, this approach does not deny that individuals often think and learn on their own, but rather that in situations of collaborative activity it is informative to study how processes of learning and cognition take place at the group level.

Thus, the question of group cognition can be viewed as largely a methodological, rather than ontological issue: it is a call to analyze case studies of collaboration at the group unit of analysis, rather than a claim that some kind of group mind exists beyond the situated and transient group discourse itself. As Stahl (2003) argued, one can identify processes of meaning-making or knowledge-building in the interaction that cannot be attributed to any individual group members, although the participation of the individuals in the group process is necessary as sources of contributed utterances and as interpreters of the shared meaning.

In fact, analysis at the group level of description often demonstrates that even when someone learns or thinks in seeming isolation, this activity is essentially conditioned or mediated by important social considerations. This was a general claim of Vygotsky (1930/1978): that intersubjective or inter-psychological or group learning generally preceded individual or intra-psychological learning, which resulted from the internalization of what took place socially. Koschmann points out that Vygotsky—one of the principle theoretical sources for CSCL—proposed the "zone of proximal development" as "a mechanism for learning on the inter-psychological plane" (p. 12).

Vygotsky (1930/1978) contrasted his conception of potential social development to the traditional psychological focus on individual learning, saying, "In studies of children's mental development it is generally assumed that only those things that children can do on their own are indicative of mental abilities" (p. 85). Vygotsky's alternative social conception of development was meant to measure a child's position in the "*process by which children grow into the intellectual life of those around them*" (p. 88; italics in original), as opposed to their mental position in doing tasks on their own.

The italicized phrase is strikingly similar to the definition of situated learning by Lave & Wenger (1991)—another central source of CSCL's theory of learning. Related foundations of the CSCL paradigm include Hutchins' (1996) presentation of distributed cognition and Suchman's (1987) discussion of situated action. Despite the attempt by these traditions within CSCL to overcome the traditional focus of educational and psychological theories on the individual as cognitive agent, none of them have worked out a satisfactory theory of group cognition.

Stahl (2003) drew on the aforementioned and other sources to argue for taking *meaning* that is constructed in successful processes of collaboration as a shared group product, which is, however, necessarily subject to *interpretation* by the individuals involved. As much as the writings on situated action, distributed cognition, social constructivism, activity theory, social practice, etc. have foregrounded the social nature of learning and thinking, it is still hard for most people to overcome their individualistic conceptual traditions and come to terms with group learning or group cognition. This Investigation is an attempt to further that effort by considering just what is meant by shared meaning and group cognition.

The Problem of Shared Meaning

The analysis by Stahl (2003) tried to provide insight into the nature of the *group perspective*. In particular, its Chapter 16 argued for a view of both shared group *meaning* and individual *interpretation*. Shared meaning was not reduced to mental representations buried in the heads of individuals. Such mental contents could only be inferred from introspection and from interpretation of people's speech and behavior, whereas socially shared meaning can be observed in the visibly displayed discourse that takes place in group interactions, including non-verbal communication and associated artifacts. This approach does not result in a behaviorist denial of human thought in bracketing out inferred mental states and focusing on observable interaction, because of the methodological recognition of interpretive perspectives. People are considered to be interpreting subjects, who do not simply react to stimuli but understand meanings.

It is true that only individuals can interpret meaning. But this does not imply that the group meaning is just some kind of statistical average of individual mental meanings, an agreement among pre-existing opinions, or an overlap of internal representations. A group meaning is constructed by the interactions of the group's members, not by the individuals on their own. It is an emergent property of the discourse and interaction. It is not necessarily reducible to assumed personal opinions or isolated understandings of individuals.

Stahl (2004) presented an example of how this works. The discourse transcribed there is strikingly elliptical, indexical and projective; that means that it implies and requires a (perhaps open-ended) set of references to complete its meaning. These references are more a function of the history and circumstances of the discourse than of intentions attributable to specific participants. The words in the analyzed collaborative moment refer primarily to each other, to characteristics of the artifacts discussed and to group interactions. In fact, one can only attribute well-defined opinions and intentions to the individual students after one has extensively interpreted the meanings of the discourse as a whole.

As seen in the example transcript, the shared meaning was collaboratively created by the group as a whole. But the establishment of that meaning as shared involved a process of negotiation through which the individual group members had to interpret the meaning from their own personal perspectives, to display their understanding of the meaning and to affirm that meaning as shared. The collaborative process itself entailed corresponding individual processes. In a sense, one can say both that the individuals learned as a result of the group learning, and that the group could only learn by ensuring that the individuals learned.

Of course, the kind of "learning" that happens in a brief interaction is not the kind of learning that educators look for over months. It is perhaps better referred to as "knowledge building," in which some word or utterance takes on a new shared meaning. To understand what takes place in collaborative interactions, it seems important to become clearer about the nature of shared knowledge—how it is produced, negotiated, distributed and internalized.

The major difficulty in understanding shared knowledge and group cognition is that it is habitual to attribute thoughts and intentions to individual actors—and to reduce group phenomena to actions of the individual group members. One typically assumes that a speaker's words are well defined in advance in the speaker's mind, and that the discourse is just a way for the speaker to express some preconceived meaning and to convey it to the listeners. This reveals a conflict. If meaning is socially constructed, why do researchers feel compelled to treat it as private property; if it takes place in isolated minds, how can it ever be shared and understood collaboratively? The possibility of shared meaning must be somehow explained. This is particularly important in cases of collaborative learning, where the knowledge that is constructed must be shared among the learners (or may be shared first, before it can become part of an individual's knowledge).

The term "shared knowledge" is ambiguous. It can refer to:

- Similarity of individuals' knowledge: The knowledge in the minds of the members of a group happens to overlap and their intersection is "shared."
- Knowledge that gets shared: Some individuals communicate what they already knew to the others, who then "share" it.
- Group knowledge: Knowledge is interactively achieved in discourse and may not be attributable as originating from any particular individual. It is part of a "shared" world.

The ambiguity of this term corresponds to different paradigms of viewing group interaction: whether it is taken to be a result of individual knowledge, reducible to knowledge held by individual thinkers or an emergent property of the group discourse as an irreducible unit for purposes of analysis. If CSCL is to be conceived as a fundamentally new educational form, rather than just a technique for fostering individual learning, than it seems that something like the third reading of “shared knowledge” needs to be explicated.

A Conflict of Paradigms

Research on learning and education is troubled to its core by the conflict of paradigms we are considering. Sfard (1998) reviewed some of the history and consequences of this conflict in terms of the incompatibility of the acquisition metaphor (AM) of learning and the participation metaphor (PM). AM conceives of education as a transfer of knowledge commodities and their subsequent possession by individual minds. Accordingly, empirical research in this paradigm looks for evidence of learning in changes of mental contents of individual learners. PM, in contrast, locates learning in intersubjective, social or group processes, and views the learning of individuals in terms of their changing participation in the group interactions. AM and PM are as different as day and night, but Sfard argues that we must learn to live in both complementary metaphors.

The conflict is particularly pointed in the field of CSCL. Taken seriously, the term “collaborative learning” can itself be viewed as self-contradictory given the tendency to construe learning as something taking place in individual minds. Having emerged from the paradigm shift in thinking about instructional technology described by Koschmann (1996), the field of CSCL is still enmeshed in the paradigm conflict between opposed cognitive and socio-cultural focuses on the individual and on the group (Kaptelinin & Cole, 2002). In his keynote at the CSCL ’02 conference, Koschmann (2002a) argued that even exemplary instances of CSCL research tend to adopt a theoretical framework that is anathema to collaboration. Koschmann recommended that talk about “knowledge” as a thing that can be acquired should be replaced with discussion of “meaning-making in the context of joint activity” in order to avoid misleading images of learning as mental acquisition and possession of knowledge objects.

Although Koschmann’s alternative phrase can describe the intersubjective construction of shared meanings achieved through group interaction, the influence of AM can re-construe meaning making as something that must perforce take place in individual human minds, because it is hard for most people to see how a group can possess mental contents. Stahl (2003) argued in effect that both Koschmann's language and that of the researchers he critiqued is ambiguous and is subject to interpretation under either AM or PM. A simple substitution of wording is inadequate; it is necessary to make explicit when one is referring to individual subjective understanding and when one is referring to group intersubjective understanding—and to make clear to those under the sway of AM how intersubjectivity is concretely possible.

The problem with recommending that researchers view learning under both AM and PM or that they be consistent in their theoretical framing is that our common-sense metaphors and widespread folk theories are so subtly entrenched in our thinking and speaking. The languages of Western science reflect deep-seated assumptions that go back to the *ideas* of Plato's *Meno* (350 BCE/1961) and the *ego cogito* of Descartes' *Meditations* (1633/1999). It is hard for most people to imagine how a group can have knowledge, because we assume that knowledge is a substance that only minds can acquire or possess, and that only physically distinct individuals can have minds (somewhere in their physical heads). The term *meaning* as in *shared meaning* carries as much historical baggage as the term *knowledge* in *knowledge building*.

The Range of Views

CSCL grows out of research on cooperative learning that demonstrated the advantages for individual learning of working in groups (e.g., Johnson & Johnson, 1989). There is still considerable ambiguity or conflict about how the learning that takes place in contexts of joint activity should be conceptualized. While it has recently been argued that the key issues arise from ontological and epistemological commitments deriving from philosophy from Descartes to Hegel (Koschmann, 2002b; Packer & Goicoechea, 2000), Stahl (2004) argued that it is more a matter of focus on the individual (cognitivist) versus group (sociocultural) as the unit of analysis.

Theoretical positions on the issue of the unit of learning (e.g., in the compilations of essays on shared cognition (Resnick, Levine & Teasley, 1991) or distributed cognition (Salomon, 1993)) take on values along a spectrum from individual to group. The following is an attempt to characterize possible positions along this spectrum, most of which have been advocated for in the literature:

- Learning is always accomplished by individuals, but this individual learning can be assisted in settings of collaboration, where individuals can learn from each other.

- Learning is always accomplished by individuals, but individuals can learn in different ways in settings of collaboration, including learning how to collaborate.
- Groups can also learn, and they do so in different ways from individuals, but the knowledge generated must always be located in individual minds.
- Groups can construct knowledge that no one individual could have constructed alone by a synergistic effect that merges ideas from different individual perspectives.
- Group knowledge can be spread across people and artifacts; it is not reducible to the knowledge of any individual or the sum of individuals' knowledge.
- Groups construct knowledge that may not be in any individual minds but may be interactively achieved in group discourse and may persist in physical or symbolic artifacts such as group jargon or texts or drawings.
- Learning is always a mix of individual & group processes; the analysis of learning should be done with both the individual and group as units of analysis and with consideration of the interplay between them.
- Individual learning takes place by internalizing or externalizing knowledge that was already constructed inter-personally; even modes of individual thought have been internalized from communicative interactions with other people.
- All human learning is fundamentally social or collaborative; language is never private; meaning is intersubjective; knowledge is situated in culture and history.

These different positions imply different answers to why CSCL is important. At one extreme of the spectrum, collaboration is only valued to the extent that it results in desirable learning outcomes for individual minds. At the other extreme, collaborative learning can benefit a whole community of practice by developing cultural artifacts like theories. Intermediate positions may acknowledge that benefits accrue at group and individual levels in parallel, through reciprocal influences.

The different positions listed above are supported by a corresponding range of theories of human learning and cognition. Educational research on small group process in the 1950's and 1960's maintained a focus on the individual as learner (Johnson & Johnson, 1989; see review in Stahl, 2000). Classical cognitive science in the next period continued to view human cognition as primarily an individual matter—internal symbol manipulation or computation across mental representations inside an individual's brain, with group effects treated as secondary boundary constraints (Simon, 1981; Vera & Simon, 1993).

In reaction to these views, a number of socio-cultural theories have become prominent in the learning sciences in recent decades. To a large extent, these theories have origins in much older works that conceptualized the situated-ness of people in practical activity within a shared world (Bakhtin, 1986; Heidegger, 1927/1996; Husserl, 1936/1989; Marx, 1867/1976; Schutz, 1967; Vygotsky, 1930/1978).

The following list describes some representative theories that focus on the group as a possible unit of knowledge construction. Of course, each theory is itself too complex to be summarized meaningfully in a sentence, consisting of multiple texts and redefining terms like "learning" and "knowledge" in the process of developing a theory:

- *Collaborative Knowledge Building*. A group can build knowledge that cannot be attributed to an individual or to a combination of individual contributions, but that exists as textual artifacts that can be critiqued by others (Bereiter, 2002; Donald, 1991).
- *Social Psychology*. One can and should study knowledge construction at both the individual and group unit of analysis, as well as studying the interactions between them (Fischer & Granoo, 1995; Resnick et al., 1991; Salomon, 1993).
- *Distributed Cognition*. Knowledge can be spread across a group of people and the tools that they use to solve a problem (Hutchins, 1996; Norman, 1993).
- *Situated Cognition*. Knowledge often consists of resources for practical activity in the world more than of rational propositions or mental representations (Schön, 1983; Suchman, 1987; Winograd & Flores, 1986).
- *Situated Learning*. Learning is the changing participation of people in communities of practice (Lave & Wenger, 1991; Shumar & Renninger, 2002).
- *Zone of Proximal Development*. Children grow into the intellectual life of those around them; they develop in collaboration with adults or more capable peers (Vygotsky, 1930/1978).
- *Activity Theory*. Human understanding is mediated not only by physical and symbolic artifacts, but also by the social division of labor and cultural practices (Engeström, 1999; Nardi, 1996).
- *Ethnomethodology*. Human understanding, inter-personal relationships and social structures are achieved and reproduced interactionally (Dourish, 2001; Garfinkel, 1967).

One does not have to commit to one of these theories in particular in order to gain a sense from them all of the possible nature of group knowledge.

Most of these theories hinge on the question of how it is possible for shared knowledge to be established. Despite this, none of these authors have explained how groups can learn in sufficient detail to overcome widespread resistance to thinking about learning at the group level of description.

Common Ground or Group Cognition?

Within CSCL, it is usual to refer to the theory of "common ground" to explain how collaborative understanding is possible. Baker *et al.* (1999), for instance, note that collaboration requires mutual understanding among the participants, established through a process of "grounding."

It is certainly clear that effective communication is generally premised on the sharing of a language, of a vast amount of practical background knowledge about how things work in the physical and social world, of many social practices implicit in interaction and of an orientation within a shared context of topics, objects, artifacts, previous interactions, etc. Much of this sharing we attribute to our socialization into a common culture or into overlapping sub-cultures.

Most common ground is taken for granted as part of what it means to be human. The phenomenological hermeneutics of Heidegger (1927/1996) and Gadamer (1960/1988)—building on the traditions of Dilthey and Husserl—made explicit the ways in which human understanding and our ability to interpret meaning rely upon a shared cultural horizon. It emphasized the centrality of interpretation to human existence as being engaged in the world. It also considered cases where common ground breaks down, such as in interpreting ancient texts or translating from foreign languages—e.g., how can a modern German or American understand a theoretical term from a Platonic dialogue or from a Japanese poem?

The current discussion of common ground within CSCL is, however, more focused. It is concerned with the short-term negotiation of common ground during brief interactions. Such negotiation is particularly visible when there is a breakdown of the common ground, an apparent problem in the mutual understanding. A breakdown appears through the attempt of the participants to repair a misunderstanding or lack of mutuality. For instance, in the presentations of Roschelle (1996) and Stahl (2004) much of the transcribed discourse was analyzed as attempts to reach shared understandings in situations in which the group discussion had become problematic.

It is not always clear whether repairs to breakdowns in such common ground come from ideas that existed in someone's head and are then passed on to others until a consensus is established, or whether the common ground might be constructed in the interaction of the group as a whole. It is possible that shared knowledge can sometimes be best explained in one way, sometimes another. At any rate, it seems that the question of the source of shared knowledge should generally be treated as an empirical question. This is what is proposed in the next section of this Investigation. But first, this alternative should be made a bit clearer.

The theory of common ground that Baker *et al.* (1999), Roschelle (1996) and many others in CSCL refer to is that of Clark and his colleagues. Clark & Brennan (1991) situate their work explicitly in the tradition of conversation analysis (CA), although their theory has a peculiarly mentalist flavor uncharacteristic of CA. They argue that collaboration, communication and "all collective actions are built on common ground and its accumulation" (p. 127). The process of updating this common ground on a moment-by-moment basis in conversation is called "grounding." Grounding, according to this theory, is a collective process by which participants try to reach mutual belief. It is assumed that understanding (i.e., mutual belief) can never be perfect (i.e., the participants can never have beliefs that are completely identical). It suffices that "the contributor and his or her partners mutually believe that the partners have understood what the contributor meant to a criterion sufficient for current purposes" (p. 129). Clark & Brennan (1991) then show how various conversational moves between pairs of people can conduct this kind of grounding and achieve a practical level of mutuality of belief. They go on to show how different technologies of computer support mediate the grounding process in different ways.

Clark's contribution theory—where one participant "contributes" a personal belief as a proposed addition to the shared common ground and then the participants interact until they all believe that they have the same understanding of the original belief, at which point their common ground is "updated" to include the new contribution—is articulated in the language of individual mental beliefs, if not to say in the jargon of computer models of rational memories. Thus, it is not surprising that Schegloff (1991) responds polemically to Clark & Brennan (1991) by opposing the tradition of ethnomethodology and CA to this theory of mental beliefs: Schegloff points out that Garfinkel (1967) "asked what exactly might be intended by such notions as 'common' or 'shared' knowledge. In the days when computers were still UNIVACS, Garfinkel viewed as untenable that notion of common or shared knowledge that was more or less equal to the claim that separate memory drums had identical contents" (p. 151f). Schegloff then presented an analysis of repair in talk-in-interaction that contrasted with Clark's by construing what took place as a social practice following social patterns of interaction. According to Schegloff's approach, repair is a form of socially shared cognition that takes place in the medium of discourse (in the broad sense of social interaction-in-talk), following established conversational patterns, rather than a transfer and comparison of beliefs between rationalist minds.

In a later critique of Clark's contribution theory of common ground, Koschmann & LeBaron (2003) present video data of an interaction in an operating room. A resident, an attending doctor and an intern are discussing the location of internal organs as viewed indirectly through a laparoscopic camera. Koschmann & LeBaron argue that the discourse that takes place does not match Clark's rubric, and that the very notion of belief contributions to some kind of common ground storage space is not useful to understanding the construction of shared understanding in this situation. Although the medical operation is successful and although technology-supported collaborative learning takes place, the beliefs of the individual participants afterwards do not agree in Clark's sense. Thus, there seems to be a group shared understanding, which is effective in the practice of the operation, but which does not correspond to the understanding of any of the individual participants when considered outside their working team—as Clark's theory of common ground would have it.

Perhaps the case of the operating room (OR) illustrates Vygotsky's contrast between a person's individual developmental level and their social developmental level (separated by the zone of proximal development). The intern was able to participate in the collaborative activity even though he could not correctly identify key items on his own afterwards, outside the group. This might indicate that what takes place in group interactions cannot reliably be reduced to behaviors of the individuals involved. The knowledge and abilities of people in individual and group settings are quite different. The group cognition of the OR team would then not be a simple sum of the individual cognitive acts of its members; the group understanding would not be a simple intersection or overlap of individual beliefs, as identifiable outside of the group context.

Of course, the OR situation was a special case which differed in significant ways from most everyday conversation. Often, interaction can be adequately analyzed as the exchange of personal beliefs. This is particularly true of dyadic conversations, such as those in Clark's examples, rather than in the more complex interactions of small groups of three or more in the OR—or in CSCL generally. The question for CSCL is: can sets of students be transformed into groups that learn collaboratively in ways that encourage the emergence of collaborative group cognition in a significant sense? This is, above all, an empirical question, although it requires a clear conceptual framework for defining and interpreting the data.

Empirical Inquiry into Group Cognitive Practices

At Drexel University, an interdisciplinary group of researchers and staff of the Math Forum—a popular online site with resources and problems related to K-12 school mathematics—undertook a research project to investigate empirically whether knowledge sharing in community contexts can construct group knowledge that exceeds the individual knowledge of the group's members. Their hypothesis was that precisely such a result is, in fact, the hallmark of collaborative learning, understood in an emphatic sense, as a vision of the future. This research is based on earlier work that indicated the possibility of observing group cognition in recordings or transcripts of team discourse.

Roschelle's (1996) study of two students constructing a new (for them) conception of acceleration can be construed as an analysis of shared knowledge building. As Koschmann (2002a) pointed out, the analytic paradigm of that paper is ambiguous. Its focus on the problematic of convergence posits the conceptual change as taking place in the minds of the two individual students, while at the same time raising the issue of the possibility of shared (i.e., convergent) knowledge. The study reported by Stahl (2004) was an attempt to analyze knowledge building at the group level by a group of five students. That analysis was in some respects similar to Roschelle's.

Our proposed new research at the Math Forum takes Stahl's (2004) study as a pilot study and aims to generate a corpus of group interactions in which problem solving and knowledge building can be most effectively observed at the group level.[6] Like many studies of collaborative learning (but unlike the proposed study), the pilot study involved face-to-face interaction with an adult mentor present. Close analysis of student utterances during an intense interaction during that study suggested that the group developed an understanding that certainly could not be attributed to the utterances of any one student. In fact, the utterances themselves were meaningless if taken in isolation from the discourse and its activity context.

There were a number of limitations to the pilot study:

(1) Although the mentor was quiet for the specific interaction analyzed, it might be possible to attribute something of the group knowledge to the mentor's guiding presence.

(2) The digital videotape was limited in capturing gaze and even some spoken wording.

(3) The data included only two sessions, too little to draw conclusions about how much individual students understood of the group knowledge before, during or after the interaction.

To overcome such limitations, in our proposed study:

(1) Mentors are not active in the collaborative groups—although the groups work on problems that have been carefully crafted to guide

[6] This discussion is largely drawn from an early proposal to the National Science Foundation for funding what became the Virtual Math Teams (VMT) Project from 2003-2015. It reflects the author's understanding of theoretical issues of CSCL in the early 2000s. For the original proposals, see (Stahl, 2010).

student inquiry and advice can be requested by email from Math Forum staff.

(2) The online communication is fully logged, so that researchers have a record of the complete problem-solving interaction, essentially identical to what the participants see online.

(3) Groups and individuals are studied during longer, more multi-faceted problem-solving sessions—and in some cases over multiple sessions.

Despite its limitations, the pilot study clearly suggested the feasibility of studying group knowledge. It showed how group knowledge can be constructed in discourse and how discourse analysis can "make visible" that knowledge to researchers. We want to study this in more detail.

We are investigating not only whether computer-supported collaborative learning can construct novel group knowledge, but also what community contexts are favorable to fostering such an outcome. We are doing this by designing and implementing an experimental service in the Math Forum. Students visiting the site are invited to join small virtual teams to discuss and solve math problems collaboratively online. We analyze the interactions in these teams to determine how they build shared knowledge within the Math Forum virtual community.

We are addressing the issue of the nature of shared understanding by studying online collaborative learning in the specific context of Math Forum problems, with the aim of presenting empirical examples of concrete situations in which groups can be seen to have knowledge that is distinct from the knowledge of the group members. By analyzing these situations in detail, we will uncover mechanisms by which understanding of mathematics passes back and forth between the group as the unit of analysis and individual group members as units of analysis.

One example might be a group of five middle-school students collaborating online. They solve an involved algebra problem and submit a discussion of their solution to the Math Forum. By looking carefully at the computer logs of their interactions in which they collaboratively discussed, solved and reflected upon the problem, we can see that the group solution exceeds the knowledge of any individual group members before, during or after the collaboration. For instance, there may be some arguments that arose in group interaction that none of the students fully understood but that contributed to the solution. Or a mathematical derivation might be too complicated for any of the students to keep "in mind" without reviewing preserved chat archives or using an external representation the group developed on an online whiteboard. By following the contributions of one member at a time, it may also be possible to find evidence of what each student understood before, during and after the collaboration, and thereby to follow individual trajectories of participation in which group and individual understandings influenced each other.

While we do not anticipate that group knowledge often exceeds that of all group members under generally prevailing conditions, we hypothesize that it can do so at least occasionally under particularly favorable conditions. We believe that we can set up naturalistic conditions as part of a Math Forum service and can collect sufficient relevant data to demonstrate this phenomenon in multiple cases. The analysis and presentation of these cases should help to overcome the AM/PM paradigm conflict by providing concrete illustrations of how knowledge can be built through group participation as distinct from—but intertwined with—individual acquisition of part of that knowledge. It should also help to clarify the theoretical framing of acts of meaning making in the context of joint activity.

Student discourse is increasingly recognized as of central importance to science and math learning (Bauersfeld, 1995; Lemke, 1990). Discourse analysis is a rigorous human science, going under various names: conversation analysis, interaction analysis, micro-ethnography, ethnomethodology (Garfinkel, 1967; Heritage, 1984; Jordan & Henderson, 1995; Sacks, 1965/1995; Streeck & Mehus, 2003). This method of analysis will allow us to study what takes place through the collaborative interactions. We will be looking for evidence of learning at the micro level, where shared meanings are developed and knowledge is built up as part of solving a challenging math problem.

The focus on discourse suggests a solution to the confusion between individual and group knowledge, and to the conceptual conflict about how there can be such a thing as group knowledge distinct from what is in the minds of individual group members. One way of putting it is that meaning is constructed in the group discourse. The status of this meaning as shared by the group members is itself something that must be continually achieved in the group interaction; frequently the shared status "breaks down" and a "repair" is necessary. In the pilot study, the interaction of interest centered on precisely such a repair of a breakdown in shared understanding among the discussants.

While *meaning* inheres in the discourse, the individual group members must construct their own *interpretation* of that meaning in an on-going way. Clearly, there are intimate relationships between the meanings and their interpretations, including the interpretation by one member of interpretations by other members. However, it is also true that language can convey meanings that transcend the understandings of the speakers and hearers. It may be precisely through divergences among different interpretations or among various connotations of meaning that collaboration gains much of its creative power (Stahl, 2003).

These are questions that we will investigate as part of our micro-analytic studies of collaboration data, guided by our central working hypothesis:

- H0 (collaborative learning hypothesis): A small online group of learners can—on occasion and under favorable conditions—build group knowledge and shared meaning that exceeds the knowledge of the group's individual members.

We believe that such an approach can maintain a focus on the ultimate potential in CSCL, rather than losing sight of the central phenomena of collaboration as a result of methods that focus exclusively on statistical trends (Stahl, 2002).

Issues for Investigation

While we believe that it is possible to clarify the nature of shared knowledge and group cognition by serious reflection upon the existing theoretical discussions and case studies that touch on these concepts (many of which have been referenced here), we are convinced that significant progress and convincing arguments will require further empirical research.

Collaborative success is hard to achieve and probably impossible to predict. CSCL represents a concerted attempt to overcome some of the barriers to collaborative success, like the difficulty of everyone in a group effectively participating in the development of ideas with all the other members, the complexity of keeping track of all the inter-connected contributions that have been offered, or the barriers to working with people who are not visually co-located. As appealing as the introduction of technological aids for communication, computation and memory seem, they inevitably introduce new problems, changing the social interactions, tasks and physical environment. Accordingly, CSCL study and design must take into careful consideration the social composition of groups, the collaborative activities and the technological supports.

In order to observe effective collaboration in an authentic educational setting, we are adapting a successful math education service to create conditions that will likely be favorable to the kind of interactions that we want to study. We must bring together groups of students who will work together well, both by getting along with and understanding each other and by contributing a healthy mix of different skills. We must also carefully design mathematics curriculum packages that lend themselves to the development and display of deep math understanding through collaborative interactions—open-ended problems that will not be solved by one individual, but that the group can chew on together in online interaction. Further, the technology that we provide to our groups must be easy to use from the start, while meeting the communicative and representational needs of the activities.

As part of our project, we will study how to accomplish these group-formation, curriculum-design and technology-implementation requirements. This is expressed in three working hypotheses of the project: H1, H2 and H3. Two further working hypotheses define areas of knowledge building that the project itself will engage in based on our findings. H4 draws conclusions about the interplay between group and individual knowledge, mediated by physical and symbolic artifacts that embody knowledge in persistent forms. H5 reports on the analytic methodology that emerges from the project:

- H1 (collaborative-group hypothesis): Small groups are most effective at building knowledge if members share interests but bring to bear diverse backgrounds and perspectives.
- H2 (collaborative-curriculum hypothesis): Educational activities can be designed to encourage and structure effective collaborative learning by presenting open-ended problems requiring shared deep understanding.
- H3 (collaborative-technology hypothesis): Online computer-support environments can be designed to facilitate effective collaborative learning that overcomes limitations of face-to-face communication.
- H4 (collaborative-cognition hypothesis): Members of collaborative small groups can internalize group knowledge as their own individual knowledge and they can externalize it in persistent artifacts.
- H5 (collaborative-methodology hypothesis): Quantitative and qualitative analysis and interpretation of interaction logs can make visible to researchers the online learning of small groups and individuals.

We believe that the theoretical confusion surrounding the possibility of group knowledge presents an enormous practical barrier to collaborative learning. Because students and teachers generally believe that learning is necessarily an individual matter, they find the effort at collaborative learning to be an unproductive nuisance. For researchers, too, the misunderstanding of collaborative learning distorts their conclusions, leading them to look for effects of pedagogical and technological innovation in the wrong places.

If these people understood that groups can construct knowledge in ways that significantly exceed the sum of the individual contributions and that the power of group learning can feed back into individual learning, then we might start to see the real potential of collaborative learning realized on a broader scale. This project aims to produce rigorous and persuasive empirical examples of collaborative learning to help bring about the necessary public shift in thinking.

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Investigation 21. Academically Productive Interaction

Abstract. Studies of computer-supported collaborative learning have begun to explore processes of online group cognition—such as small-group methods of problem solving—and how they can be mediated by various technological and interactional mechanisms to promote academically productive discourse. This Investigation first presents (1) an analysis of *co-presence* as a foundational aspect of online interaction in an excerpt of chat discourse. Based on how the students in this excerpt actually interact, it develops (2) a notion of *intersubjective shared understanding* as necessary for the possibility of collaborative knowledge-building dialog. The essay concludes with (3) a discussion of consequences for the design of computer support of academically productive online *group cognition*.

Keywords. Co-presence, shared understanding, group cognition, software agent, invasiveness, over-scripting.

An Excerpt of Computer-Supported Discourse

The studies of digital interaction by virtual math teams presented in (Stahl, 2009) adopt an ethnomethodological interest in how interaction is actually carried out in particular online contexts. They assume that the member methods or group practices of computer-mediated interaction developed by small groups of students may differ significantly from commonsense assumptions of researchers based on experience with face-to-face interaction. If this is true, then it is important to explore actual instances of digital interaction before designing interventions in such settings.

The first section of this Investigation reviews how a team of three students collaboratively achieved a cognitive accomplishment as a distributed online group. The log of their interaction makes visible mechanisms by which academically productive discourse or accountable talk (Michaels, O'Connor & Resnick, 2008) can arise naturally in settings of computer-supported collaborative learning, or CSCL (Stahl, Koschmann & Suthers, 2006). The data analysis presented in this initial section is not intended as an illustration of pre-existing theories; rather, the theory in the next section emerges through the analysis of this and similar data.

"Wait…. I don't really see"—Establishing Co-presence

Figure 1 shows a screenshot of the Virtual Math Teams (VMT) software environment, being used by three middle-school students. They volunteered to participate in this online, synchronous math activity with other students from around the world. The students are collaboratively investigating mathematical patterns (combinatorics) related to sequences of geometric figures. In the lower right of the whiteboard is a stair-step pattern of blocks remaining on the board from their previous day's session. Currently, the students are considering a pattern of regular hexagons, which they will visualize in a grid of triangles they construct in the lower left.

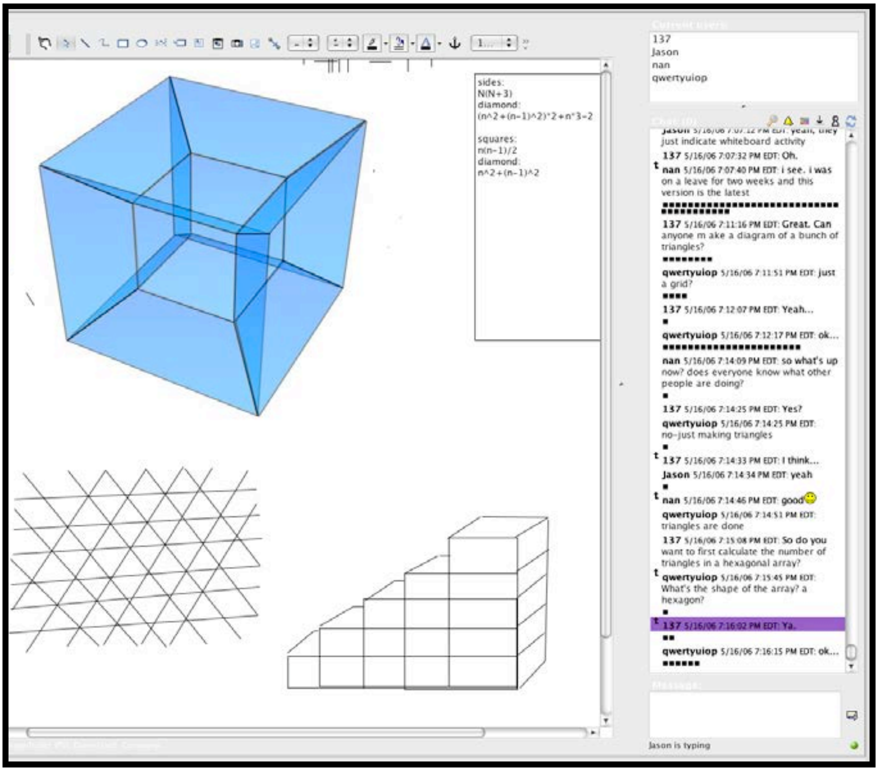

Figure 1. The VMT interface near the beginning of the excerpt.

VMT is a prototypical CSCL environment, with a text-chat tool integrated with a shared whiteboard. Log 1 shows a chat excerpt. Three students—whose online names are 137, Qwertyuiop and Jason—are chatting.

Log 1. Log of the chat excerpt.

705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance
714	19:18:53	137	How do you color lines?
715	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:12	Jason	it's the fifth one from the right
717	19:19:20	137	Thanks.
718	19:19:21	Jason	there ya go :-)
719	19:19:48	137	Er... That hexagon.
720	19:20:02	Jason	so... should we try to find a formula i guess
721	19:20:22	Jason	input: side length; output: # triangles
722	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.
723	19:20:48	137	Like this?
724	19:21:02	qwertyuiop	yes
725	19:21:03	Jason	yup
726	19:21:29	qwertyuiop	side length is the same...
727	19:22:06	Jason	yeah
728	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
729	19:22:19	137	Each one has 1+3+5 triangles.
731	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles
732	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?
733	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...
734	19:24:49	137	And there are n terms so... n(2n/2)
735	19:25:07	137	or n^2
736	19:25:17	Jason	yeah
737	19:25:21	Jason	then multiply by 6
738	19:25:31	137	To get 6n^2

In line 705, student 137 poses a math question of potential interest to the small group. Then Qwertyuiop seeks to understand the math shape that 137 proposed. Qwertyuiop next draws the grid of triangles to see if he understands what 137 means by "hexagonal array."

Jason effectively halts the discussion (line 709) to seek help in seeing the hexagonal form that 137 and Qwertyuiop see. Jason's posting is designed to bring the group work to a halt because he does not see what 137 and Qwertyuiop are talking about. This is an important collaboration move, asking the others to clarify what they are talking about. Jason is referring to the group meaning-making process and halting it so he can fully participate.

Jason phrases his request in terms of "seeing" what the others "mean." This seeing should be taken literally, in terms of vision and graphics. Jason asks the others to "highlight the hexagonal array on the diagram" so he can see it in the graphics.

137 outlines a large hexagon with extra lines, as shown in Figure 2. This provides what Jason needs to be part of the group problem-solving effort. Jason not only says, "Okay" but he contributes a next step (line 712) by proposing a math result and giving a visible demonstration of it with a highlighted small hexagon. In giving a next step, Jason shows his understanding and also takes the shared idea further. Jason points from his chat posting. Note the green rectangle highlighting a small hexagon and the line connecting Jason's current chat posting (713) to this highlighted area; this is an important feature of the VMT system supporting online pointing or deixis [see Investigation 22]. Pointing is a critical function for focusing shared understanding and establishing joint attention—and must be supported explicitly in a digital environment, where bodily gestures are not visible to others.

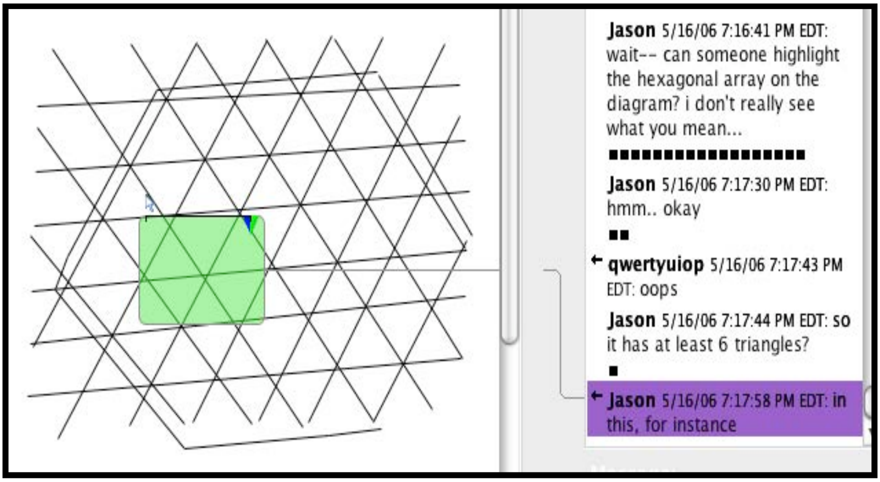

Figure 2. The VMT interface near the middle of the excerpt.

After Jason draws the visual attention of the other participants to a particular example of a smallest hexagon, consisting of six triangles, 137 asks Jason how to change the color of lines in the whiteboard. In line 715, Jason responds and 137 changes the color of the lines outlining the larger black-and-white hexagon. Color becomes an effective method for orienting the team to a shared object. This use of colored lines to help each other see focal things in the whiteboard will become an important group practice in the team's continuing work. In line 719, 137 outlines a larger hexagon, with edge of three units.

At this point, the group has established an effective *co-presence* at a mathematical object of interest. Through a variety of interactional practices—which the group members have adapted from past experiences or constructed on the spot—the group has regulated its interaction and focused its common vision into a "being-there-together" [see Investigation 17] with the object that they have constituted as a hexagonal array. The group is now in a position to explore this object mathematically.

"Like this…."—Building Intersubjective Shared Understanding

In line 720, Jason explicitly proposes finding a formula for the number of elemental triangles in a hexagonal array with side-length of N. Qwertyuiop suggests a way of seeing the hexagonal array as consisting of 6 identical sectors, which he ambiguously refers to as "the 6 smaller triangles." 137 checks what

Qwertyuiop means by asking him, "Like this?" and then dividing up the large hexagon with 3 red lines, forming 6 triangular forms inside of the blue outline (see Figure 3). This is a move by Qwertyuiop to *see* the representation of their problem as a much simpler problem. As Jason notes, now they only have to compute the number of elemental triangles in one of the 6 identical triangular sectors and then multiply that result by 6 to get the total. Furthermore, the simpler problem can be solved immediately by just looking. As Jason says, each sector has 1+3+5 triangles. The human eye can recognize this at a glance, once it is properly focused on a relevant sector.

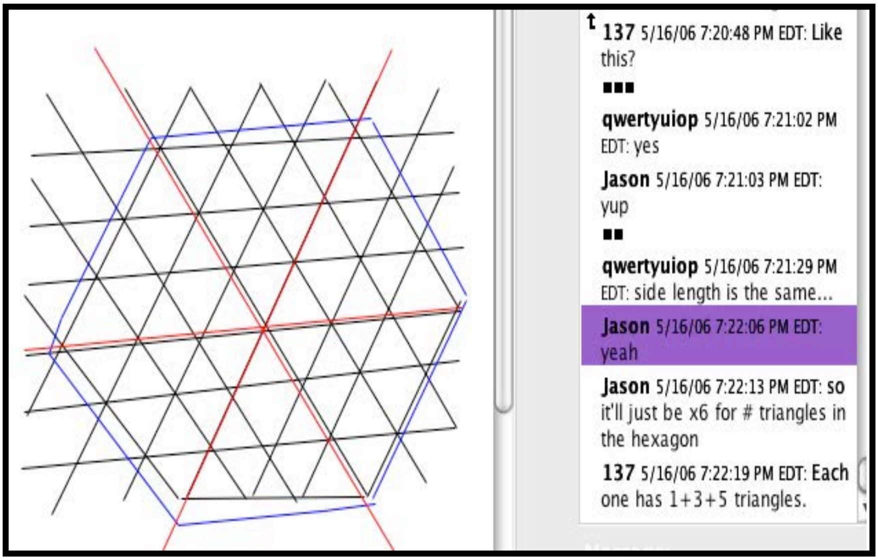

Figure 3. The VMT interface near the end of the excerpt.

The important mathematical problem-solving move here is to see the problem in a new way. Qwertyuiop sees the hexagon *as* a set of 6 symmetrical sectors. The important discourse move is to share this new view with the team. This is accomplished collaboratively in lines 722-725: Qwertyuiop proposes a new way of seeing the array; 137 outlines it, using their new technique of colored lines; and Jason aligns with them. They each participate in seeing the same thing (seeing the hexagon as composed of 6 triangles), in demonstrating to each other that they see this new way, and then in building on each other to count the small triangles visually. They thereby collectively go beyond the co-presence of seeing the same thing to actually build knowledge about the object. This group knowledge is *intersubjectively shared understanding* of the mathematical structure of the object. Through the sequence of steps outlined above, the members of the group have articulated an understanding that they share as a result of their co-presence and of their shared textual and graphical actions.

"To get $6n^2$"—Accomplishing Group Cognition

Note in the chat how the three students build on each other to construct the general formula for any size array: $6n^2$. Having collaboratively deconstructed the complicated problem into visually simple units, they now take turns in reconstructing the problem symbolically and for any size regular hexagon. They are able to work on this together because of their co-presence, which allows them to orient to the same objects, with a shared understanding of the terms (e.g., "hexagonal array," "side length"), graphics (colored border lines), procedures (spatially divide into 6, then algebraically multiply by 6) and goals ("find a formula").

The group's collaborative learning continues. Having counted the number of triangles in the array during this excerpt, the students next want to count the number of line segments. This is more complicated, but the group extends the methods we have just observed to accomplish their task. Taking advantage of multiple symmetries, they again use colored lines to break the pattern down into visually simple patterns, outline specific focal areas and attend to shared objects, where their optical systems can do the counting. Some of the smaller units are harder to visualize and there are issues of possible overlap among the sectors. But using the skills we observed and further developing those skills incrementally, the group succeeds in achieving a continuing sequence of cognitive accomplishments (for a detailed analysis, see Çakir & Stahl, 2013).

Intersubjective Shared Understanding

The establishment of shared understanding in a small group through co-attending to shared objects is essential for collaboration (Evans, Feenstra, Ryon & McNeill, 2011; Mercer & Wegerif, 1999). However, in an online context the usual techniques of body positioning, gaze and explicit pointing with fingers are not available for creating and maintaining shared attention. Virtual teams must invent new methods to coordinate attention or make use of special tools in the software that may be provided to support this.

Previous VMT studies have analyzed cases in which small groups of online students have developed methods for creating, maintaining and repairing shared understanding—similar to what was seen in the previous section. For instance, small groups working in the VMT environment have:

- Co-experienced a shared world (Stahl et al., 2011) by developing shared group practices (Medina, Suthers & Vatrapu, 2009; Stahl, 2011b)
- Used the posing of questions to elicit details needed to establish and confirm the sharing of understandings (Zhou, Zemel & Stahl, 2008).
- Built a "joint problem space" (Teasley & Roschelle, 1993)—i.e., a shared understanding about a set of topics—with ways of referencing them—an "indexical ground" (Hanks, 1992)—that is shared and supports co-attending (Sarmiento & Stahl, 2008).
- Developed group methods for bridging across temporal breaks in interaction to reestablish a group memory or shared understanding of past events. (Sarmiento & Stahl, 2007)
- Repaired their shared understanding in the face of breakdowns (Stahl, Zemel & Koschmann, 2009).
- Integrated text chat and sequences of whiteboard actions to communicate complex mathematical relationships (Çakir, Zemel & Stahl, 2009).
- Solved math problems by proceeding through logical sequences of steps collaboratively (Stahl, 2011a).

The analysis of the excerpt of interaction presented above and these other studies of VMT have identified the following features of the mediation of digital interaction: co-presence, intersubjective shared understanding and group cognition. We will now review the theoretical articulation of these three features as foundations that make possible the goals of academically productive discourse.

Co-presence

Co-presence—through co-attending as a basis for shared understanding—by a small group includes many of the basic features of an individual attending to and interpreting an object of interest. Attending to something involves focusing on it as the foreground object, assigning everything else to its background context (Polanyi, 1966). For instance, the students in the excerpt above foreground a specific hexagon against the background of the larger array of lines by coloring its outline borders or highlighting its area with the pointing tool. Attending to an object involves seeing it "as" something or some way (Goodwin, 1994; Heidegger, 1927/1996; Wittgenstein, 1953). Co-attending supports a shared interpretation, viewing or understanding by creating co-presence attending to a shared object in a shared world in a shared way. For instance, the students view the larger hexagon "as" a set of six triangular sectors by visually dividing the hexagon with red lines that outline the sectors and by texting, "it might be easier to see it as the 6 smaller triangles." (Note that the terminology Qwertyuiop naturally uses here explicitly involves "to see it as….")

Intersubjective Shared Understanding

One can distinguish two paradigms of shared understanding. A rationalist paradigm assumes that individuals each have a stock of propositions in their minds that represent their current beliefs or opinions. The corresponding conception of shared understanding starts from individual understanding of two people and tries to establish equivalence of one or more propositions they hold. This is sometimes called "cognitive convergence," where the goal is to converge the two mental models: Sharing as mutual individual possession.

The alternative paradigm of shared understanding—exemplified by the analysis in this Investigation—starts from the shared world and a view of intentionality as consciousness of an object, rather than as a mental construct by an ego. This is the view of situated and distributed cognition, where individuals are situated in and active with a shared, intersubjective world consisting of meaningful objects for which they care: Sharing as doing together.

Twentieth-century philosophy from Hegel (1807/1967) and Husserl (1936/1989) through Marx (1858/1939), Heidegger (1927/1996), Sartre (1968), Merleau-Ponty (1945/2002) and Wittgenstein (1953) has rejected the starting point of a transcendental ego in favor of consciousness as a social and fundamentally shared phenomenon [Investigation 18]. Now, even at the neuron level, the discovery of mirror neurons points to a physiological, specifically human, basis for shared cognition (Gallese & Lakoff, 2005). We can immediately experience the world through the eyes and body of other people. We can feel their pain when we see another person's body hurt. As Wittgenstein (1953) argued in other ways, there is no such thing as private feelings of pain or private meanings of language: We are co-present in an intersubjectively shared and commonly understood world.

Group Cognition

Vygotsky (1930/1978) claimed that inter-subjective (group) cognition precedes intra-subjective (individual) cognition. He conducted controlled experiments to show that children were able to accomplish cognitive tasks in collaboration with others at an earlier developmental age than they were able to accomplish the same tasks on their own. Individual-cognitive acts are often preceded by and derivative from group-cognitive acts. For instance, individual reasoning or action (dividing a figure, coloring a border) by a student in the VMT data may be based upon earlier group practices. According to Vygotsky, individual mental thinking is fundamentally silent self-talk. Thus, individual-student reasoning can often be seen as reflective self-talk about what the group accomplished. In such cases, self-reports about individual cognition—through think-aloud protocols, survey answers or interview responses—are what Suchman (2007) refers to as post-hoc rationalizations. They are reinterpretations by the individual (responsive to the interview situation) of group cognitions. In this reading of Vygotsky, group cognition has a theoretical priority over individual cognition. If one accepts this, then the theoretical analysis of shared understanding and the practical promotion of it become priorities. The emerging technologies of networked digital interaction provide promising opportunities for observing and supporting the establishment of shared understanding in online educational environments.

Based on experiments in computer support of small-group knowledge building from 1995-2005, (Stahl, 2006) proposed the construct of *group cognition* to begin to define the relevant focus on group-level cognitive achievements. Analyses of studies from 2006-2009 continued to explore the practicalities of supporting group-level cognition in (Stahl, 2009). From 2010-2015, the VMT Project focused on sessions of student groups learning collaborative dynamic geometry, resulting in studies of extended interaction in which the teams adopted group practices, reported in (Stahl 2013c; 2016).

Group cognition is not a physical thing, a mental state or a characteristic of all groups. It is a unit of analysis. What it recommends is that analysts who are scientifically studying digital interaction should look at the small-group unit of analysis (Stahl, 2010). Too often, collaborative-learning researchers reduce group-level phenomena either to individual psychological constructs or to societal institutions and societal practices (Stahl, 2013b). However, one can also identify group methods and processes taking place at the small-group unit of analysis that are not reducible to the mental behaviors of an individual or to the institutions of a community.

For instance, the three students working on hexagon patterns collaboratively solved their problem through a sequence of postings that elicit and respond to each other. Qwertyuiop proposed the view of the hexagon as 6 sectors; 137 summed the series of triangles in one sector to n^2; Jason provided the answer by multiplying the value for one sector by the number of sectors. The result was a group product of the group interaction. If one student had derived this result, we would call it a cognitive achievement of that student. Since the group derived it, it can be called an achievement of group cognition. This does not mean there is some kind of "group mind" at work or anything other than the interaction of the three students. Rather, it means that the analysis of that cognitive achievement is most appropriately conducted at the group unit of analysis, in terms of the interplay of the posting and drawing actions shared by the group.

The absolute centrality of public discourse and shared understanding to the success of group cognition—successful knowledge building at the group level—in the context of digital interaction implies the need for productive forms of talk within the group. Digital environments to support collaborative knowledge building must be carefully designed to foster co-presence, intersubjective shared understanding and group cognition through supporting academically productive talk.

Consequences for Computer Support of Discourse

The theory of academically productive discourse—or accountable talk (Michaels, O'Connor & Resnick, 2008)—has been primarily oriented toward affecting individual cognition in contexts of face-to-face instruction. Accordingly, it is based on the paradigm of cognitive convergence, trying to guide individual students to converge their individual understandings with the understandings of other students, the teacher or the community. In the alternative paradigm presented in this Investigation, for group cognition in online contexts one tries to maintain and build on intersubjective shared understanding and then guide the group of students to articulate clearly, explicitly and scientifically its largely tacit shared group understanding.

Computer technology suggests many tools for supporting group cognition. Computers can provide computational supports, such as spreadsheets and graphing calculators, for assisting individuals and groups in computing tasks. They can provide digital media for communication (text, audio, video, drawing, mapping, etc.). They can provide domain-specific visualizations and work environments, such as the multi-user dynamic-geometry system that VMT has recently incorporated (Stahl, 2013c; Stahl & Powell, 2012). Perhaps most importantly, computers allow people to interact with others around the world. This suggests the possibility of global collaboration.

A particularly intriguing potential of computer technology is to have software agents that interact directly with groups of people—in analogy with human teachers or tutors who support face-to-face groups. For instance, an accountable-talk agent could interact with students to prompt them to engage in accountable-talk moves. As promising as this sounds, it is equally problematic. Detailed studies of online interaction by small groups of students in the VMT environment show that students are creative at adapting their subtle linguistic skills to the characteristics of online media. They are able to achieve impressive accomplishments of group cognition in exploring mathematical phenomena through dialogic interaction. However, this interaction is fragile and easily disrupted by external interventions of educators and surrogate educators. In particular, software agents—designed to guide groups of students to maintain focus and to engage in productive discourse—can be particularly distracting.

This concluding section of the Investigation will address three issues related to the potential of using software agents to promote accountable talk within small online groups of students: invasiveness, automated agency and over-scripting—which respectively threaten to disrupt co-presence, shared understanding and group cognition.

Invasiveness

We have seen above that a primary cognitive need is to maintain focal attention; for group cognition, this means maintaining shared attention. Software agents and other scaffolds can distract attention from what the group has created as its focus. An automated agent might raise issues at inopportune moments, interrupting the flow of discourse and group problem solving. We call this possibility "invasiveness."

If software agents are introduced as participants in a group interaction and their status is left ambiguous in order to catch the fancy of students, this will likely raise false expectations. Students may assume that the agent knows answers, has teacher powers or understands student intentions. The agent can itself become the focus of attention, distracting from both the peer interaction and the problem solving, as students try to test, fool or relate to the agent.

Collaboration involves following the lead of the students (individually and as a group); but software agents are not good at understanding student thinking, let alone group cognition. In experiments investigating the use of software agents in the VMT environment to scaffold and guide group cognition, we have seen how problematic accountable-talk agents can be (Stahl, 2013a). Agents were sometimes distracting, confusing, disruptive. The agents did not always listen well to the students or follow their lead. While some of the problems in our initial experiments with agents were substantially reduced through re-programming the agents in response to detailed analyses of the results by multiple researchers (Suthers, Lund, Rosé & Law, 2013), agents may be ultimately incapable of being well "situated" in a group's shared world. Since they are not co-present, attending to the shared object of attention in human ways, but are following generic algorithms designed outside of the current context of interaction, their contributions can disrupt the delicate focus of group co-attention.

Automated Agent

Agents and other automated techniques for guiding student groups to achieve academic goals are often modeled on the role of an excellent teacher. But even trained, experienced teachers find the task of orchestrating student discussion overwhelming. Teachers should ideally anticipate student misconceptions, monitor their ideas and have them presented to the class in a strategic sequence (Stein, Engle, Smith & Hughes, 2008). This requires a shared understanding by the teacher of the students' articulations of ideas.

It is unlikely that software agents will soon be able to effectively engage in anticipating, monitoring, selecting, sequencing and making connections between student responses. It is not just a matter of the high level of sophistication required in understanding the students. In theory, it is questionable whether software agents can ever participate as human peers in small-group interaction. They cannot be situated in the world or understand meaning the ways that humans do—largely based on human bodily presence in the physical world (Lakoff, 1987) and intersubjective experiences (Vygotsky, 1930/1978).

Suchman (2007, p.179) derived "three outstanding problems for the design of interactive machines" in her groundbreaking empirical study of interactions with intelligent help systems in copier machines. These problems centered on the lack of a shared understanding between the machines and the humans. Suchman stressed that the limits of software supports should be made very clear to users, to avoid unrealistic expectations that lead to problems of interaction with the systems. While it is possible to address some of these concerns, it is probably important to make explicit to the users the limits of agents and other software functions. For instance, anthropomorphizing the agent with a human-sounding name and having the agent use colloquial-sounding speech forms may be counter-productive.

Over-scripting

A danger of automated guidance and scripted support (Kobbe et al., 2007), such as prompts to be answered, is that they miss the engagement of when a listener really wants an explanation. Dillenbourg (2002) noted the problem of scripted agents distracting from the student-centered nature of collaborative learning; they may appear superficially collaborative, but may fail to trigger the cognitive, social and emotional mechanisms that are expected to occur during collaboration. If academic discourse moves are not well situated in student discourse, the effect may be disruptive to authentic group-cognitive processes.

Three implications for research on the computer support of academically productive discourse and for the design of effective supports follow from the discussion in this Investigation:

- It is possible to observe and analyze in chat logs how online small groups establish co-presence, maintain intersubjectivity and accomplish group-cognitive tasks. This can often reveal cognitive processes and the effects on them of different media more clearly than in studies of individuals or face-to-face groups.
- Digital collaboration environments can support co-attention, shared understanding and group cognition in online modes that are essentially different from situations of physical embodiment. However, this requires careful design of technology, pedagogy and interventions based on iterative trials.
- Usage analysis is needed to compare the results of different approaches to the use of mechanisms such as software agents or other scaffolding. The results are often unintuitive, since they may differ from analogous effects in the context of individual cognition or face-to-face interaction.

These conclusions tend to support key hypotheses proposed in Investigation 20, motivating the research reported here.

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Investigation 22. Supporting Group Cognition with a Cognitive Tool

Gerry Stahl

Abstract. The Virtual Math Teams Project is exploring how to create, structure, support and assess an online chat-based collaborative community devoted to mathematics discourse. It is analyzing the forms of group cognition that emerge from the use of shared cognitive tools with specific functionalities. Centered on a case study of a synchronous online interchange, this Investigation discusses the use of a graphical referencing tool in coordination with text chat to achieve a group orientation to a particular mathematical object in a shared whiteboard. Deictic referencing is seen to be a critical foundation of intersubjective cognitive processes that index objects of shared attention. The case study suggests that cognitive tools to support group referencing can be important in supporting group alignment, intentionality and cognition in online communities such as this one for collaborative mathematics.

Keywords. Referencing, deixis, cognitive tool, gesture, common ground, boundary objects and intentionality, group cognition, collaborative knowledge building, cognition, intentionality, reference, sense making, temporality, learning, epistemology, adjacency pair.

The Problem of Supporting Group Cognition

Suppose one wanted to establish a collaborative community with a certain focus, say to explore mathematics (e.g., the kind of math taught in school or accessible to interested students). How might one go about doing this? How would one invite people, where would they congregate, how would they communicate, what kinds of social practices would emerge, who would provide leadership, whence would knowledge appear? The obvious approach today is to build an online community of people who want to discuss math. Research in computer-supported collaborative learning and working (CSCL and CSCW) has taught us that this requires a well-integrated infrastructure, not just a simple cognitive tool or a generic communication medium. For instance, the following range of issues would have to be addressed: how should the software environment be designed; what kind of curriculum or domain content should be included; how are working groups to be formed; how will participants be recruited? The design of cognitive tools to support such an online collaborative community would involve many inter-related considerations, most of which are not yet well understood.

Cognitive tools for collaborative communities are essentially different from cognitive tools for individuals. A number of publications detail the following considerations (Dillenbourg & Traum 2006; Jones, Dirckinck-Holmfeld & Lindström, 2006; Stahl, 2006a):

1. The use of cognitive tools by a collaborative community takes place through many-to-many interactions among people, not by individuals acting on their own.
2. The cognition that the tools foster is inseparable from the collaborative interaction that they support.
3. The relevant cognition is the "group cognition" that is shared at the small-group unit of analysis; this is a linguistic phenomenon that takes place in discourse, rather than a psychological phenomenon that takes place in an individual's mind.
4. The tools may be more like communication media than like a hand calculator—they do not simply amplify individual cognitive abilities; they make possible specific new forms of group interaction.
5. Rather than being relatively simple physical artifacts, tools for communities may be complex infrastructures.
6. Infrastructures do not have simple, fixed affordances designed by their creators; they are fluid systems that provide opportunities that must be specified by users and enacted by them.
7. The community must interpret the meanings designed into the tools, learn how to use the tools, share this understanding and form social practices or methods of use.

8. Analyzing the effectiveness of these tools requires a special methodology that can analyze the methods developed by the community for taking advantage of the infrastructure to accomplish its collaborative activities.

9. The community with its tools forms a complex, non-linear system that cannot be modeled through simple causal relationships, because the whole is both over-determined and open-ended; the community is made possible by its infrastructure, but also interprets the meaning of its tools and adapts their affordances.

This Investigation tries to respond to these considerations without having the space to present them in depth. It reports on an effort to develop a cognitive tool for an online community of mathematics discourse. Experience—along with the preceding considerations—has shown that the design of software tools for collaborative learning must consider above all else how people will actually use the tool. Therefore, our design effort was structured as a design-based research experiment, in which a relatively simple solution is first tried out in a realistic small-scale setting. The results of actual usage are analyzed to assess what worked and what barriers were encountered. Successive re-design cycles attempt to overcome the barriers that users encountered and to evolve a tool and approach (conceptualizations, theory, methodology) that provide increasingly effective support for a gradually emergent online community. This user-centered approach—applied to a growing community of users rather than to subjects representing an imagined "typical" individual user—focuses on the details of how the community interacts through the tool.

More specifically, we will look at a cognitive tool that was recently added to the infrastructural support for the Math Forum, a community of mathematics teachers and students. The tool allows users to relate work in a text-chat stream with work done in a shared whiteboard drawing area. The tool draws lines from a chat message to other chat messages and/or to areas in the whiteboard. We call this tool a "graphical referencing tool" because it supports the ability of a message to reference an item already existing in the online environment by drawing a line from the message to the graphical item.

After briefly describing our research project and discussing our methodology for analyzing usage, we will present a case study of how students used the cognitive tool for referencing. Close analysis of a brief excerpt from an actual student interaction using the tool will illustrate both how complex the achievement of shared references can be and how crucial referencing can be for the group cognition that takes place. Findings of the case study will then motivate consideration of conceptual issues in understanding referencing: reflections on the epistemology and pedagogy of referencing will provide insight into issues of gesture, common ground, boundary objects and intentionality in group cognition.

An Experiment in Designing an Online Chat Community

The Virtual Math Teams (VMT) project at the Math Forum is a research project to explore some of the issues posed above. In order to understand the experience of people and groups collaborating online in the VMT service, the researchers in the project look in detail at the interactions as captured in computer logs. In particular, the project studies groups of three to six middle- or high-school students discussing mathematics in chat rooms. The logs that are collected capture what the participants see to a close approximation.

The VMT project was designed to foster, capture and analyze instances of "group cognition" (Stahl, 2006a). The project is set up so that every aspect of the communication can be automatically captured when student groups are active in the online community, so that the researchers have access to virtually everything that enters into the communication and that is shared by the participants. All interaction takes place online, so that it is unnecessary to videotape and transcribe. Each message is logged with the name of the user posting it and the time of its submission. Similarly, each item placed in the shared whiteboard is tagged with the name of its creator and its creation or modification time. The chat is persistent, and the history of the whiteboard can also be scrolled by participants, and later by researchers.

Although many things happen "behind the scenes" during chat sessions—such as the production of the messages, including possible repairs and retractions of message text before a message is sent, or things that the participants do but do not mention in the chat—the researcher sees practically everything that the participants share, and all see. While the behavior of a participant may be influenced on an individual basis—such as by interactions with people outside of the chat or by the effects of various social and cultural influences—the researchers can generally infer and understand these influences to the same extent as the other participants (who typically do not know each other outside of the chats). These "external" factors (including the participants' age, gender, ethnicity, culture) only play a role in the group interaction to the extent that they are somehow brought into the discourse or "made relevant" in the chat. In cases where they play a role in the group, then, they are also available to the researchers in the same ways as to the participants.

In particular, the sequentiality of the chat messages and of the actions in the whiteboard are maintained so that researchers can analyze the phenomena that take place at the group level. The other way in which the group interaction may be influenced from outside of activities recorded in the chat room is through general background knowledge shared by the participants, such as classroom culture, pop culture or linguistic practices. If the participants meet on the Internet and do not all come from the same school and do not share any history from outside of the VMT chats, then researchers are likely to share with the participants most of the background understanding that the participants themselves share.

This is not to say that the researchers have the same experience as the participants, but their resources for understanding the chat are quite similar to the resources that the participants had for understanding and creating the chat, despite the dramatic differences between the participant and researcher perspectives. Participants experience the chat in real time as it unfolds on their screen. They are oriented toward formulating their messages to introduce into the chat with effective timing. Researchers are engaged in analyzing and recreating what happened, rather than participating directly in it. They are oriented toward understanding why the messages were introduced when and how they were. They are behaving under very different motivations, timings and constraints.

We want to understand how groups construct their shared experience of collaborating online. While answers to many questions in human-computer interaction have been formulated largely in terms of individual psychology, questions of collaborative experience require consideration of the group as the unit of analysis. Naturally, groups include individuals as contributors and interpreters of content, but the group interactions have structures and elements of their own that call for different analytic approaches. In particular, the solving of math problems in the chat environment gets accomplished collaboratively, interactionally. That is, the cognitive work is done by the group.

We call this accomplishment *group cognition*—a form of distributed cognition that may involve advanced levels of cognition like mathematical problem solving, and that is visible in the group discourse where it takes place. It is possible to conduct informative analyses of chats at the group unit of analysis, without asking about the individuals—e.g., their motivations, internal reflections, unexpressed feelings, intelligence, skills, etc.—beyond their participation in the group interaction. Of course, there are also fascinating questions about the interplay between group cognition and individual cognition, but we will not be considering those here.

The VMT project is studying how small groups of students do mathematics collaboratively in online chat environments. We are particularly interested in the new *group methods* that the teams must develop to conduct their interactions in an environment that presents new affordances for interaction. “Member methods” (Garfinkel, 1967) are interactional patterns that participants in a community adopt to structure and give meaning to their activities. A paradigmatic example of member methods is the set of conventions used by speakers in face-to-face conversation to take turns talking (Sacks, Schegloff & Jefferson, 1974). The use of such methods is generally taken-for-granted by the community and provides the social order, meaning and accountability of their activities. Taken together, these member methods define a group culture, a shared set of ways for people interacting to make sense together of their common world. The methods adopted by VMT participants are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to them. These methods help define the nature of the collaborative experience for the small groups that develop and adopt them. Through the use of these methods, the groups construct their collaborative experience. The chat takes on a flow of interrelated ideas for the group, analogous to an individual's stream of consciousness. The referential structure of this flow provides a basis for the group's experience of intersubjectivity and of a shared world.

As designers of educational chat environments, we are particularly interested in how small groups of students construct their interactions in chat media that have various specific technical features. How do the students learn about the affordances that designers embedded in the environment, and how do they negotiate the methods that they adopt to turn technological possibilities into practical means for mediating their interactions? Ultimately, how can we design with students the technologies, pedagogies and communities that will result in desirable collaborative experiences for them? Our response to the question of how cognitive tools mediate collaborative communities is to point to the methods that interactive small groups within the community spontaneously co-construct to carry out their activities using the tools.

To explore this complex topic within the confines of this Investigation, we will look at a brief excerpt of one dyad of students within an online small group using the affordances of the technological environment of the VMT project at one point in its development. Specifically, we look at how the students reference a particular math object in the virtual environment. We will see a number of methods—or group practices [Investigation 16]—being used within a 16-line excerpt. We will also mention other methods that we have observed students employing for referencing in similar chat sessions.

Technology for Referencing in a Chat Environment

In our design-based research at the Virtual Math Teams project (Stahl, 2005), we started by conducting chats in a variety of commercially available environments, including AOL Instant Messenger, Babylon, WebCT, Blackboard. Based on these early investigations, we concluded that we needed to include a shared whiteboard for drawing geometric figures and for persistently displaying notes. We also found a need to minimize “chat confusion” by supporting explicit referencing of response threads (Cakir, Xhafa, Zhou & Stahl, 2005; Fuks, Pimentel & de Lucena, 2006). We decided to adopt and adapt ConcertChat, a research chat environment with special referencing tools (Mühlpfordt & Wessner, 2005). By collaborating with ConcertChat’s developers, our educational researchers have been able to successively try out versions of the environment (see Figure 1) with groups of students and to gradually modify the environment in response to what we find by analyzing the chat logs.

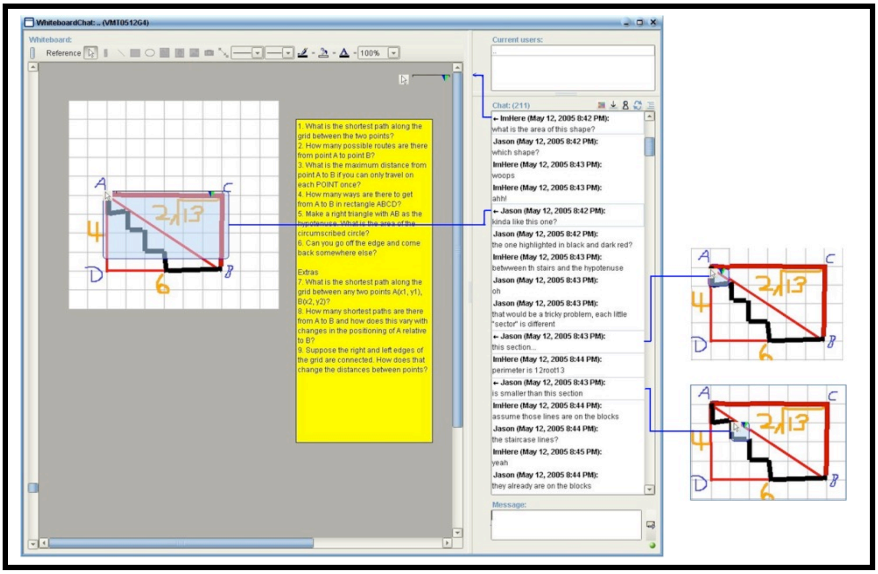

Figure 1. Screen view of ConcertChat with referencing. The image has been modified to show graphical references from chat lines 1, 5, 10 and 12 to the whiteboard. The drawing from the whiteboard has been duplicated in the figure’s margin twice to accommodate this. Only the reference from a single selected chat line would actually appear at any given time.

The ConcertChat environment allows for a variety of referencing methods in math chats:

- *Referencing the whiteboard from a posting*. When someone types a new chat message, they can select and point to a rectangular area in the whiteboard. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the area of the drawing (see Figure 1).

- *Referencing between postings*. A chat message can point to one or more earlier textual postings with a bold connecting line, like whiteboard references. ConcertChat includes a threaded view of the chat postings that, based on the explicit references between postings, displays them like a typical threaded discussion with responses indented under the posting that they reference.

- *Referencing a recent drawing*. The shared whiteboard allows chat participants to create drawings. As new objects are added to the drawing by participants, an implicit form of referencing occurs. Participants typically refer with a deictic term in their textual chat to a new addition to the drawing, whose recent appearance for the group makes it salient.

- *Linguistic referencing*. Of course, one can also make all the usual verbal references to an object on the whiteboard or posting in the chat stream: using deictic terms (*that*, *it*, *his*, *then*); quoting part of an earlier posting; or citing the author of a previous posting.

In May 2005, we conducted a series of chats using ConcertChat (VMT SpringFest 2005). We formed five virtual math teams, each containing about four middle-school students selected by volunteer teachers at different schools across the USA. The teams engaged in online math discussions for four hour-long sessions over a two-week period. They were given a brief description of a non-traditional geometry environment: a grid-world where one could only move along the lines of a grid (Krause, 1986). The students were encouraged to come up with their own questions about the grid-world, such as questions about shortest paths between points A and B in this world (as in Figure 1).

The chats were each facilitated by a member of our research project team. The facilitator welcomed students to the chat, pointed them toward the task, briefly demonstrated the graphical referencing tool and then kept generally quiet until it was time to end the session. We then analyzed the resultant chat logs in order to draw design implications for revising the tools and the service.

An Analysis of a Case of Referencing

The chat log excerpt visible in Figure 1 is reproduced in Log 1 (with line numbers added to enable referencing in this Investigation). In this interactional sequence, two students discuss parts of a drawing that has already been constructed in the shared whiteboard by the larger group to which they belong. The group had created the drawing as part of discussions about shortest paths between points A and B in a grid-world. In particular, a red triangle, ABD, was drawn with sides of length 4, 6 and 2√13. A thick black staircase line was drawn as a path on the grid from A to B. In this excerpt, the students propose a math problem involving this drawing.

1	ImH:	what is the area of this shape? [REF TO WB]
2	Jas:	which shape?
3	ImH:	woops
4	ImH:	ahh!
5	Jas:	kinda like this one? [REF TO WB]
6	Jas:	the one highlighted in black and dark red?
7	ImH:	between th stairs and the hypotenuse
8	Jas:	oh
9	Jas:	that would be a tricky problem, each little "sector" is different
10	Jas:	this section [REF TO WB]]
11	ImH:	perimeter is 12root3
12	Jas:	is smaller than this section [REF TO WB]
13	ImH:	assume those lines are on the blocks
14	Jas:	the staircase lines?
15	ImH:	yea
16	Jas:	they already are on the blocks

Log 1. Line numbers have been added and names anonymized. Graphical references to the whiteboard are indicated by "[REF TO WB]."

The message in line 1 of the chat excerpt (see Log 1) makes a bid at proposing a mathematical question for the group to consider: "What is the area of this shape?" This is accompanied by a graphical reference to the whiteboard. The reference does not indicate a specific area—apparently ImH did not completely succeed in properly using this new referencing tool. Line 2 raises the question, "Which shape?" pointing out the incompleteness of the previous message's reference. The proposal bid in line 1 calls for a proposal response, such as an attempt to answer the question. However, the question was incompletely formed because its reference was unclear, so it received a call for clarification as its immediate response. Lines 3 and 4 display a recognition and agreement of the incomplete and problematic character of the referencing.

Lines 5 and 6 offer a repair of line 1's problem. First, line 5 roughs in the area that may have been intended by the incomplete reference. It includes a complete graphical reference that points to a rectangular area that includes most of the upper area of rectangle ACBD in the drawing. The graphical referencing tool only allows the selection of rectangular areas, so line 5 cannot precisely specify a more complicated shape. The text in line 5 ("kinda like this one?") not only acknowledges the approximate nature of its own referencing, but also acknowledges that it may not be a proper repair of line 1 (by ending with a question mark) and accordingly requests confirmation from the author of line 1. At the same time, the *like* reflects that this act of referencing is providing a model of what line 1 could have done. Peer instruction in the use of the software is taking place among the students as they share the group's growing understanding of the new chat environment.

Line 5 is accompanied by line 6, which provides a textual reference or specification for the same area that line 5 pointed to: the one highlighted in black (the staircase line) and dark red (lines AC and CB). The inexact nature of the graphical reference required that it be supplemented by this more precise textual reference. Note how the sequence of indexical attempts in lines 1, 2, 5 and 6 successively focuses shared attention on a more and more well-defined geometric object. This is an interactive achievement of the group. The reference was not a simple act of an individual. Rather, it was accomplished through an extended interaction between ImH and Jas, observed by others and situated among the math objects constructed by the whole group of students in the chat room environment.

Lines 5 and 6 were presented as questions calling for confirmation by ImH. Clarification follows in line 7 from ImH: "between the stairs and the hypotenuse." Line 8's "Oh" signals mutual understanding of the evolving reference and the establishment of an agreed upon boundary object (Star, 1989) for carrying on the mathematical investigation incompletely proposed in line 1. Now that the act of referencing has been successfully completed by the group, the group can use the referenced area as a mathematical object whose definition or meaning is intersubjectively understood. Viewed at the individual unit of analysis, the referenced area can serve as a boundary object shared among the interpretive perspectives of the interacting individuals. In other words, it becomes part of the common ground (Clark & Brennan, 1991) shared by the students. The referencing interaction established or grounded this. Note, however, that what took place was not an aligning of pre-existing individual opinions—as the theory of common ground is often taken to imply—but a group process of co-constructing a shared reference through a complex interaction involving many resources and social moves.

Now that a complete reference has been constructed to a math object that is well enough specified for the practical purposes of carrying on the chat, Jas launches into the problem solving by raising an issue that must first be dealt with. Line 9 says that calculating the area now under consideration is tricky. The tricky part is that the area includes certain little "sectors" whose shapes and areas are non-standard. Line 9 textually references "each little 'sector'." *Little* refers to sub-parts of the target area. *Each* indicates that there are several such sub-parts—and *sector*, put in scare quotes, is proposed as a name/description of these hard-to-refer-to sub-parts.

Clarification of the reference to sectors is continued by lines 10 and 12. These lines compare two sectors, demonstrating that they are different by showing that one is smaller than the other. Lines 10 and 12 reference two different sectors, both with the same textual, deictic description: *this section*. It is possible to use the identical description twice here because the text is accompanied by graphical references that distinguish the two sectors. Line 10 points to the small grid square inside of rectangle ACBD in the upper left-hand corner adjacent to point A. Line 12 points to the next grid down the hypotenuse (see Figure 1).

Because of the limitations of the graphical reference tool, lines 10 and 12 can only indicate the squares of the grid, not the precise odd-shaped sectors that are of concern in the group discourse. On the other hand, the textual clause, *this section* has been given the meaning of the odd-shaped sub-areas of the area "between the stairs and the hypotenuse," although it cannot differentiate easily among the different sections. The carefully constructed combination of graphical and textual referencing accomplished in lines 10 and 12 was needed to reference the precise geometric objects. The combination of the two textual lines, with their two contrasting graphical references, joined into one split sentence was necessary to contrast the two sectors and to make visible the tricky circumstance. In this way, the discourse succeeded in constituting the complicated geometric sectors despite the limitations of the tool on its own and of textual description by itself.

Line 13 responds to the tricky issue by treating it as a non-essential consequence of inaccurate drawing. By proposing that the group "assume those lines are on the blocks," this posting treats the difference among the sectors as due to the inaccuracy of the drawing of the thick black staircase line in not precisely following the grid lines. Physical drawings are necessarily rough approximations to idealized mathematical objects in geometry. Lines drawn with a mouse on a computer screen tend to be particularly rough representations. The implication of line 13 is that the tricky issue is due to the inaccurate appearance of the lines, but that the faults of the physical drawing do not carry mathematical weight and can be stipulated away. However, line 14 questions this move. It first makes sure that line 13's reference to *those lines* was a reference to *the staircase lines* that form part of the perimeter of the target area and of its different-sized sectors. When line 15 confirms that line 13 indeed referenced the staircase lines, line 16 responds that "they already are on the blocks"—in other words, the tricky situation was not due to inaccuracies in the drawing, but the staircase lines were indeed already *taken as* following the grid for all practical purposes. The problem was still seen to be a tricky one once the mathematical object was clearly referenced and specified.

We see here that referencing can be a complex process in online mathematical discourse. In a face-to-face setting, the participants could have pointed to details of the drawing, could have gesturally described shapes, could have traced outlines or shaded in areas either graphically or through gestures with ease. Conversationally, they could have interrupted each other to reach faster mutual orientation and understanding. Online, the interaction is more tightly constrained and burdensome due to the restricted nature of the affordances of the software environment. On the other hand, we have seen that middle-school students who are new to the graphical tools of ConcertChat, as well as to online collaborative mathematics, can call upon familiar resources of textual language, drawing, pointing and school mathematics to construct interaction methods that are seen to be amazingly sophisticated, efficient, creative and effective when analyzed in some detail.

Methods of Making Referential Sense

We have here only been able to look at what took place in a single effort to reference a mathematical object. In the series of chats that this effort was taken from, we observed groups of students engaging in a variety of other referencing methods within this version of ConcertChat. (For additional uses of the referencing tool, see Mühlpfordt & Wessner, 2005). Common methods used by groups in our chats included the following:

- Graphical references to previous messages were sometimes used to make salient a message from relatively far back in the chat. Without the graphical referencing functionality, this would have required a lengthy textual explanation justifying change of topic and quoting or describing the previous message.

- Some students used graphical references to previous messages to specify a recipient for their new posting. If a student wanted to address a question to a particular student rather than to the group as a whole, he or she would accompany the question with a graphical reference to a recent posting by that student. (This was a use of graphical referencing not at all anticipated by the ConcertChat software tool designers or VMT researchers.)

- It is common in chat for someone to spread a single contribution over two or more postings (e.g., lines 10 and 12). In conversation, people often retain their turn at talk by indicating that they are not finished in various ways, such as saying “ummm.” In generic chat systems, people often end the first part of their contribution with an ellipsis (…) to indicate that they will continue in a next posting. In ConcertChat, students sometimes graphically referenced their first posting while typing their second. Then the two parts would still be tied together even if someone else's posting (like line 11) appeared in the meantime.

- Similarly, students graphically referenced their own previous posting when repairing a mistake made in it. The reference indicates that the new posting is to replace the flawed one.

- In chat, where the flow of topics is not as constrained as in conversation, it is possible for multiple threads of discussion to be interwoven. For instance, line 11 starts to discuss perimeter while area is still being discussed. Graphical references are used to tie together contributions to the same thread. For instance, line 12 might have referenced line 10 graphically.

- The graphical referencing tool is treated as one of many available referencing resources. Deictic terms are frequently used—sometimes in conjunction with graphical referencing (e.g., line 5).

- In textual chat, as in spoken conversation, sequential proximity is a primary connection. By default, a posting is a response to the immediately preceding post. Chat confusion arises because sequentiality is unpredictable in chat; people generally respond to the most recent posting that they see when they start to type, but by the time their response is posted other postings may intervene. Interestingly, the recency of drawings may function as a similar default reference. Students frequently refer to a line that was just added to the whiteboard as *that line* without needing to create a graphical reference to it.

- Of course, purely textual references are also widely used to point to postings, people, groups, drawings, abstractions and math objects.

The many forms of referencing in chat tie together the verbal and graphical contributions of individual participants into a tightly woven network of shared meaning. Each posting is connected in multiple ways—explicit and implicit—to the flow of the shared chat (Stahl, 2005). The connections are highly directional, granting a strong temporality to the chat experience (hard to fully appreciate from a static log).

The being-there-together in a chat is temporally structured as a world of future possible activities with shared meaningful objects. The possibilities for collaborative action are made available by the social, pedagogical and technical context (world, situation, activity structure, network of relevant significance) (Heidegger, 1927/1996, §18). While the shared context is opened up, enacted and made salient by the group in its chat, aspects of the discourse context appear as designed, established or institutionalized in advance. They confront the participants as a world filled with meanings, priorities, resources and possibilities for action. An online environment is a world whose features, meanings and co-inhabitants are initially largely unknown.

We are interested in providing cognitive tools to help groups of students navigate worlds of online collaborative mathematical discourse. We want to support their efforts to build collaborative knowledge. Since the Greeks and especially following Descartes, the issue of how people can know has been called "epistemology." We have seen in our case study that methods of referencing can play an important role in grounding the construction of shared knowledge in an environment like VMT. Conceptually, referencing can be seen as a key to the question of how groups can construct collaborative knowledge, how they can know.

Epistemology of Referencing

Referencing is a primary means for humans to establish joint attention and to make shared meaning within a (physical or virtual) world in which they find themselves together. Vygotsky, in a particularly rich passage, described the interactional origin of pointing as an example of how gestures become meaningful artifacts for individual minds through social interaction:

> A good example of this process may be found in the development of pointing. Initially [e.g., for an infant], this gesture is nothing more than an unsuccessful attempt to grasp something, a movement aimed at a certain object which designates forthcoming activity…. When the mother comes to the child's aid and realizes this movement indicates something, the situation changes fundamentally. Pointing becomes a gesture for others. The child's unsuccessful attempt engenders a reaction not from the object he seeks but from another person. Consequently, *the primary meaning* of that unsuccessful grasping movement *is established by others*…. The grasping movement changes to the act of pointing. As a result of this change, the movement itself is then physically simplified, and what results is the form of pointing that we may call a true gesture. (Vygotsky, 1930/1978, p. 56, italics added)

The pointing gesture is perhaps the most fundamental form of deictic referencing. In its origin where the infant begins to be socialized into a shared world, the meaning of the gesture emerges interactionally as the participants orient to the same object and recognize that they are doing so jointly. This fundamental act of collaborative existence simultaneously comes to be symbolized for them by the pointing gesture, which is practiced, repeated and abstracted by them together over time and thereby established as meaningful. The mother and infant become an organic small group, caring for shared objects by being-in-the-world-together and understanding as collaborative practice the symbolic meaning of the physical gesture as a referencing artifact.

In grasping, the infant's being-in-the-world is intentionally directed at the object; the existence of the pointing infant is a being-at-the-object (Husserl, 1929/1960). When the mother joins the infant by transforming his individual grasp into a joint engagement with the object, the intentionality of the infant's grasp becomes intersubjective intentionality, constituting the infant and mother as being-there-together-at-the-object (Heidegger, 1927/1996, §26). For Husserl, consciousness is always consciousness-of-something. Human consciousness is intentional in the sense that the conscious subject intends an object, so that the subject as consciousness is at the object (i.e., not "in the head"). Heidegger transformed this idealist conception into an embedded analysis of human being-there as being involved in the world. Heidegger's analysis builds up to the brink of a foundational social philosophy of being-there-together, but then retreats to an individualistic concern with the authentic self (Nancy, 2000; Stahl, 1975). Vygotsky points the way to a fully social foundation, interpreting Marx' social *praxis* in social-psychological terms, such as in the intersubjective interaction of the infant-mother bonding.

Epistemology as a philosophic matter is a consequence of the Platonic and Cartesian separation of mind and meaning from the physical existence of objects in the world. The "problem of epistemology" is the question of how the mind can know facts—how one can bridge the absolute gulf that Plato (Plato, 340 BCE) and Descartes (1633) drew between the mental realm of ideas and the physical world of matter.

Vygotsky's social philosophy overcomes this problem by showing how interactions among people achieve shared involvement in the world. In Descartes' system, there was no way to put together the mother's understanding, the infant's understanding, the physical grasp and the symbolic meaning of pointing. In Vygotsky's analysis, the interaction between mother and infant creates the shared meaningfulness of the pointing grasp as an intersubjectively achieved unity. There is no longer any reason to ask such questions as: where the meaning of the gesture is, how does the mother know the infant's intention, or whether there is common ground. These are pseudo-problems caused by trying to reduce a social phenomenon at the group unit of analysis to issues at an individual unit of analysis.

These philosophical issues are intimately related to issues of empirical methodology. They imply that certain matters should be analyzed as group phenomena and not reduced to individual psychic acts or mental representations.

As researchers, we can empirically observe new referencing gestures being created within interactions among collaborating people, particularly when their interaction is taking place via a new medium that they must learn how to use. In the analysis above, a chat posting—"What is the area of this shape?"—constitutes the participants in the chat as a group by designating them as the intended collective recipient and as the expected respondent to the question (Lerner, 1993). The group is the intended agent who will work out the mathematics of the proposal to compute the area. Simultaneously, by referencing a mathematical object ("this shape"), the posting constitutes the group as a being-there-together-at-the-object—at an object that is constituted, identified, referenced and made meaningful by the group interaction.

We saw how both these aspects of being a group and maintaining the group's joint attention necessitated considerable interactional work by the participants. Before the elicited answer about area could be given in response to the question, the group had to negotiate what it as a group took the object to be. Also, it required a number of actions for group participants to co-construct the shared object and their being-there-together-at-the-object. In attempting to do this, they constituted themselves as a group and they established referential gestures and terms that took on the shared meaning of intending the new math object.

The interactional work of the group involved making use of the resources of the environment that mediated their interaction. This is particularly noticeable in online interaction. Vygotsky's infant and mother could use fingers, gaze, touch, voice. Online participants are restricted to exchanging textual postings and to using features of the mediating software (Garcia & Jacobs, 1999; Stahl, 2006b). The chat participants must explicitly formulate through text, drawings or graphical references actions that can be observed by their fellow group members. These actions are also available to researchers retroactively.

The textual interactions in the chat excerpt as the cognitive actions of the group are in intimate contact with the details of the drawing as the physical intentional object. For instance, as we saw above, in the interchange in lines 13 to 16 the group attention is focused on a particularly interesting and ambiguous drawn line. Group methods of proceeding often involve *adjacency pairs*, sequences of utterances by different people that construct group meaning and social order through their paired unity. The meaning is constituted at the group unit of analysis by means of the interaction of the pair of utterances, not as a presumed pre-interactional meaning in the heads of individuals.

Line 13 is a bid at opening up a math proposal adjacency pair (Stahl, 2006b) [Investigation 25]: it offers a new step for mathematical discussion and elicits an uptake response from the rest of the group. Line 16 is the elicited response that takes up the bid with a kind of repair. It indicates that the proposed assumption is unnecessary and thereby attempts to re-establish a shared understanding of the situation. Lines 14 and 15 form a question/answer adjacency pair inserted in the middle of the proposal pair in order to make sure that the group really is together at the same detail of their shared math object.

The issue that is worked out by the group as it looks carefully at the drawing together, illustrates the subtlety of abstract mathematical thinking that the group is engaged in as a group. The issue involves the lines that were drawn with the whiteboard's rough cognitive tools for drawing and whether or not these lines coincide with lines of the grid (i.e., if the group should "assume those lines are on the blocks"). The issue is not one that is resolved by a close analysis of the actual pixels on the screen. Rather, it is a conceptual question of the meaning of those lines for the group: What do they mean in the drawing and how should they be taken by the group in its math discourse?

In being together at the lines, the group makes sense of the meaning of the lines. There is no separation of fact and meaning here—or, if there is, the group interaction engages in meaning-making processes that fluidly overcome such a gulf. This is particularly important in math discourse, where rough sketches are used to represent (mean and reference) abstract objects. Maintaining a shared understanding by a group of students working in a mathematical context like this is a subtle and intricate matter.

As designers of online education, we are interested in understanding how students collaboratively create new communicative gestures or interactional methods, including ways of referencing objects for joint consideration. More generally, an interactional understanding of referencing and meaning making leads to a theory of group cognition—rather than individual cognition based on mental representations—as a basis for studying collaborative learning (Stahl, 2006a).

All the technical terms like *cognition*, *intentionality*, *reference*, *sense making*, *temporality* and *learning* needed to articulate a theory of group cognition must be re-conceptualized at the group unit of analysis. In some cases, the nature of these phenomena is actually easier to see at the group level, where participants have to make things visible to each other in order to coordinate their actions as group activities, as was the case with referencing in the excerpt discussed above.

Pedagogy of Referencing Math Objects

Our case study suggests that cognitive tools for referencing can be important supports for group cognition and collaborative knowledge building, particularly in a setting of computer-supported collaborative mathematics.

In the investigation reported here, we tried to encourage relatively open-ended explorations of mathematical inquiry by online teams of math students. We presented them with a non-traditional form of geometry in which notions like distance, area or shortest-path have to be renegotiated—i.e., the meanings of these terms must be jointly constructed anew. While trains of inquiry can go in many directions, in a collaborative effort each step of the path may be clarified and shared. New math objects emerge and develop out of the discourse, including both geometric figures (the tricky area) and terminology ("distance along the grid").

In this Investigation, the analysis of a snippet of a group-cognitive process in a concrete empirical case has suggested the centrality of joint referencing to collaboration. This may serve as an additional clarification of what is meant by defining collaboration as "a continued attempt to construct and maintain a shared conception of a problem … an emergent, socially-negotiated set of knowledge elements that constitute a Joint Problem Space" (Roschelle & Teasley, 1995, p. 70) and what goes into actually doing such a thing.

The persistent whiteboard serves as a "group external memory" that plays a useful role in grounding shared understanding at the scale of analysis of CSCL problem solving (Dillenbourg & Traum 2006, p. 122f). The view of the whiteboard as the group's evolving joint problem space contrasts with Clark & Brennan's (1991) psycholinguistic version of common ground as located in individual minds. The intertwining uses of the dual workspaces of whiteboard and chat mirror the intertwining of content space and problem space that is characteristic of collaborative learning (Barron, 2003, p. 310).

Given the complexity resulting from dual spaces—whether split for work vs. reflection (Fischer et al., 1998; Schön, 1983) or transitory vs. persistent (Dillenbourg & Traum 2006, p. 143f)—and the concomitant substantially increased burden of coordination within the group, we can clearly see the importance of cognitive tool support for referencing from one space to the other—e.g., from text chat to graphical workspace.

Referencing in mathematical worlds has its own domain-specific characteristics and priorities. Widespread conceptions of math learning as the memorization of "math facts" or the mastery of formulaic algorithmic solutions are oriented to the routine application of arithmetic rather than to the creative process that inspires mathematicians. The history of mathematics as a branch of scientific inquiry and knowledge building is a systematic unfolding of new domains through the shared construction of new math objects, like complex numbers, fractals, curved spaces. To share these created math objects as boundary objects within their discourse community, mathematicians have had to define new vocabularies, symbols and representations for referencing objects that do not exist as such in the physical world. Referencing such abstractions presents special cognitive challenges.

People who do not understand mathematical references can scarcely be expected to share the wonder and excitement that mathematicians feel who can see what is being referenced (Lakoff & Núñez, 2000). It is likely that much of the general population simply does not share the understanding of what is referenced in most mathematical proofs and discussions. Since our goal is to increase mathematical appreciation and participation through opportunities for online math discourse, we are keen to support shared referencing in our environments with effective cognitive tools.

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Investigation 23. Sustaining Interaction in a CSCL Environment

Gerry Stahl

Abstract. Learning takes place over long periods of time that are hard to study directly. Even the learning experience involved in solving a single challenging math problem in a collaborative online setting can be spread across hundreds of brief postings during an hour or more. Such long-term interactions are constructed out of posting-level interactions, such as the strategic proposing of a next step. This Investigation identifies a pattern of exchange of postings that it terms *math-proposal adjacency pair* and describes its characteristics. Drawing on the methodology of conversation analysis, the Investigation adapts this approach to examining mathematical problem-solving communication and to the computer-mediated circumstances of online chat. Math proposals and other interaction methods constitute the collaborative group as a working group, give direction to its problem solving, and help to sustain its intersubjective meaning making or group cognition. Groups sustain their online social and intellectual work by building up longer sequences of math proposals, other adjacency pairs and a variety of interaction methods. Experiences of collaboration and products of group cognition emerge over time.

Keywords. Longer sequences, math-proposal adjacency pair, failed proposal, design-based research, sustaining interaction.

Sustaining Interaction in a CSCL Environment

Research in learning has traditionally focused on psychological processes at the individual unit of analysis. With the shift to socio-cultural approaches in recent years, the community unit of analysis came to the fore. In my writings on group cognition, I identified small groups as defining a middle ground between individual people and communities of practice:

> Small groups are the engines of knowledge building. The knowing that groups build up in manifold forms is what becomes internalized by their members as individual learning and externalized in their communities as certifiable knowledge. At least, that is a central premise of this book. (Stahl, 2006b, p. 16)

The concept of group cognition, however, retains a certain ambiguity of scale. On the micro level, it is based on the discovery by conversation analysis that a smallest element of interaction in discourse is the adjacency pair, a product of interaction within a dyad or small group, and not an expression of one or more individuals' cognition (Duranti, 1998; Schegloff, 1991). On the macro level, it is a vision of collaborative knowledge building, where knowledge arises through community, interpersonal or social interaction (Scardamalia & Bereiter, 1996; Vygotsky, 1930/1978; Lave & Wenger, 1991, respectively). Taking one approach or the other, we can analyze how participants in a small group of students build a detailed point of shared knowledge, how they develop their mediated cognition or how they apprentice participation in the community of math discourse. The question remains: how can we understand not just the results, but also what happens in the group at the interesting meso level of the small group itself during a one-hour math chat consisting of many detailed interactions but perhaps not measurably increasing the group's knowledge, skills or community participation?

This Investigation tries to address the gap in the methodology of the learning sciences in a preliminary way. It begins with a detailed analysis of a particular interaction that actually occurred in a student chat. It then gradually broadens the discussion of online math chat sessions, discussing various aspects of how the elemental adjacency pairs in such a momentary interaction contribute to a sustained group experience over a somewhat longer period of time. The presentation proceeds through these steps:

- The setting of online math chats, which provide the experimental context for our observations, is first motivated and described.
- The concept of adjacency pairs from conversation analysis is adapted to the situation of online math chats and is particularized as "math-proposal adjacency pairs."
- A specific adjacency pair is analyzed as a "failed proposal," which by contrast sheds light on the nature of successful proposals.

- We then describe our design-based research approach in which we revise our software and pedagogy in response to issues observed during a sequence of evolving trials.
- Next, we look at a more extended interaction that occurred in our revised chat environment, involving methods of computer-supported deictic referencing that build from adjacency pairs to longer sequences of cognitive work.
- To extrapolate beyond one or two detailed interactions and analyze more extended sessions with some generality would require volumes of exposition. We therefore rely on our other studies, our general impressions from observing and participating in many online math chats, and from related work by others to discuss a number of relevant aspects of sustained group cognition.
- We conclude with reflections on how groups construct and sustain their on-going sense of shared experience. This points to future work.

Doing mathematics together online

Technology-enhanced learning offers many opportunities for innovation in education. One of the major avenues is by supporting the building of collaborative meaning and knowledge (Stahl, 2006b). For instance, it is now possible for students around the world to work together on challenging math problems. Through online discussion, they can share problem-solving experiences and gain fluency in communicating mathematically. Research on mathematics education stresses the importance of student discourse about math (NCTM, 2000; Sfard, 2002), something that many students do not have opportunities to practice face-to-face.

While much research on computer-supported collaborative learning (CSCL) has analyzed the use of asynchronous threaded discussion forums, there has been relatively little research on the use of synchronous chat environments in education. The research reported here suggests that chat has great promise as a medium for collaborative learning if the medium and its use are carefully configured. Here, we investigate how math discourse takes place within the chat medium and how we use our analyses to inform the design of effective math chat environments.

In the beginning of the Virtual Math Teams (VMT) research project at the Math Forum, we invited middle-school students to participate in online chats about interesting problems in beginning algebra and geometry. The following math problem, discussed in the chat excerpt analyzed below, is typical:

> If two equilateral triangles have edge-lengths of 9 cubits and 12 cubits, what is the edge-length of the equilateral triangle whose area is equal to the sum of the areas of the other two?

We rely on a variety of approaches from the learning sciences to guide our research and to analyze the results of our trials, including coding along multiple dimensions (Strijbos & Stahl, 2005), analysis of threading (Cakir *et al.*, 2005) and ethnography (Shumar, 2006). In particular, we have developed an ethnomethodologically informed (Garfinkel, 1967; Heritage, 1984) chat analysis approach based on conversation analysis (Pomerantz & Fehr, 1991; Psathas, 1995; Sacks, 1992; Sacks, Schegloff, & Jefferson, 1974; ten Have, 1999) to understand the structure of interactions that take place in student chats. In this paper, we adapt a finding of conversation analysis to math chats and analyze a specific form of adjacency pairs that seem to be important for this context. Before presenting these findings, it may be useful to describe briefly how the notion of adjacency pairs differs from naïve conceptions of conversation.

There is a widespread commonsense or folk-theory (Bereiter, 2002; Dennett, 1991) view of conversation as the exchange or transmission of propositions. This view was refined and formalized by logicians and cognitive scientists as involving verbal "expression" in meaningful statements by individuals, based on their internal mental representations (Shannon & Weaver, 1949). Speech served to transfer meanings from the mind of a speaker to the mind of a listener, who then interpreted the expressed message. Following Wittgenstein (1953) in critiquing this view, speech act theory (Austin, 1952; Searle, 1969) argued that the utterances spoken by individuals were ways of acting in the world and were meaningful in terms of what they accomplished through their use and effects. Of course, the expression, transmission and interpretation of meaning by individuals can be problematic, and people frequently have to do some interactional work in order to re-establish a shared understanding. The construction of common ground has been seen as the attempt to coordinate agreement between individual understandings (Clark & Brennan, 1991).

Conversation analysis takes a different view of conversation. It looks at how interactional mechanisms, like the use of adjacency pairs, co-construct inter-subjectivity.

> *Adjacency pairs* are common sequences of utterances by different people—such as mutual greetings or question/answer interchanges—that form a meaningful speech act spanning multiple utterances, which cannot be attributed to an individual or to the expression of already formed mental states. They achieve meaning in their very interaction.

We are interested in what kinds of adjacency pairs are typical for math chats. The topic of adjacency pairs is taken up extensively in two sections below. Stahl (2006b) further discusses the implications that viewing adjacency pairs as the smallest elements of interactional meaning making has for the intersubjective foundation of group cognition, a process of jointly constructing meaning in discourse.

The medium of online chat has its own peculiarities (Lonchamp, 2006; Mühlpfordt & Wessner, 2005; O'Neill & Martin, 2003). Most importantly, it is a text-based medium, where interaction takes place by the sequential response of brief texts to each other (Livingston, 1995; Zemel, 2005). As a quasi-synchronous medium (Garcia & Jacobs, 1999), chat causes confusion because several people can be typing at once and their texts can appear in an order that obscures to whom or to what they are responding. Furthermore, under time pressure to submit their texts so that they will appear near the post to which they are responding, some chat participants break their messages into several short texts. Because of these peculiarities of chat, it is necessary for researchers to carefully reconstruct the implicit threading of texts that respond to each other before attempting to interpret the flow of interaction (Cakir *et al.*, 2005; Strijbos & Stahl, 2005).

Math chats differ from ordinary informal conversation in a number of additional ways. They are focused on the task of solving a specific problem, and they take place within a somewhat formal institutional setting. They involve the *doing* of mathematics (Livingston, 1986). And, of course, they are computer-mediated rather than face-to-face. The approach of conversation analysis is based on ethnomethodology (Garfinkel, 1967), which involves the study of the *methods* that people use to accomplish what they are doing. So, we are interested in working out the methods that are used by students in online math chats. In this paper, we discuss a particular method of collaboration in math chats that we have elsewhere called *exploratory participation*: participants engage each other in the conjoint discovery and production of both the problem and possible solutions (Wegerif, 2006; Zemel, Xhafa, & Stahl, 2005).

Math-proposal adjacency pairs

In order to begin to analyze the methods that students use in math chats, we take a close look at an excerpt from an actual chat. Log 1 shows an excerpt from near the beginning of the log of one of our first online collaborative math problem-solving sessions. Three students—named Avr, Sup and Pin—have just entered the chat room, said hello to each other and read the problem involving three triangles.

1. Avr (8:21:46 PM): Okay, I think we should start with the formula for the area of a triangle
2. Sup (8:22:17 PM): ok
3. Avr (8:22:28 PM): A = 1/2bh
4. Avr (8:22:31 PM): I believe
5. pin (8:22:35 PM): yes
6. pin (8:22:37 PM): i concue
7. pin (8:22:39 PM): concur*
8. Avr (8:22:42 PM): then find the area of each triangle
9. Avr (8:22:54 PM): oh, wait
10. Sup (8:23:03 PM): the base and heigth are 9 and 12 right?
11. Avr (8:23:11 PM): no
12. Sup (8:23:16 PM): o
13. Avr (8:23:16 PM): that's two separate triangles
14. Sup (8:23:19 PM): ooo
15. Sup (8:23:20 PM): ok
16. Avr (8:23:21 PM): right
17. Avr (8:23:27 PM): i think we have to figure out the height by ourselves
18. Avr (8:23:29 PM): if possible
19. pin (8:24:05 PM): i know how
20. pin (8:24:09 PM): draw the altitude'
21. Avr (8:24:09 PM): how?
22. Avr (8:24:15 PM): right
23. Sup (8:24:19 PM): proportions?
24. Avr (8:24:19 PM): this is frustrating
25. Avr (8:24:22 PM): I don't have enough paper
26. pin (8:24:43 PM): i think i got it
27. pin (8:24:54 PM): its a 30/60/90 triangle
28. Avr (8:25:06 PM): I see
29. pin (8:25:12 PM): so whats the formula

Log 1. Excerpt of 3½ minutes from a one-hour chat log. Three students chat about a geometry problem. Line numbers have been added and screen-names anonymized; otherwise, the transcript is identical to what the participants saw on their screens.

The first thing to notice here is a pattern of proposals, discussions and acceptances similar to what takes place in face-to-face discourse. Bids for proposals about steps in solving the math problem are made by Avr in lines 1, 3, 8, 17 and by Pin in lines 20, 27. These proposals are each affirmed by someone else in lines 2, 6, 10, 19, 22, 28, respectively.

To avoid chat confusion, note that line 21 responds to line 19, while line 22 responds to line 20. The timestamps show that lines 20 and 21 effectively overlapped each other chronologically: Avr was typing line 21 before he/she saw line 20. Similarly, lines 24 and the following were responses to line 20, not line

23. We will correct for these confusions later, in Log 2, which reproduces a key passage in this excerpt.

In Log 1, we see several examples of a three-step pattern:

a) A proposal bid is made by an individual for the group to work on: "I think we should…."

b) A proposal acceptance is made on behalf of the group: "Ok," "right."

c) There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step.

The three-step pattern consists of a pair of postings—a bid and an acceptance—that form a proposal about math, and some follow-up effort. This suggests that collaborative problem solving of mathematics may often involve a particular form of adjacency pair. We will call this a *math-proposal adjacency pair*.

Here are six successful math-proposal adjacency pairs from Log 1:

1. Avr: Okay, I think we should start with the formula for the area of a triangle
2. Sup: ok
3. Avr: A = 1/2bh
6. pin: i concue
8. Avr: then find the area of each triangle
10. Sup: the base and heigth are 9 and 12 right?
17. Avr: i think we have to figure out the height by ourselves
19. pin: i know how
20. pin: draw the altitude'
22. Avr: right
27. pin: its a 30/60/90 triangle
28. Avr: I see

Note from the line numbers that the response is not always literally immediately adjacent to the bid in the chat log due to the complexities of chat posting. But the response is logically adjacent as an up-take of the bid.

Many varieties of adjacency pairs allow for the insertion of other pairs between the two parts of the original pair, delaying completion of the original pair. For instance, a question/answer pair may be delayed by utterances seeking clarification of the question. As we will see below, the clarification interaction may itself consist of question/answer pairs, possibly with their own clarifications—this may continue recursively. With math-proposal adjacency pairs, the subsidiary pairs seem to come after the completion of the original pair, in the form of secondary proposals, questions or explanations that start to do the work that was proposed in the original pair. This characteristic leads to their role in sustaining group inquiry.

Math proposals tend to lead to some kind of further mathematical work as a response to carrying out what the proposal. Often—as seen in the current example—that work consists of making further proposals. In this way, the three-step structure of the math-proposal adjacency pair starts to sustain the group interaction. The proposal bid by one person calls forth a proposal response by someone else. If the response is one of acceptance, it in turn calls forth some further work to be done or a bid for another proposal. If the response is a rejection, it may lead to justification, discussion and negotiation. Since the "preferred" response to a proposal is a statement of acceptance, a response of rejection tends to require some further work.

It is striking that the work proposed by a proposal is not begun until there is agreement with the proposal bid. This may represent consent by the group as a whole to pursue the proposed line of work. Of course, this idea is not so clear in the current example, where there are only three participants, and the interaction often seems to take place primarily between pairs of participants. As confirmed by other chat examples, however, the proposal generally seems to be addressed to the whole group and opens the floor for other participants to respond. The use of "we" in "we should" or "we have to" (stated or implied) constitutes the multiple participants as a plural subject—an effective unified group (Lerner, 1993)[7]. Anyone other than the proposer may respond on behalf of the group. The fact that the multiple participants are posited as a group for certain purposes, like responding to a proposal bid, by no means rules out their individual participation in the group interaction from their personal perspectives, or even their independent follow-up work on the math. It simply means that the individual who responds to the bid may be doing so on behalf of the group.

Moreover, there seems to be what in conversation analysis is called an interactional *preference* (Schegloff, Jefferson, & Sacks, 1977) for acceptance of the proposal. That is, if one accepts a proposal, it suffices to briefly indicate agreement: "ok." If one wants to reject a proposal, however, then one has to account for this response by giving reasons. If the group accepts the bid, one person's response may serve on behalf of the group; if the group rejects the bid, several people may have to get involved.

[7] The fact that the students use "we" indicates that they are taking the set of participants to be a functional group. From an ethnomethodological perspective, this justifies the analysis of the chat as a group product since the participants themselves adopt this stance.

We would like to characterize in more detail the method of making math-proposal adjacency pairs. Often, the nature of an interactional method is seen most clearly when it is breached (Garfinkel, 1967). Methods are generally taken-for-granted by people; they are not made visible or conducted consciously. It is only when there is a *breakdown* (Heidegger, 1927/1996; Winograd & Flores, 1986) in the smooth, tacit performance of a method that people focus on its characteristics in order to overcome the breakdown. The normally transparent method becomes visible in its breach. In common-sense terms we say, "The exception proves the rule," meaning that when we see why something is an exceptional case it makes clear the rule to which it is an exception. Heidegger made this into an ontological principle, whereby things first become experience-able during a breakdown of understanding. Garfinkel uses this, in turn, as a methodological fulcrum to make visible that which is commonly assumed and is effective but unseen.

We can interpret Sup's posting in line 23 as a *failed proposal*. Given the mathematics of the triangle problem, a proposal bid related to proportionality, like Sup's, might have been fruitful. However, in this chat, line 23 was effectively ignored by the group. While its character as a failed proposal did not become visible to the participants, it can become clear to us by comparing it to successful proposal bids in the same chat and by reflecting on its sequential position in the chat in order to ask why it was not a successful bid. This will show us by contrast what the characteristics are that make other proposal bids successful.

A failed proposal

Let us look at line 23 in its immediate interactional context in Log 2. We can distinguish a number of ways in which it differed from successful math proposal bids that solicited responses and formed math-proposal adjacency pairs:

17, 18. Avr (8:23:29 PM): i think we have to figure out the height by ourselves … if possible
19. pin (8:24:05 PM): i know how
21. Avr (8:24:09 PM): how?
20. pin (8:24:09 PM): draw the altitude'
22. Avr (8:24:15 PM): right
24. Avr (8:24:19 PM): this is frustrating
23. Sup (8:24:19 PM): proportions?

Log 2. Part of the chat log excerpt in Log 1, with order revised based on threading of the postings.

(a) All the other proposal bids (1, 3, 8, 17, 20, 27) were stated in relatively complete sentences. Additionally, some of them were introduced with a phrase to indicate that they were the speaker's proposal bid (1. "I think we should ..."; 17. "I think we have to ..."; 20. "i know how ..."; and 27. "i think i got it ..."). The exceptions to these were simply continuations of previous proposals: line 3 provided the formula proposed in line 1 and line 8 proposed to "then" use that formula. Line 23, by contrast, provided a single word with a question mark. There was no syntactic context (other than the question mark) within the line for interpreting that word and there was no reference to semantic context outside of the line. Line 23 did not respond in any clear way to a previous line and did not provide any alternative reference to a context in the original problem statement or elsewhere. For instance, Sup could have said, "I think we should compute the proportion of the height to the base of those equilateral triangles."

(b) The timing of line 23 was particularly unfortunate. It exactly overlapped a line from Avr. Because Avr had been setting the pace for group problem solving during this part of the chat, the fact that she was involved in following a different line of inquiry spelled doom for any alternative proposal around the time of line 23. Pin either seemed to be continuing on his own thread without acknowledging anyone else at this point, or else he was responding too late to previous postings. So, a part of the problem for Sup was that there was little sense of a coherent group process—and what sense there was did not include him. If he was acting as part of the group process, for instance posing a question in reaction to Pin and in parallel to Avr, he was not doing a good job of it and so his contribution was ignored in the group process. It is true that a possible advantage of text-based interaction like chat over face-to-face interaction is that there may be a broader time window for responding to previous contributions. In face-to-face conversation, turn-taking rules may define appropriate turns for response that expire in a fraction of a second as the conversation moves on. In computer-based chat, the turn-taking sequence is more open. However, even here if one is responding to a posting that is several lines away, it is important to make explicit somehow the post to which one is responding. Sup could have said, "I know another way to find the height: using proportions." His posting does not do anything like that; it relies purely upon sequential timing to establish its context, and that fails in this case.

(c) Sup's posting 23 came right after Pin's proposal bid 20: "draw the altitude." Avr had responded to this with 22 ("right"), but Sup seems to have ignored that. Pin's proposal had opened up work to be done, and both Avr and Pin responded after line 23 with contributions to this work. So, Sup's proposal bid came in the middle of an ongoing line of work without relating to it. In sequential terms, he made a bid for a proposal when it was not time to make a proposal. Sup's proposal bid was not positioned within the group effort to sustain a promising line of inquiry. It is like trying to take a conversational turn when there is not a pause that creates a turn-taking opportunity. Now, it is possible—especially in chat—to introduce a new proposal at any time. However, to do so effectively, one must make a special effort to bring the on-going work to a temporary halt and to present one's new proposal as an alternative. Simply saying "proportions?" will not do it. Sup could have said, for instance, "Instead of drawing the altitude, let's use proportions to find it."

(d) To get a proposal response to a proposal bid, one can elicit at least an affirmation or recognition. Again, this is a matter of pre-structuring a sustained interaction. Line 23 does not really solicit a response. For instance, Avr's question, 21: "how?" called for an answer—that was given by Pin in line 20, which actually appeared in the chat window just prior to the question and with the same time stamp. But Sup's posting does not call for a specific kind of answer. Even Sup's own previous proposal bid in line 10 ended with "right?"—requesting agreement or disagreement. Line 10 elicited a clear response from Avr, line 11 ("no") followed by an exchange explaining why Sup's proposal was not right.

(e) Other proposal bids in the excerpt are successful in contributing to sustaining the collaborative knowledge building or group problem solving in that they open up a realm of work to be done. One can look at Avr's successive proposal bids on lines 1, 3, 8 and 17 as laying out a work strategy. This elicits a proposal response from Sup trying to find values to substitute into the formula and from Pin trying to draw a graphical construction that will provide the values for the formula. Sup's proposal bid in line 23, however, neither calls for a response nor opens up a line of work. There is no request for a reaction from the rest of the group, and the proposal bid is simply ignored. Since no one responded to Sup, he could have continued by doing some work on the proposal himself. He could have come back and made the proposal more explicit, reformulated it more strongly, taken a first step in working on it, or posed a specific question related to it. But he did not—at least not until much later—and the matter was lost.

(f) Another serious hurdle for Sup was his status in the group at this time. In lines 10 through 16, Sup had made a contribution that was taken as an indication that he did not have a strong grasp of the math problem. He offered the lengths of the two given triangles as the base and height of a single triangle (line 10). Avr immediately and flatly stated that he was wrong (line 11) and then proceeded to explain why he was wrong (line 13). When he agreed (line 15), Avr summarily dismissed him (line 16) and went on to make a new proposal that implied his approach was simply wrong (lines 17 and 18). Then Pin, who had stayed out of the interchange, re-entered, claiming to know how to implement Avr's alternative proposal (lines19 and 20) and Avr confirmed that (line 22). Sup's legitimacy as a source of useful proposals had been totally destroyed at precisely the point just before he made his ineffective proposal bid. Less than two minutes later, Sup tries again to make a contribution, but realizes himself that what he says is wrong. His faulty contributions confirm repeatedly that he is a drag on the group effort. He makes several more unhelpful comments later and then drops out of the discourse for most of the remaining chat. Sustaining a math chat discourse involves work to maintain an ongoing social interaction as well as work to continue the math inquiry. Proposal bids and other postings are constrained along multiple dimensions of efforts to sustain the activity.

The weaknesses of line 23 as a proposal bid suggest (by contrast, exception, breach or breakdown) some characteristics for successful proposals:

- A clear semantic and syntactic structure,
- Careful timing within the sequence of postings,
- A firm interruption of any other flow of discussion,
- The elicitation of a response,
- The specification of work to be done and
- A history of helpful contributions.

In addition, there are other interaction characteristics and mathematical requirements. For instance, the level of mathematical background knowledge assumed in a proposal must be compatible with the expertise of the participants, and the computational methods must correspond with their training. Additional characteristics become visible in other examples of chats. Successful proposals contribute in multiple ways to sustaining the group cognitive process.

As we have just seen, the formulation of effective bids for math proposals involves carefully situating one's posting within the larger flow of the chat. This is highly analogous to taking a turn in face-to-face conversation (Sacks *et al.*, 1974). Where conversation analysis developed a systematics of turn taking, we are discovering the systematics of chat interaction. This describes how math proposals and other chat methods must be designed to fit into—and thereby contribute to—the sustained flow of group interaction.

So far in this Investigation, the notion of math-proposal adjacency pairs has been illustrated in just a single chat log excerpt. But in our research, we have seen both successful and failed math proposals many times. Other researchers have also noted the role of successful and failed proposals in collaborative problem solving (Barron, 2003; Cobb, 1995; Dillenbourg & Traum, 2006; Sfard & McClain, 2003).

Each proposal bid and uptake is unique—in its wording and its context. The interactional work that it does and the structuring that it employs are situated in the local details of its sequential timing and its subtle referencing of unique and irreproducible elements of the on-going chat. Each group of students develops somewhat different methods of engaging with math problems and making math proposals. Even within a given chat, each posting pair that might be a proposal must be analyzed as a unique, meaning-making interaction in order to determine if it is in fact a math-proposal adjacency pair. That is why case studies provide the necessary evidence. The essential details of interaction methods are lost in aggregation, in the attempt to overcome what Garfinkel (1967) terms the "irreducible indexicality" of the event. To the extent that identifying proposal pairs is a useful analytic approach, it is important to determine what interactional methods of producing such proposals are effective (or not) in fostering successful knowledge building and group cognition, as we have begun to do here.

An understanding of methods like proposal making can guide the design of activity structures for collaborative math. As we are collecting and analyzing a corpus of chat logs under different technological conditions, we are evolving the design of computer support through iterative trials and analyses.

Designing computer support

If the failure of Sup's proposal about proportions is considered deleterious to the collaborative knowledge building around the triangles problem, then what are the implications of this for the design of educational computer-based environments? One response would be to help students like Sup formulate stronger proposals. Presumably, giving him positive experiences of interacting with students like Avr and Pin, who are more skilled in chat proposal making, would provide Sup with models and examples from which he can learn—assuming that he perseveres and does not drop out of the chat.

Another approach to the problem would be to build functionality into the software and structures into the activity that scaffold the ability of weak proposal bids to survive. As students like Sup experience success with their proposals, they may become more aware of what it takes to make a strong proposal bid.

Professional mathematicians rely heavily upon inscription—the use of specialized notation, the inclusion of explicit statements of all deductive steps and the format of the formal proof to support the discussion of math proposals—whether posted on an informal whiteboard, scrawled across a university blackboard or published in an academic journal. Everything that is to be referenced in the discussion is labeled unambiguously. To avoid ellipsis, theorems are stated explicitly, with all conditions and dependencies named. The projection of what is to be proven is encapsulated in the form of the proof, which—at least since Euclid—starts with the givens and concludes with what is proven. Perhaps most importantly, proposals for how to proceed are listed in the proof itself as theorems, lemmas, etc. and are organized sequentially. (This view of proof is an idealization that abstracts from unstated tacit background knowledge of the mathematical community, as Livingston (1999) and Wittgenstein (1944/1956) before him have demonstrated.)

One could imagine a chat system supplemented with a window containing an informal list of proposals analogous to the steps of a proof. After Sup's proposal, the list might look like Figure 1. When Sup made a proposal in the chat, he would enter a statement of it in the proof window in logical sequence. He could cross out his own proposal when he felt it had been convincingly argued against by the group (see dashed lines in Figure 1 crossing out the proposal that base and height = 9 and 12).

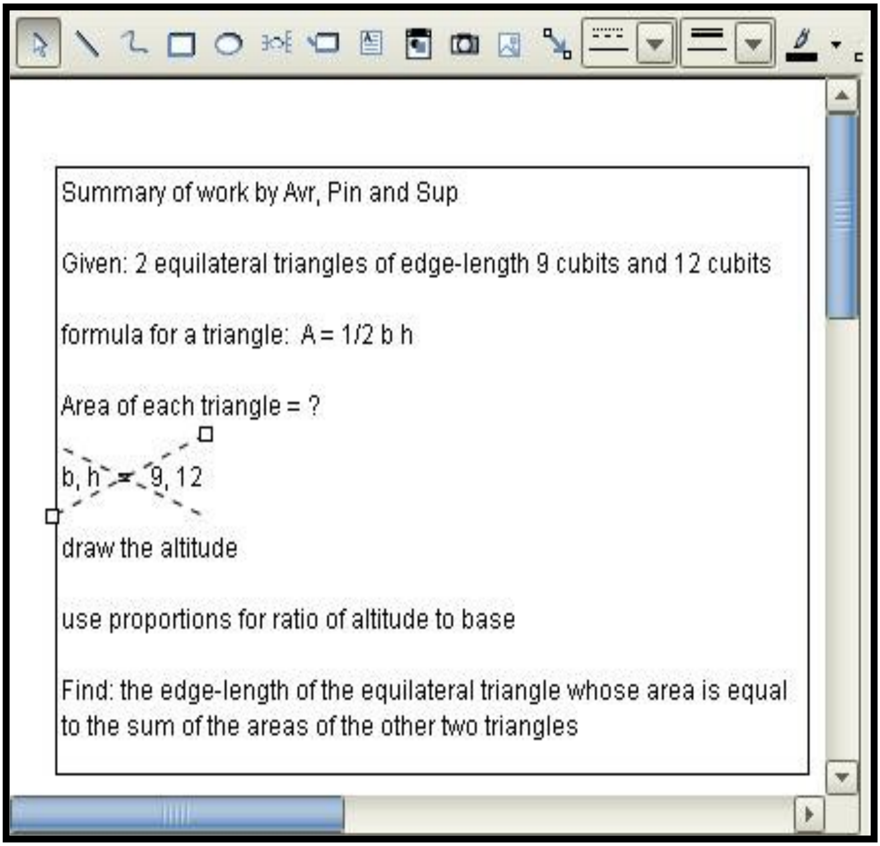

Figure 1. A software interface window with a list of proposals.

The idea is that important proposals that were made would be retained in a visible way and be shared by the group. Of course, there are many design questions and options for doing something like this. Above all, would students understand this functionality and how would they use it? The design sketch indicated in Figure 1 is only meant to be suggestive.

Another useful tool for group mathematics would be a shared drawing area. In the chat environment used by Sup, Pin and Avr, there was no shared drawing, but a student could create a drawing and email it to the others. Pin did this twelve minutes after the part of the interaction shown in the excerpt. Before the drawing was shared, much time was lost due to confusion about references to triangles and vertices. For math problems involving geometric figures, it is clearly important to be able to share drawings easily and quickly. Again, there are many design issues, such as how to keep track of who drew what, who is allowed to erase, how to point to items in the drawing and how to capture a record of the graphical interactions in coordination with the text chatting.

Because we are designing a computer-supported experience that has never before existed and because we want our design to be based on detailed study of how students actually create their collaborative experience in the environment we are designing, we follow a highly iterative try-analyze-redesign cycle of design-based research (Design-Based Research Collective, 2003), in order to gradually approach an effective computer-supported environment and math discourse community.

We started with a simple online service. We used AOL's IM commercial chat system that was already familiar to many students. We invited students into chat rooms and presented a problem from the Math Forum's well-established Problem of the Month service. An adult facilitator was present in the room to help with any technical problems. When we saw how necessary a shared whiteboard was, we tried an open-source solution and also WebCT's and Blackboard's interactive classrooms. Eventually we collaborated with researchers in Germany to use and further develop ConcertChat. Together, we have gradually evolved ConcertChat into a sophisticated environment for students, instrumented to capture student interaction for researchers.

Since the early AOL-based chat analyzed above, we have gone through many cycles of design, trial and analysis. In addition to designing wiki support for persistent summaries of work (such as that in Figure 3) and a shared whiteboard for constructing geometric drawings (discussed in the following section), we have incorporated the following: a referencing tool; a way for users to explicitly thread their chat postings; several forms of social awareness; tutorials on how to use the new features; a help system on using the tools, collaborating and problem-solving; and a lobby to support group formation. We have also experimented extensively with how best to formulate math problems or topics and how to provide feedback to students on their work.

References and threading

The more we study chat logs, the more we see how interwoven the postings are with each other and with the holistic Gestalt of the interactional context that they form. There are many ways in which a posting can reference elements of its context. The importance of indexicality to creating shared meaning was stressed by Garfinkel (1967). Vygotsky also noted the central role of pointing for mediating intersubjectivity in his analysis of the genesis of the infant-and-mother's pointing gesture (1930/1978, p. 56). Our analysis of face-to-face collaboration emphasized that spoken utterances in collaborative settings tend to be elliptical, indexical and projective ways of referencing previous utterances, the conversational context and anticipated responses (Stahl, 2006b, chapter 12).

Based on these practical and theoretical considerations—and working with the ConcertChat developers—we evolved the VMT-Chat environment. As shown in Figure 2, it not only includes a shared whiteboard, but has functionality for referencing areas of the whiteboard from chat postings and for referencing previous postings [Investigation 22]. The shared whiteboard is necessary for supporting most geometry problems. (This will save Avr the frustration of running out of paper, and also let Pin and Sup see what she is drawing and add to it or reference it.) Sharing drawings is not enough; students must be able to reference specific objects or areas in the drawing. (For example, Sup could have pointed to elements of the triangles that he felt to be significantly proportional.) The whiteboard also provides opportunities to post text where it will not scroll away. (Sup could have put his failed proposal in a text box in the whiteboard, where he or the others could come back to it later.) The graphical references (see the bold line from a selected posting to an area of the drawing in Figure 2) can also be used to reference one or more previous postings from a new posting in order to make the threads of responses clearer in the midst of "chat confusion" (Fuks, Pimentel, & de Lucena, 2006).

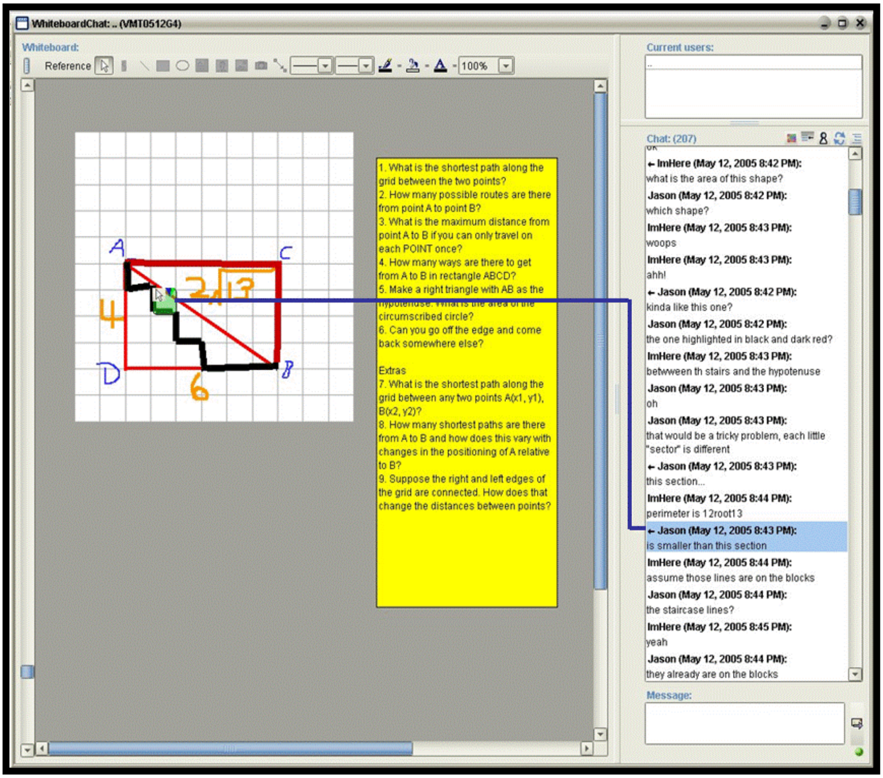

Figure 2. Screen view of VMT-Chat with referencing. Line 12 of the chat is selected.

In one of our first chats using VMT-Chat, the students engaged in a particularly complex interaction of referencing a figure in the whiteboard whose mathematics they wanted to explore [Investigation 22]. The chat log from Figure 2 (graphical references to the whiteboard are indicated by "[REF TO WB]" in the log) is listed in Log 3.

1	ImH:	what is the area of this shape? [REF TO WB]
2	Jas:	which shape?
3	ImH:	woops
4	ImH:	ahh!
5	Jas:	kinda like this one? [REF TO WB]
6	Jas:	the one highlighted in black and dark red?
7	ImH:	between th stairs and the hypotenuse
8	Jas:	oh
9	Jas:	that would be a tricky problem, each little "sector" is different
10	Jas:	this section [REF TO WB]
11	ImH:	perimeter is 12root3

12	Jas:	is smaller than this section [REF TO WB]
13	ImH:	assume those lines are on the blocks
14	Jas:	the staircase lines?
15	ImH:	yeah
16	Jas:	they already are on the blocks

Log 3. Chat log from Figure 2.

Line 1 of the chat in Log 3 textually references an abstract characteristic of a complex graphical form in the whiteboard: "the area of this shape." The software function to support this reference failed, presumably because the student, ImH, was not experienced in using it and did not cause the graphical reference line to point to anything in the drawing. Line 5 provides a demo of how to use the referencing tool. Using the tool's line, a definite textual reference ("this one") and the use of line color and thickness in the drawing, lines 5 and 6 propose an area to act as the topic of the chat. Line 7 makes explicit in text the definition of a sub-area of the proposed area. Line 8 accepts the new definition and line 9 starts to work on the problem concerning this area. Line 9 references the problem as "that" and notes that it is tricky because the area defined does not consist of standard forms whose area would be easy to compute and add up. It refers to the non-uniform sub-areas as little "sectors." Line 10 then uses the referencing tool to highlight (roughly) one of these little sectors or "sections." Line 12 continues line 10, but is interrupted in the chat log by line 11, a failed proposal bid by ImH. The chat excerpt continues to reference particular line segments using deictic pronouns and articles as well as a growing vocabulary of mathematical objects of concern: sectors, sections, lines, blocks.

Progress is made slowly in the collaborative exploration of mathematical relationships, but having a shared drawing helps considerably. The students use multiple textual and graphical means to reach a shared understanding of mathematical objects that they find interesting but hard to define. In this excerpt, we start to get a sense of the complex ways in which brief textual postings weave dense webs of relationships among each other and with other elements of the collaborative context.

This example shows how creating shared meaning can require more than a simple adjacency pair. In order to establish a reference to "this shape" that could allow the two participants to discuss that math object, the dyad had to construct a complex involving nested question/answer pairs, math proposal pairs, a failed proposal bid, drawing, coloring, labeling, pointing, multiple repairs, computations. Here we see a more sustained group-cognitive process. Across 16 postings and considerable coordinated whiteboard activity during two minutes, the student dyad defines a math object for investigation. The definition is articulated by this whole sequence of combined and intricately coordinated textual and graphical work.

Sustaining the group interaction

The goal of our research is to provide a service to students that will allow them to have a rewarding experience collaborating with their peers in online discussions of mathematics. We can never know exactly what kind of subjective experience they had, let alone predict how they will experience life under conditions that we design for them. For instance, it is methodologically illegitimate to ask if ImH already "intended" or "had in mind" in line 1 the shape that the group subsequently arrived at. We know from the log that ImH articulated much of the explicit description, but he only did this in response to Jas. If we interviewed ImH afterwards he might quite innocently and naturally project this explicit understanding back on his earlier state of mind as a retrospective account or rationalization (Suchman, 1987).

Our primary access to information related to the group experiences comes from chat logs (including the whiteboard history). The logs capture most of what student members see of their group on their computer screens. They therefore constitute a fairly complete record of everything that the participants themselves had available to understand their group interaction. We can even replay the logs so that we see how the session unfolded sequentially in time. Of course, we are not engaged in the interaction the way the participants were, and recorded experiences never quite live up to the live version because the engagement is missing. To gain some first-hand experience, we researchers do test out the environments ourselves and enjoy the experience, but we experience math and collaboration differently than do middle-school students. We also interview students and their teachers, but teenagers rarely reveal much of their life to adults.

So, we try to understand how collaborative experiences are structured as interpersonal interactions that are sustained over time. The focus is not on the individuals as subjective minds, but on the human, social group as constituted by the interactions that take place within the group. Although we generally try to ground our understanding of interaction through close, detailed analysis of excerpts from chat recordings, we do not have room to document our analysis of longer-scale structures at that level of detail here. During Spring Fest 2005, we collected over 50 hours of small-group chat about math. We engage in weekly collaborative data sessions (Jordan & Henderson, 1995) to develop case studies of unique chat excerpts. A number of published papers arising from these sessions are available. The discussion in the remainder of this Investigation is a high-level summary based on what we have observed.

Replies, up-take, pairs and triplets

Figure 3 provides a diagram of the responses of postings in the chat discussed above involving Avr, Pin and Sup (Log 1). The numbers of the posts by each participant are placed in chronological order in a column for that participant. Math-proposal adjacency pairs are connected with solid arrows and other kinds of responses are indicated with dashed arrows. Note that Sup's failed proposal bid (line 23) is isolated. Most of the chat, however, has coherence, flow or motion because most postings are responses to previous messages. This high level of responses is due to the fact that many postings elicit responses or up-take, the way that a greeting invariably calls forth another greeting in response, or a question typically elicits an answer. In a healthy conversation, most contributions by one participant are taken up by others. Conversationalists work hard to fit their offerings into the timing and evolving focus of the on-going interaction. In chat, the timing, rules and practices are different, but the importance of up-take remains [Investigation 4].

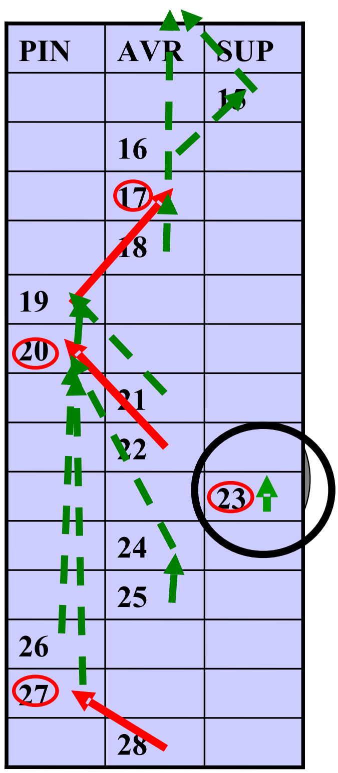

Figure 3. Threading of adjacency pairs and other uptake.

The fact that the group process and the cross ties between people are central to collaborative experiences does not contradict the continuing importance of the individuals. The representation of Figure 3 uses columns to indicate the connections and implicit continuity within the sequence of contributions made by an individual (compare the representation in Sfard & McClain, 2003). We may project psychological characteristics onto the unity of an individual's postings, attributing this unity to personal interests, personality, style, role, etc. Such attributions may change as the chat unfolds. The point is that the individual coherence and unfolding of each participant's contributions adds an important dimension of implicit sustaining connections among the postings.

Adjacency pairs like math proposals, greetings and questionings provide important ties that cut across the connections of individual continuities. They form the smallest elements of interaction precisely by binding together postings from different people. A proposal bid that is not taken up is not a meaningful proposal, but at best a failed attempt at a proposal. A one-sided greeting that is not recognized by the other is not an effective greeting. An interrogative expression that does not call for a response is no real questioning of another. These adjacency pairs are all interactional moves whose meaning consists in a give-and-take between two or more people. When we hear something that we recognize as a proposal bid, a greeting or a question, we feel required to attempt an appropriate response. We may ignore the proposal bid, snub the greeter or refuse to answer the question, but then our silence is taken as a response of ignoring, snubbing or refusing—and not simply a lack of response. The first part of an adjacency pair opens up a space of possibility for other group members to respond. The space if structured to allow certain kinds of utterances and not others. Some of the permitted responses are preferred and others are dis-preferred, requiring additional elaboration.

The way that a response is taken is an integral part of the interaction itself. In discussing the building of common ground, Clark argues that shared understanding by A and B of A's utterance involves not only B believing that he understands A, but also A agreeing that B understands (Clark & Brennan, 1991). This may require an interaction spanning multiple utterances. For instance, the most prevalent interaction in classroom discourse is when a teacher poses a question, a student provides an answer demonstrating understanding and then the teacher acknowledges the student response as such an understanding (Lemke, 1990). Here, the elemental cell of interactional meaning making is a sequence of contributions by different people.

It is clear in this analysis that the meaning is constructed through the interaction of multiple people, and is not a simple expression of pre-existing mental representations in any one individual's head. This is the philosophical importance of the concept of adjacency pair: that meaning in groups is made through the interaction of multiple people, not completely by an individual's mental activity. In calling this "group cognition," we extend the term "cognition" from individual psychology to apply to processes in which small groups through their discourse construct meaning structures like logical arguments or mathematical proofs—that is, they engage in processes which are considered thinking when conducted by individual people. This approach is consistent with dialogical theories that actually view higher-level thinking by individuals as derivative of such intersubjective meaning making (Bakhtin, 1986; Linell, 2001; Stahl, Koschmann, & Suthers, 2006; Vygotsky, 1930/1978; Wegerif, 2006).

Longer sequences

Although much attention has been given to adjacency pairs in conversation analysis and although such pairs can be thought of as the elements of meaning making in collaborative interaction, they form only one of many levels of analysis. For instance, there are *longer sequences* (Sacks, 1992, vol. II, p. 354), *episodes* (Linell, 2001) and *topics* in dialogs and chats (Zemel, Xhafa, & Cakir, 2005) that provide layers of structure and sense. An hour-long chat is not a homogeneous interchange. A typical math chat might start with a period of introductions, greetings, socializing. Then there could be some problem-solving work. This might be periodically interrupted by joking, playing around, or silliness. People may come and go, requiring catching up and group reorganization, or even bridging across sessions. Each of these episodes has boundaries during which the group members must negotiate whether or not to stop what they were doing and start something else. These transitions may themselves be longer sequences of interaction, especially in large groups. We have barely begun to explore these different layers.

In social conversation, people work hard to strike up conversations, to propose new topics of mutual interest and to keep the conversation going. Online math chats face similar challenges. Students hesitantly greet each other and get things started. Math proposals are often used to introduce new topics and to carry forward a train of thought together. Finally, participants engage in considerable interaction work to sustain their sessions, intertwining humor, flirting, socializing and math inquiry—often using one of these modes to sustain others. Eventually every group decides to disband, at least until a future session.

The above referencing excerpt from a VMT-Chat was from the second hour-long session in a series of four chats by the same group. The sessions referred back to previous sessions and prepared for future ones. We hope to foster a community of Math Forum users who come back repeatedly to math chats, potentially with their friends. Their chats will reference other chats and different online experiences, building connections at the community level. This adds more layers of interconnections. It may sustain group interaction, inquiry and reflection over more significant periods of time.

Constructing proofs

Learning math involves becoming skillful in the social practices of the math community (Livingston, 1999). The math community is an aspect of the world-historical global community. The most central participants are the great mathematicians, who have invented new mathematical objects and developed new forms of mathematical practice (Sfard & Linchevski, 1994). Most of the population has low math literacy and participates on the periphery of the math discourse community. They are unable to manipulate math concepts fluidly in words or mathematical symbolism (Sfard, 2002). Nevertheless, they can use basic arithmetic methods for practical purposes (Lave, 1988). One of the most fundamental methods of math is counting, which young children are drilled at extensively. Formal math assumes that the practitioner is skilled at following rules, such as the non-formalized rules of numeric sequencing or counting (Wittgenstein, 1944/1956).

In our chats, students work on math problems and themes. In solving problems and exploring math worlds or phenomena, the groups construct sequences of mathematical reasoning that are related to proving. Proofs in mathematics have an interesting and subtle structure. To understand this structure, one must distinguish:

- The problem statement-and-situation;
- The exploratory search for a solution;
- The effort to reduce a haphazard solution path to an elegant, formalized proof;
- The statement of the proof; and
- The lived experience of following the proof (Livingston, 1986, 1987).

Each of these has its own structures and practices. Each implicitly references the others. To engage in mathematics is to become ensnarled in the intricate connections among them. To the extent that these aspects of doing math have been distinguished and theorized, it has been done as though there is simply an individual mathematician at work. There has been virtually no research into how these could be accomplished and experienced collaboratively—despite the fact that talking about math has for some time been seen as a priority in math education (NCTM, 1989; Sfard, 2002).

The stream of group consciousness

Psychologists like William James and novelists like Jack Kerouac have described narratives that we tell ourselves silently about what we are doing or observing as our stream of consciousness. This "inner voice" rattles on even as we sleep, making connections that Sigmund Freud found significant (if somewhat shocking in his day). In what sense might online chats—with their meanderings, flaming, associative referencing, unpredictable meaning making and unexpected images—deserve equal status as streams of (group) consciousness? Group cognition can be self-conscious: The group discourse can talk about the existence of the group discourse itself and comment on its own characteristics.

Our sense of sustained time and the rhythms of life are largely reliant upon the narratives we tell ourselves (Bruner, 1990; Sarmiento, Trausan-Matu, & Stahl, 2005). We know that we have already lived through a certain part of the day or of our life because our present is located within a nexus of ties to the past or hopes for the future. In similar ways, a chat's web of references that connects current postings to prior ones to which they respond and to future postings that they elicit defines a temporality of the chat. This is experienced as a lived sense of time that is shared by the group in the chat. Like our individual internal clocks, the group temporality is attuned to the larger world outside—the world of family life that calls the students away from the chat for dinner or the world of school that interrupts a chat with class changes or homework pressures. The temporality that defines a dimension of the collaborative experience is constrained by the nature of the social situation and by the functionality of the technological environment.

Constructing the group experience

Groups constitute themselves (Garfinkel, 2006, pp. 189ff; Sacks, 1992, vol. I, pp. 144-149). We can see how they do this in the chat logs. At one level, the VMT service brings several students together and locates them in a chat room together. It may supply a math problem for them to work on and it may provide a facilitator who introduces them to the environment. At this point, they are a potential group with a provisionally defined membership. The facilitator might say something like, "Welcome to our first session of Virtual Math Teams! I am the facilitator for your session. . .. As a group, decide which question you would like to work on." (This is, in fact, part of the facilitator script from the session involving ImH and Jas excerpted above.) Here we can see that the facilitator has defined the group ("as a **group** … **you**") and distinguished her own role as outside the group ("**I** am the facilitator … **your** session"). The potential group projected by the facilitator need not necessarily materialize.

Individual students may come to the setting, look around, decide it is lame and leave as individuals. However, this rarely happens. Sometimes an individual will leave without ever interacting, but as long as enough students come there, a group emerges.

Students enter the chat environment with certain motivations, expectations and experiences. These are generally sufficient to get the group started. One can see the group form itself. This is often reflected in the shift from singular to plural pronouns: "**Let's** get started. Let **us** do some math." We saw this in Avr's proposal: "**I** think **we** have to figure out the height by **our**selves." The proposal bid comes from an individual, but the projected work is for the group. Through her use of "we," Avr constitutes the group. Through her proposal bid, she constitutes the group as a recipient of the bid and elicits a response from them. Someone other than Avr must respond to the bid on behalf of the group. When Pin says, "I know how: draw the altitude," he is accepting Avr's proposal as a task for the group to work on and in so doing he makes a proposal about how the group should go about approaching this task (by making a geometric construction). In this interchange, the group (a) is projected as an agent ("we") in the math work (Lerner, 1993), and (b) is actually the agent of meaning making because the meaning of Avr's proposal is defined by the interaction within the group (e.g., by a math-proposal adjacency pair).

If the group experience is a positive one for the participants, they may want to return. Some chats end with people making plans to get together again. In some experiments, the same groups attended multiple sessions. We would like to see a community of users form, with teams re-forming repeatedly and with old-timers helping new groups to form and learn how to collaborate effectively.

The recognition that collaborative groups constitute themselves interactionally and that their sense making takes place at the group unit of analysis has implications for the design of cognitive tools for collaborative communities. The field of computer-supported collaborative learning (CSCL) was founded in the 1990s to pursue the analysis of group meaning making and the design of media to support it (Stahl, Koschmann *et al.*, 2006). We view the research described here as a contribution to this CSCL tradition.

We are designers of tools for collaborative groups. We want to design an online collaborative service, with strong pedagogical direction and effective computer support. Our goal is to design an environment that fosters exciting mathematical group experiences for students and inspires them to return repeatedly. Our ultimate vision is to foster a sustainable community of math discourse among students. We approach this by trying to understand how groups of students construct their experience in such settings.

When students enter our website now, they are confronted by a densely designed environment. The lobby to our chat rooms is configured to help students find their way to a room that will meet their needs. In the room, there is a daunting array of software functionality for posting and displaying chat notes, drawing geometric forms and annotating them, keeping track of who is doing what and configuring the space to suit oneself. There may be a statement of a math problem to solve or an imaginary world to explore mathematically. The service, problems and software are all designed to enhance the user's experience. But how can a student who is new to all this understand the meanings of the many features and affordances that have been built into the environment?

Groups of students spontaneously develop methods for exploring and responding to their environments. They try things out and discuss what happens. A new group may doodle on the whiteboard and then joke about the results. They bring with them knowledge of paint and draw programs and skills from video games, SMS and IM. The individuals may have considerable experience with single-user apps, but react when someone else erases their drawing; they must learn to integrate coordination and communication into their actions. The math problems they find in the chat rooms may be quite different from the drill-and-practice problems they are used to in traditional math textbooks and commercial "educational" software. It may take the group a while to get started in productive problem solving, so the group has to find ways to keep itself together and interacting in the meantime. There may be various forms of socializing, interspersed with attempts to approach the math. As unaccustomed as the math may be, the students always have some knowledge and experience that they can bring to bear. They may apply numerical computations to given values; try to define unknowns and set up equations; graph relationships; put successive cases in a table; use trigonometric relationships or geometric figures; draw graphical representations or add lines to an existing drawing. Mainly, they put proposals out in the chat stream and respond to the proposals of others. Sometimes the flow of ideas wanders without strong mathematical reflection. Other times, one individual can contribute substantial progress and engage in expository narrative to share her contribution with the group (Stahl, 2006a).

Groupware is never used the way its designers anticipated. The designers of VMT-Chat thought that its referencing tools would immediately clarify references to elements of drawings and transform chat confusion into logical threaded chat. But our studies of the actual use of these designed functions tell a quite different and more interesting story. The shared whiteboard with graphical references from the chat may allow more complex issues to be discussed, but they do not make pointing problem-free. We saw in a previous section how much work ImH and Jas engaged in to clarify for each other what they wanted to focus on. In the excerpt and in the longer chat, they used a variety of textual, drawing and referencing methods. Through this process, they learned how to use these methods and they taught each other their use. Within a matter of a fraction of a minute, they were able to reach a shared understanding of a topic to work on mathematically. During that brief time, they used dozens of deictic methods, some that would prove more useful than others for the future.

Chat is a highly constrained medium. Participants feel various pressures to get their individual points of view out there. In a system like VMT-Chat, there is a lot to keep track of: new postings, changes to the whiteboard, signs that people are joining, leaving, typing, drawing. Small details in how something is written, drawn or referenced may have manifold implications through references to present, past or future circumstances. Students learn to track these details; apply them creatively; acknowledge to the group that they have been recognized; check, critique and repair them. Each group responds to the environment in its own way, giving group meaning to the features of the collaborative world and thereby putting their unique stamp on their group experience.

In the process, they create a group experience that they share. This experience is held together with myriad sorts of references and ties among the chat postings and drawings. Often, what is not said is as significant as what is. Individual postings are fragmentary, wildly ambiguous, and frequently confusing. In lively chats, much of what happens remains confusing for most participants. Clarity comes only through explicit reflections, up-takes, appreciations or probing. The interactions among postings, at many levels, cohere into a stream of group consciousness, a flow of collaboration, a shared lived temporality and, with luck, an experience of mathematical group cognition.

The small groups who meet in the VMT-Chat rooms participate in the larger collaborative communities of: the VMT project, the Math Forum user community and the math discourse community at large. In general, interacting small groups mediate between their individual members and the larger communities to which they belong. The discourse within the small group evokes and collects texts, drawings and actions by different participants, who bring multiple interpretive perspectives to the shared meaning making. Enduring ambiguity, mutual inconsistency and downright contradiction pervade the resultant group cognition, with its “inter-animation of perspectives” (Bakhtin, 1986; Wegerif, 2006). Whether or not we assume that an individual’s thoughts are logically consistent and interpretively determinant, it seems that much of a group chat generally remains a mystery to both participants and researchers. Yet, from out of the shrouds of collective fog, insights are co-constructed that could not otherwise shine forth. The tension arising from conflicting or ungraspable interpretations in place of harmonious shared meaning fuels the creative work of constructing innovative group understanding.

The chat environment as incorporated in the VMT project is essentially different from familiar conversational situations, as we have seen in this paper. In general, there is little known by the participants about each other, except for what appears in the chat text or whiteboard drawings. No one's age, gender, appearance, accent, ethnicity is known. Even people's real names are replaced in the chat with anonymous login handles. Participants do not observe each other typing and correcting text until it is posted. Nor do they see what people are doing or saying in their lives outside the chat—if they have gone for a snack, are talking on the phone or are engaged in other, simultaneous online interactions. Normally, a person's history, culture and personality are conveyed through their vocal intonation and physical appearance (Bourdieu, 1972/1995); these are absent in chat. The one-hour duration of most VMT chats limits the history that can be established among participants through the available outlets of text and drawing, interaction style, word choice and use of punctuation. Yet, these drastically restricted means somehow allow incredibly rich, unique, creative and sophisticated interactions to take place. Insights take place and are shared; meaning is constructed and made sense of by groups. Perspectives and personal voices are established and acknowledged. Like characters in a Beckett play, chat participants learn to survive using radically impoverished discourse within a sensuously desolate landscape, and they sustain surprising forms of interaction for about an hour.

Conclusion

As we have seen in this paper, when students enter into one of our chats they enter into a complex social world. They typically quickly constitute a working group and begin to engage in activities that configure a group experience. This experience is conditioned by a social, cultural, technological and pedagogical environment that has been designed for them. Within this environment, they adopt, adapt and create methods of social practice for interacting together with the other students who they find in the chat environment. Over time, they explore their situation together, create shared meaning, decide what they will do and how they will behave, engage in some form of mathematical discourse, socialize, and eventually decide to end their session.

Then our job as researchers begins: to analyze what has happened and how the software tools we are designing condition, support and mediate the collaborative experiences that groups construct and sustain. We face the same poverty of knowledge about our subjects that the participants themselves face about each other. But, here too, less can be more. This record is conducive to careful, detailed analysis, without the interpretive complexities of video recording and transcription. We can analyze what happens at the group unit of analysis, with the methods of interaction adopted by the participants, because everything that could have gone into the shared understanding of the participants is available in the persistent record of the chat-room history.

We can study this record at our leisure and make explicit the influences that the group experienced tacitly in the flow of its life. We can observe how several students constitute and sustain their group cognition in the math chat environment we are designing with them. The group cognition persists in its record, indefinitely available for analysis.

We can identify successful and failed math proposals, questions, greetings and other low-level interactions. We can observe how groups construct, identify, make sense of and explore mathematical objects. But we can also see how these elementary interactions build up longer sequences of group cognition (Stahl, 2006b) [Investigation 22], intersubjective meaning making (Suthers, 2006) [Investigation 4] and sustained collaborative group experiences.

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Investigation 24. Viewing Learning and Thinking in Groups

Gerry Stahl

Abstract. This is an invited keynote talk that opened the International Conference on Computers and Education (ICCE 2009) on November 30, 2009, in Hong Kong, China. The intent of the talk was to provide a personal view of the field of computer-supported collaborative learning (CSCL) and to relate it to the Asia-Pacific audience. To do this, I tried to describe—in an informal tone—the approach I was taking to analyzing online interaction in small groups. In publishing the talk, I have tried to retain its original tone.

The field of CSCL is particularly interested in the ways small groups can build knowledge together thanks to communication and support from networking technology. I hope that CSCL environments can be designed that make possible and encourage groups to think and learn collaboratively.

In my research, my colleagues and I look at logs of student groups chatting and drawing about mathematics in order to see how they build on each other's utterances to achieve more than they would individually. To answer this important question, we must look carefully at the details of discourse in CSCL groups and develop innovative tools and theories. In this Investigation, I outline methods and levels of analysis that have resulted in the findings reported in the Virtual Math Teams research cited in the references.

Keywords. Event, session, theme, discourse move, adjacency pair, utterance, reference.

"Learning without thought is labor lost;
thought without learning is perilous."
-- Confucius 孔丘 Kong Qiu

Views of Learning and Thinking

About 2,600 years ago, Confucius viewed *learning* and *thinking* as belonging together.

The learning sciences of the twenty-first century agree with Confucius on this point. They view learning as involving meaning making by the learners (Stahl, Koschmann & Suthers, 2006). Students who just passively accept instruction without thinking about it and coming to understand it in their own way of making sense of things will be wasting everyone's time. Why? Because they will not be able to *use* the new knowledge or to *explain* it. Of course, this construction of meaning takes place over time: someone can learn something one day and make sense of it later, when they try to use it in different circumstances and to explain their use to other people and to themselves. But if they never integrate what they have learned into their own thinking and acting—by applying it where appropriate and talking about it clearly—then they will not have really learned anything important.

What sociologists of small groups like Bernstein, as presented in Hasan's overview (1999), know about social interactions and contribute to our understanding of the significance of group cognition is the way participants internalize the resources that evolve within one interactional context and then recontextualize them in the new and radically different contexts they find themselves in later. In this way, the new knowledge that is created, or the new or enhanced knowledge-building skills that are appropriated, can replicate and spread contagiously. It is the magic that, for instance, makes seemingly inconsequential interactions between mothers and children while cleaning the oven play a key role in a child's preparation for schooling (Cloran, 1999). It is precisely because of the tremendous impact the results of these interactions can have going forward that the local sacrifice that may occur in terms of efficiency of the interaction can be viewed as a small price to pay when one considers the long-term cost-benefit ratio, the profound impact of one transformational experience of group cognition.

Vygotsky *(1930/1978)* made an even stronger argument. He showed for the major forms of human psychological functioning that these individual capabilities were derived from experiences of interactions between people:

> *An inter-personal process is transformed into an intra-personal one*. Every function in the child's cultural development appears twice: first, on the social level and later, on the individual level; first *between* people (*inter-psychological*), and then *inside* the child. This applies equally to voluntary attention, to logical memory, and to the formation of concepts.

> All the higher functions originate as actual relations between human individuals. (p. 57)

Although all functions of individual cognition are derived from group cognition, the reverse is not true. As Hutchins (1996) demonstrated with his example of the bridge of a large Navy ship, not all group cognition can be internalized by an individual: "The distribution of knowledge described [in the book] is a property of the navigation team, and there are processes that are enabled by that distribution that can never be internalized by a single individual" (p. 284). Whether or not specific skills and knowledge can be mastered by individuals or only by teams, the learning of those skills or knowledge seems to rely heavily and essentially on group cognition. That is why we try to study and to promote group cognition.

What we, as learning scientists, have learned about learning and thinking in recent decades in the West is influenced by what philosophers before us said. For instance, most Western philosophers until the middle of the 1900s thought that knowledge could be expressed by propositions, sentences or explicit statements. If that were true, then the learning of knowledge could, indeed, consist simply of students individually hearing or reading the right sentences and remembering them.

However, Ludwig Wittgenstein's book, *Philosophical Investigations*, published in 1953, questioned this view of learning and thinking. It looked at math as a prime example. Mathematical knowledge can be seen as a set of procedures, algorithms or rules. Wittgenstein asked how one can learn to follow a mathematical rule (Wittgenstein, 1944/1956, Part VI; 1953, §185-243, esp. §201). For instance, if someone shows you how to count by fours by saying, "4, 8, 12, 16," how do you know how to go on? Is there a rule for applying the rule of counting by fours? (Such as, "Take the last number and add 4 to it.") And if so, how do you learn to apply *that* rule? By another rule? Eventually, you need to know how to do something that is not based on following a propositional rule—like counting and naming numbers and recognizing which numbers are larger. The use of explicit rules must be somehow grounded in other kinds of knowledge. Wittgenstein's question brought the logical view of knowledge as explicit propositions into a paradox: if knowledge involves knowing rules, then it must involve knowing how to use rules, which is itself *not* a rule. These other kinds of knowing include the tacit knowledge of how to behave as a human being in our culture: how to speak, count, ask questions, generalize, put different ideas together, apply knowledge from one situation in another context and so on. *And these are the kinds of things that one initially learns socially, in small groups or in child-parent dyads.* In the theory of group cognition, we identify many of these non-propositional ways of knowing as practices. We study how groups adopt group practices, which may become personal habits or skills of the group participants [Investigation 16].

Wittgenstein was an unusual philosopher because he said that problems like this one could not be solved by contemplation, but rather by looking at how people actually do things. He said, “Don’t think, look!” (1953, §66). In studying group cognition, I try to follow Wittgenstein’s advice. I try to view how people actually *do* things. Rather than telling you what my *views* or ideas are about learning and thinking in CSCL groups, I want to *show* you how I *view* or observe learning and thinking in CSCL groups.

The term “view” in the title of my talk has this double meaning: it means both viewing by looking at something with my eyes and also viewing in the metaphorical sense of thinking about something from a conceptual perspective. The Greek philosopher, Plato, who lived at about the same time as Confucius, made this metaphor popular in Western thought (Plato, 340 BCE/1941).

Although Wittgenstein himself did not actually look at empirical examples of how people follow rules in math, we can. By carefully setting up a CSCL session, we can produce data that allows us to view small groups of students learning how to follow math rules and thinking about the math rules. This is what I do to view learning and thinking in CSCL groups. It is the basic approach of the science of group cognition (Stahl, 2009) that I want to describe today.

The work of our research team and other colleagues involves looking closely at some rich examples of student groups learning and thinking about math. I would like to share a brief excerpt from one of these examples with you and talk about how we go about viewing the learning and thinking of this group of students. In particular, how do they construct their group cognition through collaborative meaning-making activities?

In this Investigation, we will look at the meaning-making work of a group of students, analyzing their language-based interaction at multiple levels:

- The overall *event*,
- A specific hour-long *session* of the two-week event,
- A discussion *theme* that arose in the session,
- A discourse *move* that triggered that theme,
- A pivotal *interchange* that carried out the discourse move,
- A single *utterance* that was part of the interchange and
- A particular *reference* in the utterance.

By looking at the linguistic connections, we can see how the syntax, semantics and pragmatics weave a network of meaningful references that accomplishes a set of cognitive achievements by the group interaction.

On the one hand, we can see the linguistic elements of a log of discourse and their structure of temporal and hierarchical relationships as accomplishing group cognition by, at each moment, constraining the next utterance as situated in the context of event, session, theme, discourse moves, eliciting adjacency pairs, preceding utterances and network of references. On the other hand, human actors creatively design accountable responses within the constraining situation defined by these contextual elements. That is, among the constraints on the actors is the requirement that their linguistic actions make sense in the on-going discourse and that they reveal their meaning and relevance in their linguistic design.

Although people often design their utterances to convey the impression that they are the result of psychological processes (change of mental state, expression of internal reflections), we can analyze the group cognition in terms of the linguistic effects of the observable words and drawing actions, without making any assumptions about individual mental representations. The individual students are active as linguistic processors—interpreting and designing the utterances—but the larger mathematical and cognitive accomplishments are achieved through the group discourse, which exists in the computer displays, observable by the students and—even years later—observable in the logs by analysts. We can see and make explicit how teams become teams in the ways that they manifest the contingencies and accountabilities of their unique situation, using conventional linguistic structures as resources.

The work of the Virtual Math Teams (VMT) research team—which I directed from 2003-2014—and collaborating researchers involves looking closely at some rich examples of student groups learning and thinking about math. I would like to share a brief excerpt from one of these examples with you and talk about how we go about viewing the learning and thinking in this group of students.

An Example of Learning and Thinking

I will now describe an illustrative event, session, theme, move, interchange, utterance and reference from the work of a VMT group of students.

The event: VMT Spring Fest 2006 Team B

Here, we will be talking about an online event that occurred several years ago. The interaction is preserved in a computer log, which can be replayed by researchers and is partially presented in this Investigation. Three students, probably about 16 years old, were assigned to form Team B, and they met with a facilitator in an online chat environment on May 9, 10, 16 and 18, in 2006, for about an hour in the late afternoon each day. The participants were distributed across three time zones in the US. The event was part of the VMT research project. Neither the students nor we know anything more about each other's personal characteristics or background.

The topic for this event was to explore a pattern of sticks forming a stair-step arrangement of squares (see Figure 1) and then to explore similar patterns chosen by the students themselves. (A subsequent session by the same students is discussed in Investigation 23.)

Session I

1. Draw the pattern for N=4, N=5, and N=6 in the whiteboard. Discuss as a group: How does the graphic pattern grow?

2. Fill in the cells of the table for sticks and squares in rows N=4, N=5, and N=6. Once you agree on these results, post them on the VMT Wiki

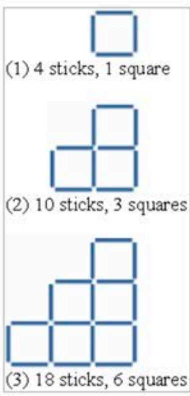
3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

Sessions II and III

1. Discuss the feedback that you received about your previous session.

2. WHAT IF? Mathematicians do not just solve other people's problems—they also explore little worlds of patterns that they define and find interesting. Think about other mathematical problems related to the problem with the sticks. For instance, consider other arrangements of squares in addition to the triangle arrangement (diamond, cross, etc.). What if instead of squares you use other polygons like triangles, hexagons, etc.? Which polygons work well for building patterns like this? How about 3-D figures, like cubes with edges, sides and cubes? What are the different methods (induction, series, recursion, graphing, tables, etc.) you can use to analyze these different patterns?

3. Go to the VMT Wiki and share the most interesting math problems that your group chose to work on.



N	Sticks	Squares
1	4	1
2	10	3
3	18	6
4	?	?
5	?	?
6	?	?
...	...	...
N	?	?

Figure 1. Topic for VMT Spring Fest 2006.

The VMT online environment consisted primarily of a synchronous chat window and a shared whiteboard. At the end of each session, the students were supposed to post their findings on a wiki, shared with other teams participating in the Spring Fest. Between sessions, the facilitator posted feedback to the students in a textbox on the whiteboard.

The session: Session 3, May 16, 7:00-8:00 pm

Let's look at an excerpt from the end of the third session. The three students had already solved the original problem of the stair-step pattern of squares. They had also made up their own problem involving three-dimensional pyramids. Now they turned to look at the problem that Team C had described on the shared VMT wiki after session 2. Team B is looking at an algebraic expression that the other team of students had derived for a diamond pattern of squares. They start to draw the pattern in their whiteboard (see Figure 2) and chat as a team about the problem of this new pattern.

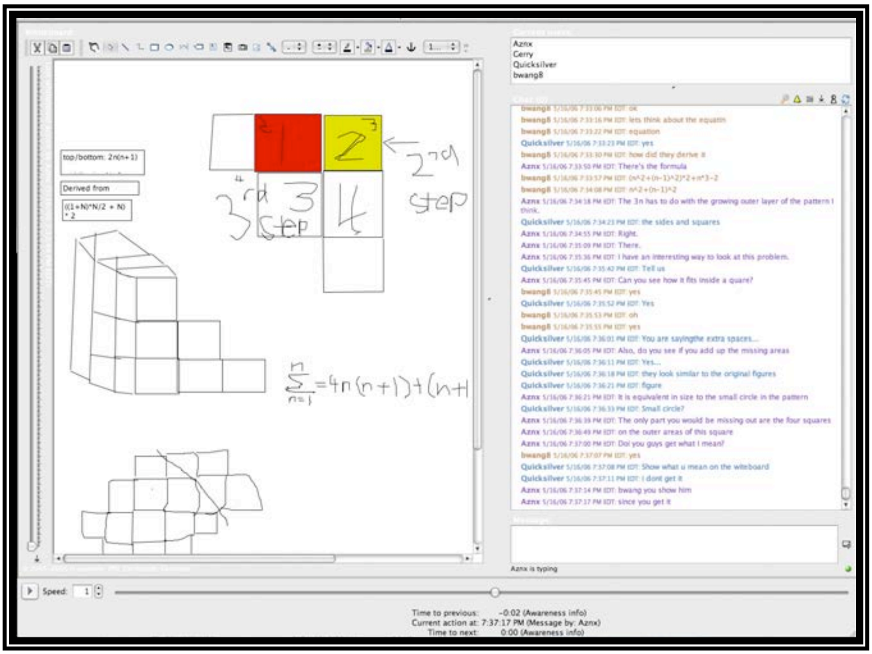


Figure 2. The VMT Replayer showing the VMT online environment.

The theme: "I have an interesting way to look at this problem"

One of the students, Aznx, begins to make a proposal on how to "look" at their problem. First, he announces, "I have an interesting way to look at this problem." Note that he uses the word "look" in the same double meaning of "view" that was mentioned above. As we will see, he means he has a new way to think about the problem mathematically—and that involves a way of observing a visual image of the problem. The group does its thinking both by typing text or algebraic expressions in the chat window and by simultaneously drawing and viewing diagrams or geometric constructions of the problem in the shared whiteboard (see Çakir, Zemel & Stahl, 2009 [Investigation 12], for an analysis of the coordination by the group of their text, symbols and drawings).

Aznx' announcement opens an opportunity for the group to discuss a way of looking at the problem. In fact, the group takes up the offer that is implicit in Aznx' statement and the students spend the next eight minutes trying to each understand it. As it turns out, they will work on this view of the problem for the rest of this session and most of their final session.

A VMT chat session can generally be analyzed as a series of themes or discussion topics. Often, themes come and go, and different themes overlap, with one wrapping up while another gets started. Researchers can identify the boundaries of a theme: when a new theme opens and an old one closes (Zemel, Xhafa & Çakir, 2009).

In this case, the group has been talking about how the diamond pattern grows as a geometric figure for a couple of minutes and then they discuss Team C's algebraic expression for a couple of minutes. As those themes get played out and there is a pause in the chat, Aznx makes a move to open a new theme for the group.

A move: Showing how to view the problem

Aznx' announcement that he has a perspective to share with the group is a way of introducing a new theme, a "pre-announcement" (Schegloff, 2007, pp. 37-44; Terasaki, 2004). Conversations often flow by new contributions picking up on something that was already being discussed. Online text chat tends to be more open than face-to-face talking; chat does not follow the strict turn-taking rules of conversation. However, it is still common to do some extra work to change themes even in chat. In a sense, Aznx is asking permission from the group to start a new theme. Quicksilver responds encouragingly right away by saying, "Tell us" (see Figure 3).

line	date	start	post	delay		
919	5/16/06	19:35:26	19:35:36	0:00:06	Aznx	I have an interesting way to look at this problem.
920	5/16/06	19:35:41	19:35:42	0:00:03	Quicksilver	Tell us
921	5/16/06	19:35:38	19:35:45	0:00:00	Aznx	Can you see how it fits inside a quare?
922	5/16/06	19:35:45	19:35:45	0:00:07	Bwang	yes
	5/16/06	19:35:49	19:35:52	0:00:00	Bwang	[user erased message]
923	5/16/06	19:35:51	19:35:52	0:00:01	Quicksilver	Yes
924	5/16/06	19:35:52	19:35:53	0:00:02	Bwang	oh
925	5/16/06	19:35:55	19:35:55	0:00:06	Bwang	yes
926	5/16/06	19:35:53	19:36:01	0:00:04	Quicksilver	You are sayingthe extra spaces...
927	5/16/06	19:35:58	19:36:05	0:00:06	Aznx	Also, do you see if you add up the missing areas

Figure 3. The move to introduce Aznx' new way of looking at the group's problem. (This log for analysis encodes the chat stream and associated awareness messages about when people started typing, along with timing data to reflect the flow of discourse.)

Actually, Aznx already starts typing his proposal before he gets Quicksilver's response, but it is not posted until afterward. The next step in his proposal is: "Can you see how it fits inside a square?" Here, he structures his contribution as a question, which elicits a response from the other members of the team. Note that he uses the term "see" in his proposal with the same double meaning as the term "look" in his prior announcement. As we shall see (in both senses), the group tries to work out and comprehend Aznx' proposal both conceptually and visually.

Both Bwang and Quicksilver respond to Aznx's proposal with "Yes." However, both modify this response. Bwang starts to type something else, but erases it; then he posts two messages: "oh" and "yes." This suggests some hesitation in responding to the proposal immediately. Quicksilver follows his initial positive response with, "You are saying the extra spaces ..." He is asking for more clarification of the proposal. While Quicksilver is typing his request for clarification, Aznx is typing an expansion of his initial proposal: "Also, do you see if you add up the missing areas ..."

The analysis of interaction moves is central to the science of group cognition. This is the level of granularity of many typical group-cognitive actions. Discourse moves are ways in which small online groups get their work done. They often follow conventional patterns—speech genres (Bakhtin, 1986) or member methods (Garfinkel, 1967)—which makes them much easier for participants to understand. Researchers can also look for these patterns to help them understand what the group is doing.

In this case, a new theme is being opened, one that will provide direction for the rest of this group's event together. This move is an example of one way in which a group can establish a shared understanding of a diagram or select a joint problem conceptualization (depending on how we take the terms "look" and "see"). Other moves that we often see in VMT logs are, for instance, defining shared references, coordinating problem-solving efforts, planning, deducing, designing, describing, solving, explaining, defining, generalizing, representing, remembering and reflecting as a group.

An interaction: Question/response: "Can you see how it fits inside a square?" / "Yes"

Interactions involve two or more people responding to each other. In conversation analysis, one typically looks for well defined "adjacency pairs" (Duranti, 1998; Sacks, 1965/1995; Schegloff, 2007) as forms that interactions often take. A prototypical adjacency pair is question/answer. Aznx' offering of a question—"Can you see how it fits inside a square?"—followed by Bwang and Quicksilver's responses—"yes," "Yes"—illustrate this structure for the simplest ("preferred") case: one person poses a yes/no question and the others respond with an affirmative answer.

Response structures of interactions are often more complicated than this. Text chat differs from talk in that people can be typing comments at the same time; they do not have to take turns and wait until one person stops talking and relinquishes the floor. They will not miss what the other person is saying, because unlike with talk, the message remains observable for a while. The disadvantage is that one does not observe how people put together their messages, with pauses, restarts, corrections, visual cues, intonations and personal characteristics. While it is possible to wait when you see that someone else is typing a message,[8] people often

[8] The VMT environment displays an awareness message under the chat tab when someone is typing. The content being typed is not displayed to other participants until the message is posted to the chat. See Figure 2 or 5.

type simultaneously, so that the two normal parts of an adjacency pair may be separated by unrelated postings. For example, Quicksilver's question (line 926 in Figure 3) separated Aznx's continuation of his line 921 posting in line 927, because 926 appeared before 927 although 927 was typed without seeing 926. So, in chat we might call these "response pairs" rather than "adjacency pairs." While they may be less sequentially *adjacent* than in talk, they are still direct *responses* of one posting to another.

Because the sequencing in online chat texting is less tightly controlled than in face-to-face talk, response pairs are likely to become entangled in the longer sequences of group moves. This may result in the common problem of "chat confusion" (Fuks, Pimentel & Pereira de Lucena, 2006; Herring, 1999). It can also complicate the job of the researcher. In particular, it makes the task of automated analysis more complicated. In convoluted chat logs, it is essential to work out the response structure (threading) before trying to determine the meaning making. The meaning making still involves participants interacting through the construction of response pairs, but in chat people have to recreate the ties among these pairs. Realizing this, the group members design their postings to be read in ways that make the response pair or threading structure apparent, as we will see (for more discussion of this, see Zemel & Çakir, 2009).

An utterance: Question: "Can you see how it fits inside a square?"

In his posting— "Can you see how it fits inside a square?"—Aznx is comparing the relatively complicated diamond shape to a simple square. This is a nice strategy for solving the group's problem. The group can easily compute the number of stick squares that fill a large square area. For instance, if there are five little squares across the width of a square area (and therefore five along the height), then there will be five-squared, or 25 little squares in the area. In general, if there are N little squares across the width, there will be N-squared to fill the area. This is a strategy of reducing the problem to a simple or already known situation—and then perhaps having to account for some differences. So Aznx' posting seems to be relevant to thinking about the math problem conceptually.

At the same time, Aznx poses his proposal in visual or graphical terms as one of "seeing" how one shape "fits inside" of the other. The group has been looking at diagrams of squares in different patterns, both a drawing by Team C in their wiki posting and Team B's own drawings in their whiteboard. So Aznx's proposal suggests visualizing a possible modification to one of the diamond drawings, enclosing it in a square figure (see the blue diamond pattern enclosed in the red square in Figure 4). He is asking the others if they can visualize this also, so that the group can use this to simplify and solve their problem with the diamond.

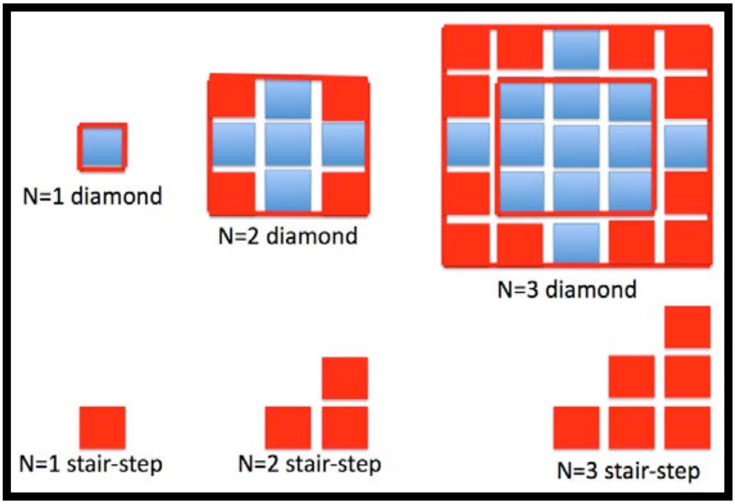


Figure 4. Blue diamond patterns and red stair-step patterns.

Aznx presents his proposal about re-thinking the problem as a question about visualizing the diagram. The group has been working in the VMT environment, going back and forth between text in the chat area and drawings in the whiteboard. They have started with problems presented graphically and have discussed these graphical problems in their text chat. They have shared different ways of viewing the relationships within the drawings and they have gradually developed symbolic algebraic ways of expressing general relationships about patterns in these drawings, working out these symbolic expressions in the chat and then storing them more persistently in the whiteboard.

We have been calling Aznx' chat posting a "problem-solving math proposal" (Stahl, 2006a, chapter 21). However, it is presented in the grammatical form of a *question*. Aznx did not simply state a proposal like, "I think we should enclose the diamond in a square, calculate the size of the square and then subtract the missing areas." Rather, he first announced that he had "an interesting way to look at this problem" and then explained his way of looking by asking if the others could "see how it fits inside a square." Presenting a proposal calls on the others to accept the proposal and to start to work on it. Of course, the others can reject the proposal, ask for clarifications about it, make a counterproposal or ignore the proposal.

But Aznx' utterance is not a full proposal that the others must accept or reject. It is another preliminary step. It asks the others if they can visualize something. It puts this to them as a question. If they say yes, then Aznx can proceed to make his proposal—or perhaps the others will see the implications of his way to look at the problem—what makes it "interesting"—and propose the strategy without Aznx having to advocate it, explain it and defend it. If they say no—that they cannot see how it fits inside a square—then he can explain his view further so they will be better prepared to accept his proposal.

Aznx' chat posting avoids articulating a complete proposal; by starting the conversation about the visualization, it involves the others in articulating the proposal *collaboratively*. In fact, in the subsequent discussion, the others do "see" the strategy that is implicit in Aznx' interesting view of the problem and they do help to articulate the strategy and then pursue it. By designing his proposal as this preliminary question about viewing the problem, Aznx succeeds in directing the group problem solving in a certain direction without his having to fully work out a detailed, explicit proposal. Aznx does not seem to be presenting a solution that he has worked out in his head. Rather, he is presenting his "interesting idea" for an approach to solving the problem so that the group will proceed to use the idea and work as a group to try to solve the problem with this approach.

A reference: "It"

Aznx' question is ambiguous at a purely syntactic level. It asks the others, "Can you see how it fits inside a square?" To what does the term "it" refer? People use pronouns like "it" rather than lengthy explicit noun phrases when the reference is clear from the context. This situates the utterance in its context—its meaning cannot be gathered from the utterance considered in isolation. Often, "it" will reference something that was recently referred to in a previous contribution that the new utterance is building on. For instance, "it" could refer to something mentioned in Aznx' previous utterance, "I have an interesting way to look at this problem." But to say that "it" refers to "this problem" does not make complete sense. The *problem* does not fit inside a square.

However, a minute earlier, when the group was discussing Team C's equations, Aznx said about part of an equation, "The 3n has to do with the growing outer layer of the pattern I think." He was referencing different aspects of the growth of the diamond pattern, particularly its "outer layer." Therefore, when he announces that he has an interesting way to view the problem, it is reasonable to assume that his new way of looking may be closely related to the observation that he had just reported about the outer layer of the diamond pattern. Because everyone in the group was following the flow of the discussion, Aznx could refer to the topic of the outer layer of the diamond pattern in the shorthand of the pronoun "it." When he typed, "Can you see how it fits inside a square?" he could assume that the readers of this posting would understand that he was referring to how some aspect of the diamond pattern can be seen as fitting inside of some square shape.

Although the reference to some aspect of the diamond pattern is relatively clear, the details are not clear about just what aspect of the diamond is to be visualized or focused on visually, where a square is to be constructed, and how the diamond fits inside the square. At this point, only a rather confusing image of a diamond pattern is visible on the whiteboard (see Figure 2). To *make sense* of "it," everyone has to follow the flow of discussion and the way in which the math topic is being developed as part of a "joint problem space," understood and visualized by the whole group.

Bwang and Quicksilver both respond initially to Aznx' question with "Yes." However, as we saw, Bwang indicates some hesitancy in his response and Quicksilver asks for further clarification. Aznx and Quicksilver discuss what they see when they fit a diamond pattern inside a square. Quicksilver notes that the "extra spaces" (colored red in Figure 4) look similar to the stair-step pattern that the team worked on previously. But Aznx goes on to talk about the four squares on the outer areas of the square, confusing Quicksilver. That is, as they each try to work out the details of Aznx' view, they display that they are not *seeing* things quite the same way. They have not yet achieved an adequate shared understanding or shared view.

Quicksilver suggests that Aznx show what he means on the whiteboard, so the ambiguity of his proposal can be resolved. Rather than drawing it himself, Aznx asks Bwang to do a drawing, since Bwang said he could see what Aznx was talking about. Bwang has in the past shown himself to be skilled at making drawings on the whiteboard, while Aznx has not tried to draw much.

Bwang draws a very clear diagram on the whiteboard for the diamond pattern when N=2 (see Figure 5). As soon as Bwang completes his drawing, he makes explicit the problem-solving proposal that is implicit in Aznx' way of viewing the problem or the pattern: "We just have to find the whole square and minus the four corners." His drawing has made this process very visible. He drew the diamond pattern with white squares and then filled in a large square that the diamond fits into by adding red squares. The red squares fill in symmetrical spaces in the four corners of the diamond pattern. The group can now look at this together in the shared whiteboard, providing a shared view of the matter to the group.

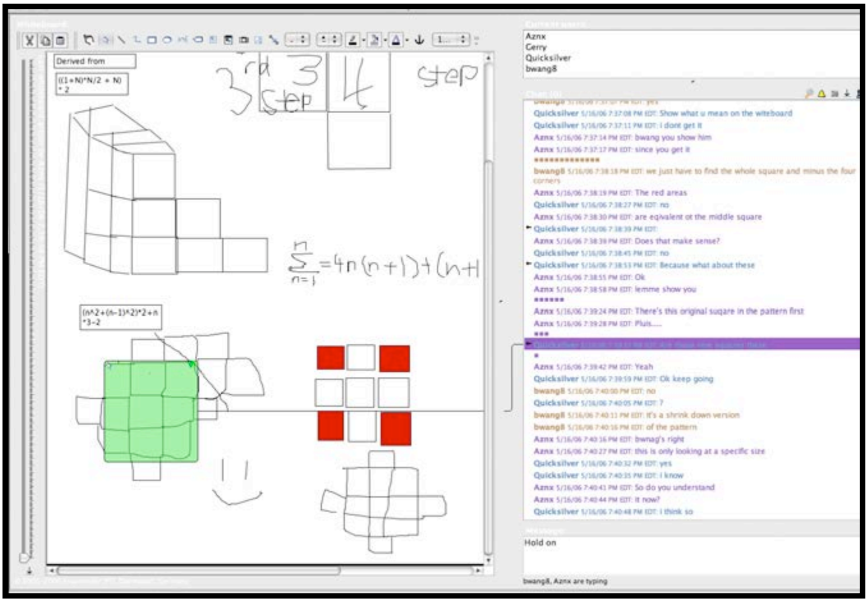


Figure 5. Bwang has drawn the white diamond for N=2 with red squares filling in the corners of an enclosing square. Quicksilver is pointing to a diamond pattern for N=3, also re-drawn lower on the whiteboard.

The group then discusses the view of the diamond pattern fitting into an enclosing square. They eventually realize that some of their observations are only true for the diamond pattern at a certain stage, like N=2.

So Bwang then draws the pattern for N=3. Here it starts to become visible to the group that the red squares in each corner follow the stair-step pattern (see Figure 6).

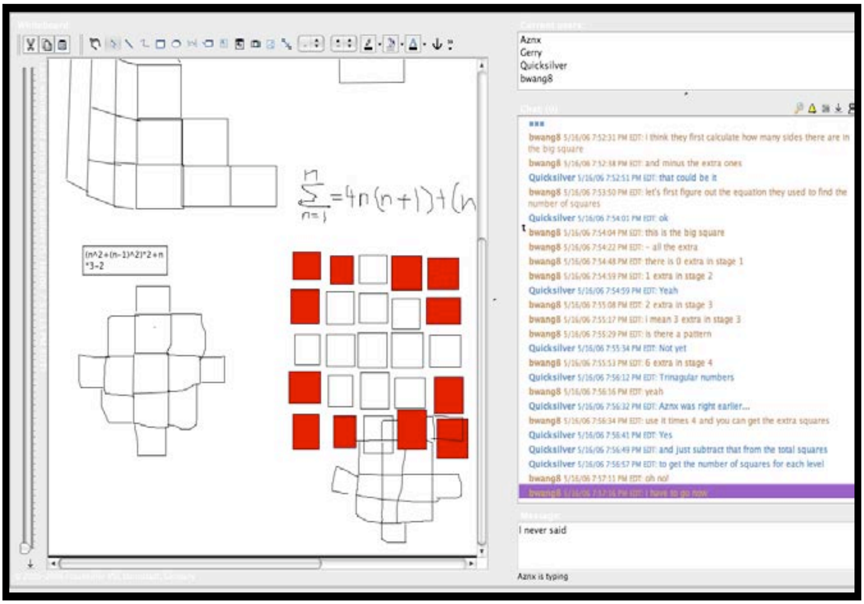


Figure 6. Bwang expanded his drawing to make the diamond for N=3. Note the red corners are now stair-step patterns.

The group has realized that viewing a graphical image of a mathematical pattern can be very helpful in thinking about the pattern. They treat the whiteboard as a shared, viewable image of aspects of the joint problem space of their collaborative work. Viewing this image and pointing out elements of it ground their chat discourse.

However, the image drawn by Bwang captures just one particular stage in the pattern, one value of N. They then start to look at images for different values of N or different stages in the growing pattern. They count the number of red squares in a corner as N increases and notice that it goes: 0, 1, 3, 6 (see Figure 4). This pattern is familiar to them from their earlier analysis of the stair-step pattern. They call this sequence “triangular numbers,” from Pascal’s triangle, which is often useful in combinatorics math problems. They know that this sequence can be generated by Gauss’ formula for the sum of the consecutive integers from 1 to N: (N+1)N/2. Unfortunately, at that point Bwang has to leave the group. But when they return in session 4, they will quickly put together the simple formula for the enclosing square minus this formula for the number of squares in each of the four corners, to solve their problem.

Viewing the Learning and Thinking

Let us pause now from all these details about the case study of three students in a VMT session and talk about how we view learning and thinking in CSCL groups. I have tried to demonstrate how we view learning and thinking in CSCL groups by *viewing* with you how a group of three students engaged in collaborative thinking and learning processes within an online environment for drawing and chatting.

We went through several levels of analysis of the group discourse (see Figure 7). We started by mentioning the overall context of the *event*. This was an online event in which Team B, consisting of three students, met in the Virtual Math Teams environment to discuss patterns of squares formed by sticks. We then focused on the smaller *session* unit, looking at Team B's third session, in which they considered a pattern that another group, Group C, had analyzed. Within this session, we identified one of several *themes* of discussion in that session, namely the one involving Aznx' "interesting way to look at this problem."

Event:	VMT Spring Fest 2006, Team B	
Session:	session 3, May 16, 7:00-8:00 pm	
Theme:	"I have an interesting way to look at this problem"	
Move:		Show how to view
Pair:		"Can you see how it fits inside a square?" "Yes"
Utterance:		"Can you see how it fits inside a square?"
Reference:		"it", diamond pattern

Figure 7. Levels of analysis of online group discourse.

Aznx introduced the theme by initiating a group problem-solving *move*. Namely, he got the group to view the problem in a certain way, as a diamond enclosed in a square. We saw how the group ended up drawing images in their shared whiteboard of diamond patterns enclosed in squares. Aznx introduced this group move in a subtle way; he did not simply come out and say, "We should analyze this pattern as partially filling an enclosing square." Rather, he first announced that he had an interesting view, involving the others in his approach to make it a group problem-solving process. Then he asked if the others could view the problem in a certain way. He did this through a question/answer response *pair*: he asked a question, which elicited a yes-or-no response from the others. By eliciting the response, he oriented the others to look at the diagram in the whiteboard in a certain way—namely in the way that his question implicitly proposed. A set of lines on the whiteboard are not immediately meaningful—they must be seen (interpreted) *as* something (Heidegger, 1927/1996, §32; Wittgenstein, 1953, §II xi).

Aznx' formulation of his question looks like a simple *utterance* in question format, but it entails selection from a number of different ways of picturing the relationships among the diamond pattern, the enclosing square and the empty corners. To begin with, one must decide what the *reference* to "it" is doing.

Indexical references like the pronoun "it" are ubiquitous in online text chat—and unavoidable according to Garfinkel (1967) [see Investigation 5]. They require the reader to understand or reconstruct the implicit threading or response structure of the chat. The difficulty of doing this often leads to confusions, which require the participants to spend time clarifying the content and structure of their discussion. For instance, in our example of the move of seeing the diamond in the square, the group had to engage in a couple minutes of chatting and drawing to co-construct a shared understanding of the problem.

Issues of shared understanding can be analyzed as linguistic problems of reference. In other words, in order to view learning and thinking in CSCL groups, we do not try to figure out what is going on in the heads of the students; rather, we try to figure out what is going on in their chat postings and their drawing actions. This is what we call the group's *interaction*. In VMT, the interaction of the virtual math team consists of sequences of chat postings and drawing actions.

Our first step in figuring out what is going on in the chat postings and drawing actions is generally to try to analyze the sequencing of these by reconstructing their response structure—what previous action each new action is responding to and what kinds of action it is eliciting, what it is opening up an interaction space for, or what kinds of responses it is making relevant as next postings. Often, this leads to some kind of threading diagram (Çakir, Xhafa & Zhou, 2009), uptake graph (Suthers, Dwyer, Medina & Vatrapu, 2010), or interaction model (Wee & Looi, 2009). This represents graphically a basic structure of the meaning-making sequencing. Then we try to understand what problem-solving work is being accomplished at each point in the sequence. This involves looking at different levels of granularity, such as the event, session, theme, move, pair, utterance and reference. Understanding the meaning that the group is co-constructing in their interaction generally involves going back and forth through these different levels and integrating partial interpretations from the different levels in a dialectic of whole and part (Gadamer, 1960/1988).

Through this process, we can gradually view the learning and thinking that takes place in the CSCL group. This learning and thinking are not something that takes place primarily in the minds of the individual participants (although the individuals in the group are each continuously using their linguistic skills to understand what is going on and to respond to it with their postings and drawings). Rather, when there is an intense collaborative process taking place in the online environment, the thinking and learning takes place in the visible text and graphical interactions.

According to the theory of group cognition, thinking in a CSCL collaborative interaction does not take place so much the way we usually think of thinking. Thoughts, or cognitive processes, are not characterized in terms of neurons connecting and firing in a brain; they are analyzed in terms of text postings and drawings referring to each other and building on each other, in the spirit of the idea of transactivity. We will look more at how this takes place in a minute. Similarly, learning does not take place the way we learned about learning. It is not viewed as a change in the amount of knowledge stored in a brain. Rather it is described as a matter of knowledge artifacts being gradually refined through sequences of text postings and graphical drawings that are interrelated and that explicate each other. The knowledge artifacts may be statements about a problem the group is working on, as viewed from a new perspective that the group has developed. The knowledge artifact might be a drawing like Bwang's in Figure 6 or an algebraic formula that sums up the group's analysis of pattern growth.

Unpacking the Group Learning and Thinking

Rather than talking about learning and thinking in the abstract, let us unpack some more how learning and thinking take place in Team B's interaction—in their text chatting and drawing together. Let's go back through the hierarchy of levels of analysis *in the opposite order* to say something about how references, utterances, response pairs, moves, themes, sessions, and events can contribute to learning and thinking in CSCL groups (see Figure 8).

Reference:	network of meaning, indexical ground, joint problem space
Utterance:	recipient design for reading's work
Pair:	projection and uptake
Move:	getting the problem-solving work done
Theme:	coherent interactional sequences
Session:	temporal structuring and re-member-ing
Event:	forming groups and co-constructing knowledge objects

Figure 8. Levels of learning and thinking in online group discourse.

Reference: Network of meaning, indexical ground, joint problem space

When one studies logs of virtual math teams, one sees that they spend a lot of time reaching shared understanding about references in their postings. Elsewhere, I review an example of this from Team B's session four, where Aznx, Quicksilver, and Bwang get quite confused about references from the chat to different equations written on the whiteboard (Stahl, Zemel & Koschmann, 2009).

The reason that people devote so much time and energy to resolving confusing references is that the network of references that they build up together plays an extremely important role in their group learning and thinking. In the theory of CSCL, there is considerable emphasis on the ideas of "common ground" (Clark & Brennan, 1991) and "joint problem space" (Teasley & Roschelle, 1993). A group establishes common ground largely by reaching a shared understanding of how references work in their discourse. As it interacts over time, a group co-constructs a network of references that can become quite complex (Sarmiento & Stahl, 2008).

This network of references defines the context or situation in which the group discourse continues to take place. Aznx' reference to "it" that we looked at contributed to a network of meaning that the group built up continuously through their interaction. This network included images of sticks in various patterns (like diamonds at stage N=2 and N=3), the relationships of the patterns (like a diamond enclosed in a square with stair-step empty corners), concepts referred to by technical terms (like "triangular numbers" or "summation"), and symbols representing mathematical operations (like equations for number of squares in a pattern).

As a group builds up its network of shared references, it can use more shortcut references to point to things without creating confusion. People can use deictic references to point to things in the network, like "this formula," "the second equation," or "it." In linguistic terms, the shared network of references provides a background for referring to things, a so-called indexical ground of deictic reference (Hanks, 1992).

In problem-solving terms, the network of references forms a joint problem space, a shared view of the topic that the group is addressing (Sarmiento & Stahl, 2008). For Team B, the joint problem space starts with the stair-step pattern and the chart of the number of sticks and squares for each stage of this pattern as presented in the topic description for the event. By the middle of session 3, it includes the diamond pattern and the view of "it" enclosed in a square, forming empty corners. It also includes triangular numbers and their associated formula, as well as several other equations from Team C and from Team B's own work. The team's interaction (the text postings and drawings) gradually creates this joint problem space and is situated within it. The work and utterances of the team can only be understood (by the participants and by us as researchers) through an ongoing understanding of the joint problem space as a network of meaningful reference. For Team B, the VMT whiteboard makes their joint problem space visible and persistent as it evolves.

Utterance: Recipient design for reading's work

While both the students who participate in the sessions and the researchers who analyze the logs need to understand the network of references, they understand them in very different ways. The students understand how to respond to what is going on the way they might know how to ride a bike. That is, they are not reasoning about it explicitly, rationally, logically, consciously. Rather, they are paying attention to what is going on and responding knowingly and intuitively. Quicksilver has not carried out any kind of analysis of Aznx' word "it" the way I did; yet he could respond to it with a sophisticated set of questions. He only had a couple of seconds to respond, whereas I, as a researcher, could spend hours going back and forth over the log reasoning about explicit interpretations.

People are incredibly skilled at using language without thinking about how they do it. In fact, even researchers are only aware of a small percentage of what people take into account almost instantaneously without being aware of it. We say that Aznx "designs" his announcement and proposal so that it will be read by Bwang and Quicksilver in a way that will lead them to understand in a complex way. They will figure out what "it" is referencing, but also realize some of the ambiguity of the reference. They will also come to think about the strategy for finding the number of squares in the diamond pattern because of this ambiguity. However, Aznx does not design his statement explicitly, through a rational sequence of logical arguments. Rather, as a skilled user of language, he gives voice to a well-designed posting that responds to the current discourse situation. It is somewhat like the way a skilled off-road biker responds to the terrain intuitively as she is speeding down a rough hillside with no time to think about what she is doing—and she somehow designs an optimal path for her journey.

Aznx was successful in designing his question so that it would be read in a certain way within the context of the group's discussion in their joint problem space. This is what ethnomethodology calls the "accountability" of utterances (Garfinkel, 1967). This simply means that utterances are designed to be understood by their recipients, by the audience for whom they are intended. That is, utterances are designed to meet the expectations of their recipients (Garfinkel, 1967). They include an "account" of how they should be read, embedded in the design of their presentation. In chat, postings are designed to be read in a certain way by the other chat readers. We call this "recipient design." This is analogous to utterances in spoken talk, which are designed to be heard, and are therefore given subtle vocal emphasis and timing. Chat postings, on the other hand, can incorporate capitalization, abbreviations, symbols, punctuation, emoticons and special fonts. They can reference previous postings that occurred further back in time because the chat text is persistent, remaining visible or retrievable for longer than speech. In chat, group work takes place as reading; chat postings must be designed to support reading's work of understanding the posted utterances in their discourse context (Livingston, 1995; Zemel & Çakir, 2009).

Response pair: Projection and uptake

An important aspect of the design of utterances or postings is how they are designed to fit into what comes before and after them. In general, an utterance performs an uptake or response to something that came before (Suthers, Dwyer, Medina & Vatrapu, 2010). At the same time, it elicits a follow-up, or at least makes relevant certain forms of subsequent utterances by others (Schegloff, 2007). Through its uptake and projection, an utterance provides continuity to the discourse—in fact, it thereby creates a temporal structure (Heidegger, 1927/1996).

The clearest and simplest example of this is the adjacency pair or response pair, such as a question/answer pair. A question elicits an answer. That is, stating a question projects that an answer will be given in response. It opens a conversational space for an answer. It makes it relevant for the next utterance to be an answer responding to the question. In other words, a question is designed to be read as something that should be responded to with an answer. A question worded like "Can you see how it fits inside a square?" is designed to be answered with a "yes" or a "no." The question-and-answer pair forms a unity, a small unit of interaction between people. The "yes" response shows that the posting it is responding to was read as a question and creates the pair as a successful question/answer interaction.

One of my first discoveries in studying virtual math teams was that math discourse is largely driven forward by what I called "math proposal response pairs" (Stahl, 2006a; 2006b). These have the following structure:

- An individual makes a bid for a proposal to the group suggesting how the group should continue to do its mathematical work.
- Another member of the group accepts (or rejects) the proposal on behalf of the group.

This is the simple, default form of the math-proposal response pair. If the proposal is accepted, then work begins on the proposal, often in the form of a follow-up proposal.

Of course, there are many variations and complications possible. The bid can be ignored or never responded to. In that case, it does not function as an effective proposal; at best it is a "failed proposal." Before a proposal response is made, there can be other response pairs inserted in the middle of the expected pair—such as a clarification question. It is also possible that someone will propose an amendment to the proposal bid before the original is accepted. Thus, a simple pair can develop a complicated recursive structure of insertions, extensions, repairs, etc.—with each of these being subject to their own insertions, extensions or repairs. Eventually, each of the intervening pairs may get closed with its anticipated response and then the original pair may be completed.

Move: Getting the problem-solving work done

Group problem-solving moves often have the structure of a longer sequence than a simple pair. Such a longer sequence may consist of a complex of response pairs embedded in one another. To identify such a structure, it may be necessary to first conduct a threading analysis to determine what is responding primarily to what. Then, it is often useful to see how this longer response sequence is built up out of simple response pairs (Stahl, 2011).

Together, these intertwining response pairs form a successful move, introducing a new theme for the group. As an example, let's look again at Aznx's move in Figure 9.

line	start	post		
919	19:35:26	19:35:36	Aznx	I have an interesting way to look at this problem.
920	19:35:41	19:35:42	Quicksilver	Tell us
921	19:35:38	19:35:45	Aznx	Can you see how it fits inside a quare?
922	19:35:45	19:35:45	Bwang	yes
	19:35:49	19:35:52	Bwang	[user erased message]
923	19:35:51	19:35:52	Quicksilver	Yes
924	19:35:52	19:35:53	Bwang	oh
925	19:35:55	19:35:55	Bwang	yes
926	19:35:53	19:36:01	Quicksilver	You are sayingthe extra spaces...

927	19:35:58	19:36:05	Aznx	Also, do you see if you add up the missing areas

Figure 9. The move to introduce Aznx' new way of looking at the group's problem.

We can see four response pairs there:

1. Aznx announces, "I have an interesting way to look at this problem" and Quicksilver responds by asking him to "Tell us."
2. Aznx asks, "Can you see how it fits inside a square?" and first Bwang responds "yes." Then Quicksilver responds, "Yes." Then Bwang responds again, more emphatically, "oh ... yes."
3. Quicksilver asks a clarification question about the proposal implicit in Aznx' question, "You are saying the extra spaces ...[?]"
4. Aznx, in parallel with Quicksilver's question asks a follow-up question, which contains an implicit further proposal about the group's work: "Also, do you see if you add up the missing areas [...?]"

As the discussion continues, Quicksilver responds to Aznx' question and the two of them continue to discuss the issues raised in both their questions.

Theme: Coherent interactional sequence

Aznx' *move* introduces the *theme* of the diagonal pattern viewed as enclosed in a square with missing spaces in the four corners. As we have just seen, the move consists of multiple response pairs that drive the work of the group to consider this theme.

As the theme evolves, the group draws and discusses some increasingly elaborate drawings to view the patterns that the theme involves. The group considers different stages of the pattern (N=1, 2, 3, 4) and how the number of missing spaces changes as the diamond pattern grows.

This leads them right to the point where they can formulate an equation to summarize their analysis of the pattern growth. Unfortunately, Bwang has to leave the session and they do not complete this work. During the fourth session two days later, the group picks up this theme and discusses it repeatedly, eventually deriving the equations for number of squares and sticks in the diamond pattern at all stages (Stahl, 2011). This theme is the basis for the equation for number of squares, which simply subtracts the number of missing spaces in the four corners of a square that encloses the diamond pattern.

Session: temporal structuring and re-member-ing

After Bwang left the third session, Aznx and Quicksilver try to review the group's accomplishments. They become confused about various equations and unsure of their ability to explain what the group has figured out. They end the session with Quicksilver saying, "then let's pick it up next time when Bwang can explain it." This ends one session and projects what will happen in a future session.

When the group meets for the fourth session, Aznx and Quicksilver do eventually get Bwang to review the derivation of the equation based on the view of the problem that Aznx introduced in the theme we just considered. The discussion in session four refers back to the group's work in session three and also to Team C's work in session two. But it does this in ways that are situated in Team B's session-four context (Sarmiento-Klapper, 2009). The team members and the memories they bring with them from the past are re-constituted in the new situation, made relevant to the current themes, problem space and available resources. The group remembering process makes the individual students who are present in the new sessions members of the group: it is a re-member-ing process, necessary at the start of many sessions, especially where the group membership has changed.

Event: Forming groups and co-constructing knowledge artifacts

At the beginning of session one, the students were not part of a particularly effective group or team. They did not build much on each other's contributions and were hesitant to make proposals, ask each other to undertake tasks, produce permanent drawings or manipulate mathematical symbols. That all changed dramatically during their four-session event. By the end, they had many graphical, narrative and symbolic representations or expressions related to their mathematical topic. They worked effectively together and solved their problems well. Problem-solving methods that one person introduced were later proposed and used by the other group members. Effective collaboration comes with interaction practice.

You may be wondering if each of the students learned mathematics. The interesting thing about looking closely at what really went on in this event is that what we traditionally consider to be the math content actually plays a relatively minor role in the group's problem solving. Yes, content is brought in: the students talk about triangular numbers, and they apply the formula for summing consecutive integers, for instance. Often, this math content is brought in quickly through proposals by individuals. It is then discussed through responses to the proposal that check that everyone understands the math content and agrees on its applicability. However, the bulk of the hard work is not accessing the traditional math content, but selecting, adapting, integrating, visualizing, sharing, explaining, testing, refining, building on and summarizing sequences of group response pairs. These proposals and discussions reference not only math content, but also various related resources that the group has co-constructed.

The learning and thinking of the group take place through the group's discourse, as a temporally unfolding multi-level structure of response pairs interwoven into larger sequences of group moves, problem-solving themes, and sessions of events. The group learns about the mathematics of its topic by building and exploring an increasingly rich joint problem space. It thinks about the mathematical relationships and patterns by following sequences of proposals, raising and responding to various kinds of questions, and engaging in other sorts of interactional moves. Some of this gets summarized in persistent knowledge artifacts like drawings, concepts, equations, solution statements and textual arguments. The building of the joint problem space generally requires a lot of work to resolve references and to co-construct a shared network of meaning (Stahl, Zhou, Çakir & Sarmiento-Klapper, 2011).

The math skills—like following certain procedures to do long division or to transform symbols—are not where the deep learning takes place and real knowledge is involved. Rather, the ability to sustain progressive inquiry through methods of group interaction is the real goal. This ability makes use of the math skills as resources for answering questions and coming up with new proposals.

If you wonder how to view learning and thinking in CSCL groups, follow Wittgenstein's advice: "Don't think, look!" My colleagues and I have tried to do this by looking at the work of virtual math teams in the way I have just described. We have been amazed to discover that collaborative learning and group cognition are a lot different than people thought.

CSCL as a New Approach to Computers in Education

Reading is learning, but applying is also learning
and the more important kind of learning at that….
It is often not a matter of first learning and then doing,
but of doing and then learning, for doing is itself learning.
--Chairman Mao 毛泽东 (1936)

Computers in education bring many advantages, even as seen within a traditional view of education:

- They give students and teachers access to all the information on the Web.
- They provide the ability to access lectures anywhere/anytime/on large scales.
- They can support testing, tutoring and scripting of learning processes.
- They offer simulations, educational gaming, virtual reality and artificial intelligence.

But networked computers in education—using CSCL software environments like VMT—also open opportunities for a radically new view of learning and thinking:

- Networking of students can let them get together with others interested in similar topics around the world.
- Effective collaborative-learning experiences help students learn how to work, think and learn in groups. Group work is a new force of production in the world, and students need to learn how to produce knowledge in teams.
- CSCL events can give students first-hand, hands-on experience in knowledge building.
- Discussing mathematics in peer groups teaches students how to do math, how to talk about math, how to make math connections, how to learn math and how to think mathematically.

In this second view of computers in education, book learning of facts and rote procedures has a place, but the more important kind of learning comes through doing. CSCL groups can provide effective learning experiences in which teams of students actually do mathematics by exploring rich problem spaces and discussing them—the way that Aznx, Quicksilver and Bwang did.

There are *two* popular approaches to CSCL theory:

- Collaborative learning can be seen as an *extension* of traditional *individual* learning. Individuals possess knowledge that they can state in sentences and can communicate to other individuals. Our commonsense concepts can describe this, and we can measure what individuals know at different times. Learning in this traditional view is an increase in individual knowledge
- Collaborative learning can be viewed as being *qualitatively* different from traditional individual learning, and we need to *discover* the nature of collaborative learning and its relation to individual learning by exploratory

> research. We need to *re-think* our ideas about learning, collaboration, education, computer support, research methodology and cognitive theory (Stahl, 2006a). We need to look carefully at data from real CSCL sessions to see what *actually* takes place there, without imposing our commonsense views.

It should be clear by now that *I view* learning and thinking in CSCL groups as a mystery to be investigated, not as something well understood to be measured. It is a new form of human existence with great potential. We must observe it to learn how it works. My colleagues and I have begun to do this, as have other researchers in CSCL. I have tried to indicate to you here how you can go about observing learning and thinking in CSCL groups.

It may be easier to understand issues of technology design and of traditional instruction when studying computers in education than to understand this new view of learning and thinking. However, I believe that if we hope to get the most benefit from computers in education and to understand how groups learn and think in CSCL groups, then we will have to closely observe the discourse and interaction in ways similar to what I have presented here.

Afterword: Notes on Group Cognition

When one studies logs of virtual math teams, one sees that the teams spend a lot of time and effort constructing *shared understanding* about references in their postings. The reason that teams and other small groups devote so much time and energy to resolving confusing references is that the network of references that they build up together plays an essential role in their group learning and thinking. In the theory of CSCL, there is considerable emphasis on the idea of "common ground" (Clark & Brennan, 1991) and "joint problem space" (Teasley & Roschelle, 1993). A group establishes common ground largely by reaching a shared understanding of how references work in their discourse. As it interacts over time, a group co-constructs a network of references that can become quite complex.

The "shared understanding" that is built up is akin to the notion of *co-orientation*, which refers to the mutual orientation of individuals in a group toward an object (knowledge, belief, attitude), and can be traced back to the interactionist social psychology of John Dewey and George Herbert Mead. Psycho-linguistic metaphors of comparing stored mental representations are unnecessary and can be misleading, reducing all knowledge to individual mental possessions. Team members share a world centered on their task; they orient as a group to the objects that populate that world, such as Aznx' proposals, Bwang's drawings and

Quicksilver's queries. *Because they share a common world*—which they co-constitute largely through their discourse, mediated by the larger common social, cultural and historical horizons of their world—*they can co-construct a shared understanding*.

The shared network of references defines the context or *situation* in which the group discourse continues to take place (Heidegger, 1927/1996, §18). Aznx' reference to "it" that we looked at contributed to a network of meaning that the group built up continuously through their interaction. This network included images of sticks in various patterns (like diamonds at stage N=2 and N=3), the relationships of the patterns (like a diamond enclosed in a square with stair-step empty corners), concepts referred to by technical terms (like "triangular numbers" or "summation") and symbols representing mathematical operations (like equations for number of squares in a pattern).

The co-construction of shared understanding by a small group is what I refer to as "group cognition."

This Investigation represents a disciplinary perspective from Computer-Supported Collaborative Learning (CSCL), an interdisciplinary field concerned with leveraging technology for education and with analyzing cognitive processes like learning and meaning making in small groups of students (Stahl et al., 2006). *Group cognition* is a theory developed to support CSCL research by describing how collaborative groups of students could achieve cognitive accomplishments together and how that could benefit the individual learning of the participants (Stahl, 2006a).

It may well be that a group of students working together manages to solve problems faster than any of the individual students may have been able to do alone—particularly when the problem is challenging for them. However, the most important benefits of group cognition are the potential for genuinely innovative solutions that go beyond the expertise of any individual in the group. It is the deeper understanding that is achieved through the interaction as part of that creative process—and the lasting impact of that deep understanding that the students take with them when they move on from that interaction—which they may then carry with them as new resources into subsequent group problem-solving scenarios. Group cognition can then be seen as what transforms groups into factories for the creation of new knowledge.

The types of problems that have been the focus of exploration within the group-cognition paradigm have not been routine, well-structured problems where every participant can know exactly what their piece of the puzzle is up-front in such a way that the team can divide up the work, *cooperate* and function as a well-oiled machine. Many critical group tasks do not fit into well-known and practiced protocols—for example, low-resource circumstances that may occur in disaster situations, where standard solutions are not an option. In acknowledgement of this, the focus within the group-cognition research has been on problems that offer groups the opportunity to explore creatively how those problems can be approached from a variety of perspectives, where the groups are encouraged to *collaborate* and explore unique perspectives.

The processes that are the concern of group-cognition research have not primarily been those that are related to efficiency of problem solving. Rather, the focus has been on the pivotal moments where a creative spark or a process of collaborative knowledge building occurs through interaction. Our fascination has been with identifying the conditions under which these moments of group inspiration are triggered, with the goal of facilitating this process of team innovation and collaborative knowledge creation.

The field of CSCL has explored what makes group discussions productive for learning under different names, such as *transactivity* (Berkowitz & Gibbs, 1983; Teasley, 1997; Azmitia & Montgomery, 1993; di Lisi & Golbeck, 1999), *uptake* (Suthers, 2006), *social modes of co-construction* (Weinberger & Fischer, 2006), or *productive agency* (Schwartz, 1998). Despite differences in orientation between the subcommunities where these frameworks have originated, the conversational behaviors that have been identified as valuable are quite similar. Specifically, these different frameworks universally value explicit articulation of reasoning and making connections between instances of articulated reasoning. For example, Schwartz and colleagues (1998) and de Lisi and Golbeck (1999) make very similar arguments for the significance of these behaviors from the Vygotskian and Piagetian theoretical frameworks, respectively. The idea of transactivity as a property of a conversational contribution originates from a Piagetian framework and requires that a contribution contain an explicit reasoning display and encode an acknowledgement of a previous explicit reasoning display. However, note that when Schwartz describes from a Vygotskian framework the kind of mental scaffolding that collaborating peers offer one another, he describes it in terms of one student using words that serve as a starting place for the other student's reasoning and construction of knowledge. This implies explicit displays of reasoning, so that the reasoning can be known by the partner and then built upon by that partner. Thus, the process is very similar to what we describe for the production of transactive contributions. In both cases, a transactive analysis would say that mental models are articulated, shared, mutually examined and potentially integrated.

Group cognition is a post-cognitive theory [Investigation 15]. Post-cognitivism is a tradition characterized by situated, non-dualistic, practice-based approaches. Cognitivism—which retains theoretical remnants of the Cartesian dualism of the mental and physical worlds—originally arose through the critique of behaviorism, with the argument that human responses to stimulae in the world are mediated by cognitive activity in the mind of the human agent. This argument was particularly strong in considerations of linguistic behavior (Chomsky, 1959). More recently, post-cognitivist theories have argued that cognitive activity can span multiple people (as well as artifacts), such as when knowledge develops through a sequence of utterances by different people and the emergent knowledge cannot be attributed to any one person or assumed to be an expression of any individual's prior mental representations (e.g., Bereiter, 2002, p. 283).

Group-cognition theory explicitly focuses on these inter-personal phenomena and investigates data in which one can observe the development of cognitive achievements in the interactions of small groups of people, often in online collaborative settings, where interactions can be automatically logged. By interaction, we mean the discourse that takes place in the group. Group cognition is fundamentally a linguistic (speech or text) process, rather than a psychological (mental) one. Thus, unlike the theory of transactivity described above, this post-cognitive approach does not assume cognitive constructs such as mental models, internal representations or retrievable stores of personal knowledge. In the online setting of VMT, cognition is analyzed by looking closely at the ways in which meaning is built up through the interplay of text postings, graphical constructions and algebraic formulations (Çakir, Zemel, et al., 2009) [Investigation 12].

There is a tension between the human sciences and the natural sciences, between *understanding* team cognition (e.g., with micro-analysis of situated case studies) and *explaining* it (e.g., modeling, confirming general hypotheses, formulating laws and specifying predictive causal relations). Group cognition in online teams involves both humans and computers—both highly situated collaborative interactions and programmed computer support. Thus, the analysis of group cognition must integrate the identification of characteristic patterns with the recognition of irreducible uniqueness of cases.

In our research, our colleagues and we look at logs of student groups chatting and drawing about mathematics in order to see if they build on each other's ideas to achieve more than they would individually. How do they understand each other and build shared language and a joint problem focus? What kinds of problems of understanding do they run into and how do they overcome those? How do they accomplish intersubjective meaning making, interpersonal trains of thought, shared understandings of diagrams, joint problem conceptualizations, common references, coordination of problem-solving efforts; planning, deducing, designing, describing; problem solving, explaining, defining, generalizing, representing, remembering and reflecting as a group? What can we say about the general methods that small groups use to learn and think as groups? How can we support and encourage this better with software support for social awareness, social networking, simulations, visualizations, communication; with intelligent software agents; with pedagogical scaffolds and guidance; with training and mentoring; with access to digital resources; with new theories of learning and thinking? To answer these complex questions, we must look carefully at the details of discourse in CSCL groups and develop innovative tools (both analytic and automated) and theories (of cognition by individuals, small groups and discourse communities).

The field of CSCL is particularly interested in the ways small groups can build knowledge together thanks to communication and support from networking technology. We hope that CSCL environments can be designed that make possible and encourage groups to think and learn collaboratively.

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Investigation 25. Structuring Problem Solving

Gerry Stahl

Abstract. To develop a science of small-group interaction in collaboration software, we need a method for analyzing the structure of computer-mediated discourse that complements our theory of group cognition. We need an approach to understanding the structure of interaction during group sessions of mathematical problem solving and similar group-cognitive activities. Conversation analysis offers an analysis of conversational talk in terms of a fine structure of adjacency pairs and offers some suggestions about longer sequences built on these pairs.

This Investigation presents a case study of students solving a math problem in an online chat environment. It shows that their problem-solving discourse consists of a sequence of exchanges, each built on a base adjacency pair and each contributing a move in the longer sequences of the solution process.

Keywords. Adjacency pair, longer sequence, conversation analysis, discourse, problem solving.

Structuring Group Cognition at Multiple Levels

Information, people and technology converge in a practical way in online collaborative problem solving. My colleagues and I have been pursuing a research agenda aimed at investigating how to support online collaborative problem solving. We have focused on the domain of school mathematics—especially beginning algebra and geometry—where students learn formal techniques and tacit practices of solving abstract problems. This is perhaps the most perspicuous occasion for observing the development of abstract thinking, including systematic problem solving. In a collaborative context, students have to demonstrate to each other what they are doing and why. As researchers studying such interactions, we find that mechanisms of group problem solving can become visible in this context.

Our research—such as that reported here—confirms that there are distinctive processes of information use in problem solving at the small-group unit of analysis. These processes should not be reduced to either the individual psychological level or the larger social community level—despite the fact that groups are physically composed of individuals and that they are embedded in socio-historical contexts. A small group of young students does not solve a problem the way that an individual adult does or the way a mathematical algorithm would indicate. Group cognition is distinct from both individual and social cognition. While an approach methodologically focused on the group unit of analysis is in line with current post-cognitive theories, it is rarely carried out consistently at that level.

We developed the Virtual Math Teams (VMT) environment and invited students to work in online groups for up to eight hour-long sessions. We presented challenging problems for them to explore together and encouraged them to pursue their own questions. The environment was instrumented to capture a complete and accurate record at the group unit of analysis—i.e., all text-chat postings, all drawing actions and all social awareness messages that were displayed to the group. As researchers, we can replay the group interaction and view it as it appeared to the group or browse it in as much detail as needed for analysis.

Because we are pursuing design-based research to improve the VMT environment, we are not oriented toward theoretical hypotheses, statistical generalizations, individual mental representations or socio-cultural influences—except to the extent that they manifest themselves within the group interaction. Rather, we try to understand the situated processes that take place at the group level of description in actual case studies. In particular, we look at the ways in which groups of math students use information and solve problems in our environment so that we can design improved socio-technical supports for their collaborative online problem solving.

We have tried a variety of research approaches in the VMT Project, including coding, statistical comparison, modeling, uptake analysis, conversation analysis, critical ethnography and discourse analysis. In general, we have found the most insightful approach to involve adapting ethnomethodologically inspired conversation analysis (CA) to our context of online text chat by math students.

In Investigation 24, I claimed that the discourse of group cognition has a hierarchical structure, typically including the following levels:

- **Group event**: E.g., Team B's participation in the VMT Spring Fest 2006.
- **Temporal session**: Session 3 of Team B on the afternoon of May 16, 2006.
- **Conversational topic**: Determining the number of sticks in a diamond pattern.

- **Discourse move**: A stage in the sequence of moves to accomplish discussing the conversational topic.
- **Adjacency pair**: The base interaction involving two or three utterances, which drives a discourse move.
- **Textual utterance**: A text chat posting by an individual participant, which may contribute to an adjacency pair.
- **Indexical reference**: An element of a textual utterance that points to a relevant resource.

The multi-layered structure corresponds to the multiplicity of constraints imposed on small-group discourse—from the character of the lifeworld and of culture (which mediate macro-structure) to the semantic, syntactic and pragmatic rules of language (which govern the fine structure of utterances). A theory of group cognition must concern itself primarily with the analysis of mid-level phenomena—such as how small groups accomplish collaborative problem solving and other conversational topics.

The study of mid-level group-cognition phenomena is a realm of analysis that is currently underdeveloped in the research literature. For instance, many CSCL studies focus on coding individual (micro-level) utterances or assessing learning outcomes (macro-level), without analyzing the group processes (mid-level). Similarly, conversation analysis (CA) centers on micro-level adjacency pairs while socio-cultural discourse analysis is concerned with macro-level identity and power, without characterizing the interaction patterns that build such macro phenomena out of micro-elements. Understanding these mid-level phenomena is crucial to analyzing collaborative learning, for it is this level that largely mediates between the interpretations of individuals and the socio-cultural factors of communities.

In the current Investigation, we will see how a small group of students collaborating online constructed a coherent longer sequence, through which they solved the problem that they had posed for themselves. In particular, we will look at the final conversational topic in Session 4 of the same virtual math team whose Session 3 is analyzed in Investigation 24.

Methodologically, it is important to note that the definition of the longer sequence—like that of the other levels of structure listed above—is oriented to by the discourse of the students and is not simply a construct of the researcher.

An Analytic Method

I have tried to apply our approach based on CA in a systematic way to the analysis of VMT chat logs. Schegloff's (2007) book on *Sequence Organization in Interaction: A Primer in Conversation Analysis* represents the culmination of decades of CA analysis. As indicated by its subtitle, it provides a useful primer in CA. My goal here is to extend the CA approach based on short sequences of utterances to analyze the larger scale interactions of group problem solving in VMT.

Schegloff's presentation highlights the central role of the adjacency pair as the primary unit of sequence construction according to CA. An adjacency pair is composed of two turns by two different people, with an interactional order, such as a question followed by an answer to the question. The simple two-turn pair can be extended with secondary adjacency pairs that precede, are inserted between or follow up on the base pair, potentially recursively. This yields "extensive stretches of talk which nonetheless must be understood as built on the armature of a single adjacency pair, and therefore needing to be understood as extensions of it" (p. 12).

These "extensive stretches of talk" are still focused on a single interaction of meaning making, and not a larger cognitive achievement like problem solving, involving multiple steps. However, both Sacks and Schegloff provide only vague suggestions about the analysis of longer sequences. These suggestions have not been extensively developed within CA. This essay is an attempt to explore them in an online text-chat context.

Schegloff (2007) briefly takes up "larger sequence structures to which adjacency pairs can give rise and of which they may be building blocks … such as sequences of sequences" (p. 12). One way in which a sequence (an extended adjacency pair) may be related to, yet separate from, a previous, completed adjacency pair "is that it implements a next step or stage in a course of action, for which the just-closed sequence implemented a prior stage" (p. 213). Note the two-way reference, with the second stage having the character of a next, but also the first stage having the character of a prior. This is analogous to the two parts of a simple adjacency pair according to Schegloff:

> Adjacency pair organization has (in addition to the backwards import just described) a powerful prospective operation. A first pair part projects a prospective relevance, and not only a retrospective understanding. It makes relevant a limited set of possible second pair parts, and thereby sets some of the terms by which a next turn will be understood—as, for example, being responsive to the constraints of the first pair part or not. (p. 16)

The adjacency-pair structure was first discussed extensively by Sacks (1965/1995, II 521-569). In these seminal lectures, he also briefly discussed long sequences. Here, his main point was to state that little is known about the structure of long sequences; that the analytic problem is in principle harder; and that, in particular, it is wrong to assume that an analysis at the level of adjacency pairs will be useful to understanding the co-construction of long sequences:

> It turns out that one central problem in building big packages is that the ways the utterances that turn out to compose the package get dealt with as single utterances or pairs of utterances or triplets of utterances, etc., may have almost no bearing on how they're to be dealt with when an attempt is made to build the larger package. (II p. 354)

The analyses provided by CA come primarily from the study of American adults conducting face-to-face, verbal, informal, social conversation, although some of the early data came from distance conversations by telephone and the field has broadened its sources considerably more recently. However, we must be careful when applying CA methods to online, text-based, learning-related discourse about mathematics by students. Along these lines, Schegloff (2007) warns about his presentation:

> Note that this discussion is focused on conversation in particular. Because different organizations of turn-taking can characterize different speech-exchange systems (Sacks, Schegloff & Jefferson, 1974, n. 11 729-731), anything that is grounded in turn-taking organization may vary with differences in the turn-taking organization. It is a matter for empirical inquiry, therefore, how the matters taken up in the text are appropriately described in non-conversational settings. (p. 15n)

As we have frequently argued (e.g., Stahl, 2006; 2009c; Stahl, Koschmann & Suthers, 2006), we believe that adapting CA to computer-mediated communication offers the best prospects for analysis of interaction in socio-technical environments like VMT. The preceding review of the topics of adjacency pairs and long sequences indicates that it is an empirical question how well this proposed adaptation might work in specific cases. We designed and conducted the VMT Project from 2003 to 2015 in order to produce a corpus of data that could be analyzed in as much detail as needed to determine the structure of group cognition, that is, of collaborative knowledge building through interaction at the group unit of analysis.

In looking at the VMT data corpus, the VMT research team has clearly seen the differences between online text chat and verbal conversation. The system of turn taking so important in CA (Sacks et al., 1974) does not apply in chat. Instead, chat participants engage in reading's work (Zemel & Çakir, 2009), in which "readers connect objects through reading's work to create a 'thread of meaning' from the various postings available for inspection" (p. 274f). The first and second parts of an adjacency pair may no longer be literally temporally adjacent to each other, but they still occur as mutually relevant, anticipatory and responsive. The task of reading's work—for both participants and analysts—typically includes reconstructing the threading of the underlying adjacency-pair response structure (Stahl, 2009b).

In CA, adjacency pairs are related to both issues of timing (turn taking) and of sequentiality (response). In chat, they retain their importance solely as sequential, in order to maintain interaction in the absence of turn taking. We have tried to explore the larger sequential structure of problem-solving chat by using the CA notion of openings and closings (Schegloff & Sacks, 1973). VMT researchers looked at several math chats from 2004, which used a simple chat tool from AOL. We coded and statistically analyzed the fine-structure threading of adjacency pairs (Çakir, Xhafa & Zhou, 2009). In addition, we defined long sequences based on when opening and closing adjacency pairs achieved changes in topic (Zemel, Xhafa & Çakir, 2009). These long sequences were graphed to show their roles in constituting the chat sessions, but their internal sequential structures were not investigated at that time.

My colleagues and I have subsequently conducted numerous case studies from the VMT corpus. We have been particularly drawn to the records of Team B and Team C in the VMT Spring Fest 2006. These were particularly rich sessions of online mathematical knowledge building because these teams of students met for over four hours together and engaged in rich explorations of interesting mathematical phenomena. However, partially because of the richness of the interactions, it was often hard for analysts to determine a clear structure to the student interactions. Despite access to everything that the students knew about each other (team members were spread across the US) and about the group interaction, it proved hard to unambiguously specify the group-cognition processes at work (Medina, Suthers & Vatrapu, 2009; Stahl, 2009b; Stahl, Zemel & Koschmann, 2009).

Therefore, in the following case study, I have selected a segment of Team B's final session, in which the structure of the interaction seems to be clearer. The interaction is simpler than in earlier segments partially because two of the four people in the chat room leave. Thus, the response structure is more direct and less interrupted. In addition, the students have already been together for over four hours, so they know how to interact in the software environment and with each other. Furthermore, they set themselves a straightforward and well-understood mathematical task. The analysis of this relatively simple segment of VMT interaction can then provide a model for subsequently looking at the more complex data and seeing if it may follow a similar pattern.

The Case Study

Three anonymous students (Aznx, Bwang, Quicksilver) from US high schools met online as Team B of the VMT Spring Fest 2006 contest to compete to be "the most collaborative virtual math team." They met for four hour-long sessions during a two-week period in May 2006. A facilitator was present in the chat room to help with technical issues, but not to instruct in mathematics.

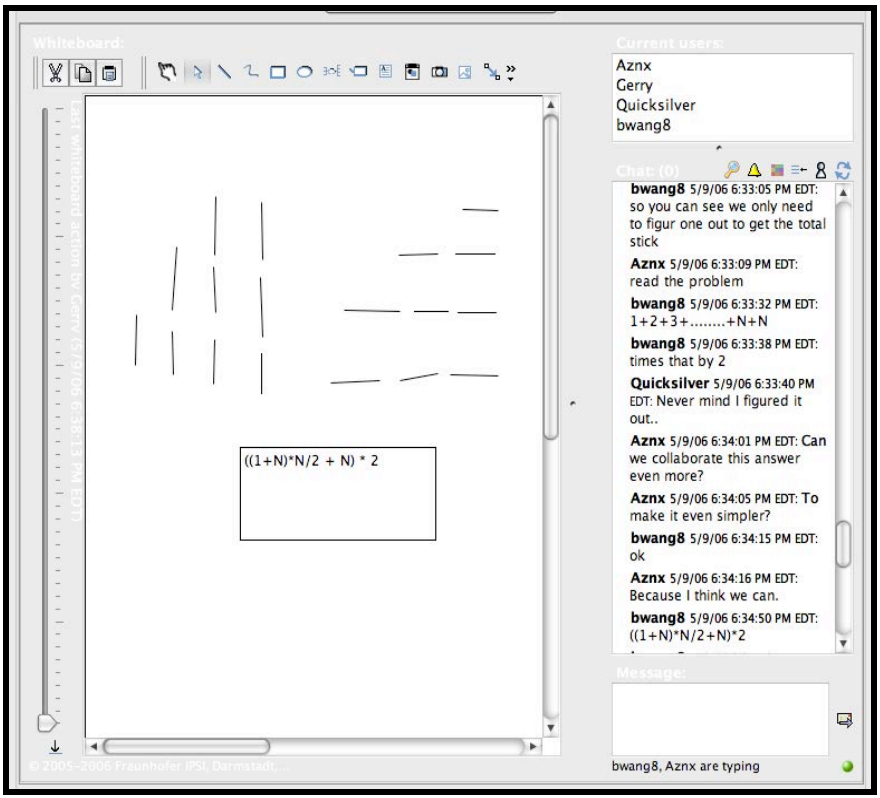

Figure 1. Screenshot of the VMT environment showing the pattern of horizontal and vertical sticks in the stair-step figure.

In their first session, they solved a given combinatorics problem, finding a mathematical formula for the growth pattern of the number of squares and the number of sticks making up a stair-step figure. They determined the number of sticks by drawing just the horizontal sticks together and then just the vertical ones (see Figure 1). They noticed that both the horizontals and the verticals formed the same pattern of $1 + 2 + 3 + \ldots + n + n$ sticks at the n^{th} stage of the growth pattern. They then applied the well-known Gaussian formula for the sum of consecutive integers, added the extra n, and multiplied by 2 to account for both the horizontal and vertical sets of sticks.

In the second session, they explored problems that they came up with themselves, related to the stair-step problem, including 3-D pyramids. Here they ran into problems drawing and analyzing 3-D structures. However, they managed to approach the problem from a number of perspectives, including decomposing the structure into horizontal and vertical sticks.

In the third session, Team B was attracted to a diamond-shaped variation of the stair-step figure, as explored by Team C in the Spring Fest. They tried to understand how the other team had derived its solution. They counted the number of squares by simplifying the problem through filling in the four corners surrounding the diamond to make a large square; the corners turned out to follow the stair-step pattern from their original problem.

In the fourth session, they discovered that the other team's formula for the number of sticks was wrong. In the following, we join them an hour and 17 minutes into the fourth session, when one of the three students as well as the facilitator had to leave.

Problem-Solving Moves

In this section, the interaction will be analyzed as a sequence of moves in the problem-solving interaction between Bwang and Aznx, the two remaining students. Each move is seen to include a base adjacency pair (changed to **bold** face in the logs), which provides the central interaction of the move and accomplishes the focal problem-solving activity. The captions given by the researchers to the log excerpts indicate the aim of the move, according to the analysis.

In line 1734 of Log 1, Bwang states that the team is close to being able to solve the problem of the number of sticks in the n^{th} stage of the diamond pattern, suggesting that they might stay and finish it. Note that this is the end of the last of the scheduled four sessions for the contest, despite some arrangements underway to allow the team to continue to meet.

Log 1. Open a Topic

LINE	TIME	AUTHOR	TEXT OF CHAT POSTING
1734	08.17.20	bwang8	i think we are very close to solving the problem here
1735	08.17.35	Quicksilver	Oh great...I have to leave
1736	08.17.39	Aznx	**We can solve on that topic.**
1737	08.17.42	Quicksilver	Sorry guys
1738	08.17.45	bwang8	oh
1739	08.17.46	Aznx	It shouldn't take much time.
1740	08.17.47	bwang8	**ok**
1741	08.17.50	Aznx	k, bye Quicksilver
1742	08.17.52	Quicksilver	Just tell me the name of the room
1743	08.17.52	bwang8	bye
1744	08.18.14	Gerry	The new room is in the lobby under Open Rooms
1745	08.18.44	Gerry	It is under The Grid World. It has your names on it
1746	08.18.49	Quicksilver	[leaves the room]
1747	08.19.00	Aznx	Alright found it.
1748	08.19.04	Aznx	Thanks.

Aznx responds in line 1736, indicating—and implicitly endorsing the suggestion—that the team could indeed continue to work on the current topic. This opens the topic for the group.

Quicksilver apologetically stresses that he must leave immediately. He just wants to know the location of the new chat room that the facilitator is setting up for the team to continue its math explorations on a future date. The facilitator supplies this information, and everyone says goodbye to Quicksilver. We ignore this other activity in our current analysis, and focus on the problem-solving interactions.

Aznx expresses uncertainty about how to proceed now that Quicksilver has gone, and the facilitator has arranged things for the future. He questions whether he and Bwang need to go as well (Log 2). Bwang then reiterates his suggestion that they could stay and finish solving the problem. He argues that it should not take much longer. Bwang directly asks Aznx if he wants to solve the problem now.

Log 2. Decide to Start

1749	08.19.12	Aznx	I guess we should leave then.

1750	**08.19.34**	**bwang8**	**well do you want to solve the problem**
1751	08.19.36	bwang8	i mean
1752	08.19.39	bwang8	we are close
1753	**08.19.48**	**Aznx**	**Alright.**
1754	08.19.51	bwang8	i don't want to wait til tomorrow
1755	08.19.53	bwang8	ok

Aznx agrees by responding to Bwang's question in the affirmative. This effects a decision by the pair of students to start working on the problem right away. Bwang continues to argue for starting on the problem now—posting line 1754 just 3 seconds after Aznx' agreement, probably just sending what he had already typed before reading Aznx' response. Bwang then acknowledges the response.

Once a decision has been made to solve the problem, the question of how to approach the problem is raised in line 1756 (Log 3). Bwang immediately lays out his approach in lines 1757, 1759, 1764 and 1765. The approach is the same as they used in the first session: visualize just the vertical or just the horizontal sticks. The two sets follow the same pattern. In fact, the diamond is also symmetric left/right and top/bottom, so the vertical sticks can be divided left/right into two identical sets and the horizontal sticks can be divided top/bottom. This produces four identical sets of sticks (color-coded in Figure 2), each having rows of 1, 3, 5, 7, ... sticks, up to (2n-1) for the n^{th} stage of the diamond pattern.

Log 3. Pick an Approach

1756	08.19.55	Aznx	**How do you want to approach it?**
1757	08.20.14	bwang8	**1st level have 1*4**
1758	08.20.20	Gerry	You can put something on the wiki to summarize what you found today
1759	08.20.29	bwang8	2st level have (1+3)*4
1760	08.20.32	Aznx	bwang you put it.
1761	08.20.35	Aznx	for the wiki
1762	08.20.37	bwang8	ok
1763	08.20.42	Aznx	we actually did quite a lot today
1764	08.20.53	bwang8	3rd level have (1+3+5)*4
1765	08.21.05	bwang8	4th level have (1+3+5+7)*4
1766	08.21.10	Gerry	This is a nice way to solve it

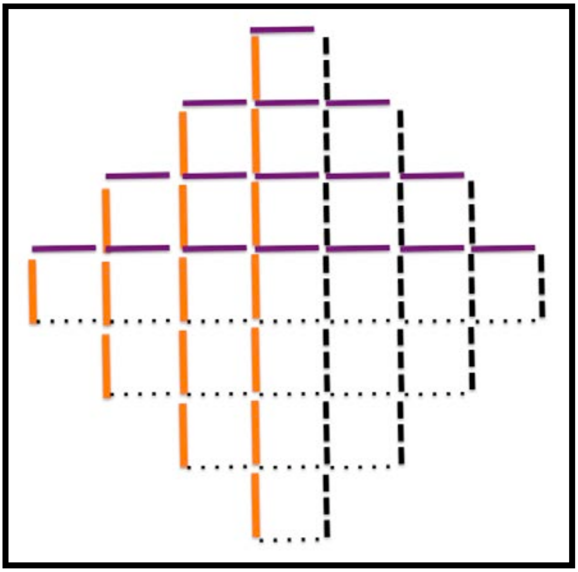

Figure 2. A representation (not from the data) of the diamond figure at stage n=4, color-coding the sticks in four identical (symmetric) sets.

Interspersed with this defining of the approach is a reminder from the facilitator to summarize the team's work on the Spring Fest wiki for other teams to see, motivating this with a word of encouragement about the team's work.

Aznx has previously been oriented toward finding patterns of growth in the mathematical objects the group has been exploring. Often, someone will create a graphical representation of the object in such a way that it makes the pattern visible. Aznx will then formulate a textual description of the pattern. Then the group will work on a symbolic representation to capture the pattern in a mathematical formula. (See (Çakir, Zemel & Stahl, 2009) [Investigation 12] for an analysis of the intertwining of graphical/visual, textual/narrative and symbolic/mathematical modes of interaction within the work of Team C.)

Here, in line 1767 (Log 4), Aznx describes the pattern as involving adding numbers that successively increase by 2. The number of sticks in a given stage of the diamond shape is a sum of numbers that start at 1 and increase successively by 2. When going from one stage to the next, one simply adds another number to this sum that is 2 more than the highest previous one.

Log 4. Identify the Pattern

1767	08.21.12	Aznx	**So it's a pattern of +2s?**
1768	08.21.15	Aznx	Ah ha!
1769	08.21.15	bwang8	**yes**

1770	08.21.20	Aznx	There's the pattern!

Aznx presented his description as a question and Bwang affirmed it at the same time as Aznx posted line 1768. Aznx then emphasized that they had discovered the pattern.

Bwang indicates that the next step in their work is to "find an equation that describes the pattern" (line 1771, Log 5). Aznx asks Bwang to let him state the equation, implicitly agreeing that this is the next step by trying to produce the equation.

Log 5. Seek the Equation

1771	08.21.39	bwang8	now we have to find a equation that describe that pattern
1772	08.21.49	Aznx	Hold on.
1773	08.21.51	Aznx	I know it.
1774	**08.21.57**	**bwang8**	**what is it**
1775	08.21.58	Aznx	But I'm trying to remember it. =P
1776	08.22.04	Aznx	and explain it as well.
1777	08.22.17	Aznx	try and think of it
1778	08.22.53	Gerry	Maybe Quicksilver can come back here tomorrow or next week to finish it with you
1779	08.23.01	Gerry	I have to go now
1780	08.23.05	Gerry	Bye!
1781	08.23.06	bwang8	ok
1782	08.23.07	bwang8	bye
1783	08.23.23	Gerry	[leaves the room]
1784	08.23.29	bwang8	ok
1785	08.23.32	bwang8	so
1786	08.23.37	bwang8	i think it is this
1787	08.23.53	Aznx	ok
1788	08.23.55	Aznx	i found it
1789	**08.24.00**	**Aznx**	**n^2**
1790	08.24.01	bwang8	(2*n)*n/2
1791	08.24.09	Aznx	or (n/2)^2

Bwang asks Aznx to state the equation and Aznx expresses difficulty in formulating an adequate and accountable answer. After a half minute of silence with still no formulation from Aznx, the facilitator suggests that Aznx and Bwang might want to wait until a future time when the whole group can work together to finish the problem. The facilitator then says goodbye and leaves the chat room.

After more than a minute since Aznx posted anything, Bwang starts to preface the presentation of his own formulation. Eventually, Aznx joins back in. Simultaneously, Aznx and Bwang post their formulae. For Aznx, it is either n^2 or $(n/2)^2$. For Bwang, it is $2n(n/2)$.

Aznx has not given any indication of how he got his proposed formula. The format of Bwang's formula suggests the use of Gauss' summation, which the students have used repeatedly in the past. According to this summation of an arithmetic sequence of integers, the result is the sum of the first and last member of the sequence times half the number of members. For a sequence of n members, $1 + 3 + 5 + ... + (2n-1)$, the sum would be $[1 + (2n-1)]*(n/2)$. Adding the 1 and the -1, yields Bwang's formula, $2n(n/2)$. Note that the n^{th} odd integer can be represented by $(2n-1)$.

It is likely that Aznx used a similar method, working on his own during his prolonged silence, but got confused about the result when he simplified his expression. As Aznx shows next, Aznx's first answer is equivalent to Bwang's answer, once Aznx simplifies it (in Log 6). His second answer is related to part of Bwang's unsimplified answer.

Log 6. Negotiate the Solution

1792	08.24.14	Aznx	I'm simplifying
1793	08.24.30	Aznx	if u simplify urs
1794	**08.24.35**	**Aznx**	**its n^2**
1795	08.24.59	Aznx	bwang
1796	08.25.01	Aznx	you there?
1797	**08.25.03**	**bwang8**	**so that's wrong**
1798	08.25.07	bwang8	yeah
1799	08.25.08	bwang8	i am here

Aznx simplifies Bwang's formula: $2n(n/2) = n^2$. This is the same as one of Aznx' proposed formulae. When Bwang does not respond to this posting, Aznx wonders if Bwang is still present online.

Bwang was apparently already typing "so that is wrong" when be received Aznx' question concerning his presence. This message in effect confirmed that Aznx' second formula, $(n/2)^2$, is wrong and his first one, which agrees with Bwang's, is correct.

Going along with this, Aznx then multiplies their agreed upon formula by 4 because there were 4 sets of horizontal or vertical sticks, each numbering 1 + 3 + ... In lines 1800-1802 (Log 7), Aznx poses his message as a question, soliciting confirmation from Bwang. By offering this next step in the symbolic representation, Aznx demonstrates that he understands where Bwang's formula came from, and he understands the larger strategy of approaching the problem that Bwang had proposed. In other words, Aznx demonstrates a level of mathematical competence and of shared understanding that he did not always display in the previous sessions.

Log 7. Check Cases

1800	08.25.11	Aznx	so
1801	08.25.13	Aznx	the formula
1802	**08.25.22**	**Aznx**	**would be 4n^2?**
1803	08.25.28	bwang8	let's check
1804	08.25.55	bwang8	Yes
1805	**08.26.00**	**bwang8**	**it actually is**
1806	08.26.02	Aznx	So we got it!

Before being ready to answer whether $4n^2$ is actually the correct formula for the number of sticks, Bwang suggests that they first check if the formula works by testing it for a number of values of n and counting the sticks in drawings of diamonds at the corresponding n^{th} stage. A half-minute later, Bwang concludes that the formula does check out. He therefore answers Aznx' question with a confident "Yes", perhaps expressed with a touch of surprise.

Aznx concludes that they got the solution for the number of sticks in the diamond pattern—a problem that Team C had posed for itself, but for which they had derived the wrong formula, without, however, realizing it. Team B had been shocked earlier to discover that the formula they had been struggling to understand from Team C had been wrong; that it did not check out for any values of n.

Their surprise and excitement at correctly solving this elusive problem is almost uncontrollable. In Log 8, they use every chat technique they know to express their joy. Their postings intertwine like a frenzied dance.

Log 8. Confirm the Solution

1807	08.26.02	bwang8	omg
1808	08.26.04	Aznx	yay!
1809	**08.26.08**	**bwang8**	**i think we got it!!!!!!!!!!!!**
1810	**08.26.12**	**Aznx**	**WE DID IT!!!!!!**
1811	08.26.12	bwang8	and it is so simple
1812	08.26.14	Aznx	YAY!!!!
1813	08.26.16	Aznx	i know
1814	08.26.17	bwang8	lol
1815	08.26.18	Aznx	lol

Once the mathematical exploration is done, it is time to write up a report of one's findings. They plan their report in Log 9.

Log 9. Present a Formal Solution

1816	08.26.34	Aznx	So you're putting it in the wiki, right?
1817	08.26.37	bwang8	yes
1818	08.26.41	Aznx	Alright then.
1819	08.26.43	bwang8	ok
1820	08.26.53	Aznx	Give an email to Gery, telling him that we got it. =)
1821	08.26.57	bwang8	ok
1822	08.26.59	Aznx	I meant Gerry
1823	08.27.04	bwang8	**are you going to do it**
1824	08.27.07	bwang8	or am i
1825	08.27.12	Aznx	**You do it.**
1826	08.27.14	bwang8	ok
1827	08.27.19	Aznx	Tell him that we both dervied n^2
1828	08.27.29	Aznx	And then we saw that pattern
1829	08.27.37	Aznx	and we got the formula

Professional mathematicians would do this in the form of a proof. When a group of mathematicians recently conducted an online collaborative analysis of a mathematical problem, it took them longer to write the publishable proof than it did to figure out the approach and solve it (Gowers & Nielsen, 2010; Polymath, 2010).

Sometime after the chat session, Bwang posted the narrative shown in Figure 3 to the Spring Fest wiki.

We then move on to understand Team C's formula for summing up the total # of sticks in n-level diamond. We first tried to used the big square and then minus the extra corners, but the corners turns out to be to hard to calculate. Then we tried to simplify Team C's equation to help as find a lead, but we found out that their stick equation is wrong. We then decide to find out a whole new equation and tried to divide the sticks up into vertical and horizontal groups like we did before with all the other problems. The groups can be further divided into 2 equal parts. We found a pattern.

1st level: 1
2nd level: 1+3
3rd level: 1+3+5
4th level: 1+3+5+7

5th level: 1+3+5+7+9
nth level: (2*n)*n/2

We then found out that each of these can be by calculated by (2*n)*n/2 which simplified into n^2. n^2 can then be multiplied by 4 and get the total of sticks in a nth leveled diamond. The final equation is 4(n^2).

Figure 3. Wiki posting by Group B after session 4.

Finally, Aznx and Bwang wrap up the conversational topic by exchanging email addresses and agreeing to meet again online with Quicksilver and pursue further mathematical adventures together (Log 10).

Log 10. Close the Topic

1830	08.27.44	Aznx	when should we meet again?
1831	08.27.49	Aznx	hat's your email?
1832	**08.27.52**	**Aznx**	**we should keep in touch**
1833	**08.27.57**	**bwang8**	**yeah**

The Sequence of Pairs

Within each of the preceding log excerpts, we have identified a base adjacency pair by means of which the work of a specific move in the problem-solving effort of the small group is interactively accomplished. In most cases, a question is posed, and a response is then given to it.

As Schegloff (2007) argues, an adjacency pair is itself a sequence. It embodies a temporal structure, with the first element of the pair projecting the opportunity and expectation of a response in the interactional immediate future. The second element constitutes an uptake of a first element that it implicitly references as in the interactional immediate past (Suthers, Dwyer, Medina & Vatrapu, 2010). In engaging in the exchange of an adjacency pair, the participants in the interaction effectively co-construct an elementary temporal structure in which future and past are constituted.

In talk-in-interaction, as analyzed by conversation analysis, the immediacy of response is intimately related to the turn-taking structure of vocal conversation (Sacks et al., 1974). As mentioned above, the completion of the adjacency pair is often postponed by insertion sequences, such as repairs of misunderstandings or clarification exchanges. The base adjacency pair can also be preceded by introductory exchanges, such as announcements of what is coming, or succeeded by follow-up exchanges or confirmations.

In chat-in-interaction, as seen in the preceding log extracts, adjacency pairs can be delayed by a more complicated response structure, in which multiple participants can be typing simultaneously and postings do not always directly follow the message to which they are responding. Thus, in Log 1, Quicksilver or Gerry can be initiating other topics in the midst of an interaction between Aznx and Bwang. Also, Aznx and Bwang can be typing to each other simultaneously as in Log 6, particularly if there has been an extended period of inactivity. This often makes textual chat harder to follow and to analyze than verbal conversation.

Nevertheless, it is generally possible to identify base adjacency pairs carrying the discourse along. In the previous section, we identified ten pairs. The discourse moves in the log excerpts (each including one of these base adjacency pairs) formed a problem-solving sequence:

- Log 1. Open the topic
- Log 2. Decide to start
- Log 3. Pick an approach
- Log 4. Identify the pattern
- Log 5. Seek the equation
- Log 6. Negotiate the solution
- Log 7. Check cases
- Log 8. Confirm the solution
- Log 9. Present a formal solution
- Log 10. Close the topic

The integrity of each of the ten moves is constructed by the discourse of the participants. Each move contains its single base adjacency pair, which drives the interaction. In addition, there may be several utterances of secondary structural importance, which introduce, interrupt or extend the base pair; there may also be some peripheral utterances by other participants.

The analysis of this essay is an attempt to make explicit the structure of adjacency pairs and a problem-solving longer sequence that is experienced by the participants and is implicit in the formulation of their contributions to the discourse. This is in contrast to analytic approaches that to some degree impose a set of coding categories based on the analyst's research interests or on an a priori theoretical framework, rather than on the perspective of the participants as evidenced in their discourse.

Lines 1795 and 1796, for instance, show the power *for the participants* of the adjacency pairings. Here, Aznx has addressed a mathematical proposal to Bwang: "If you simplify yours [expression], it is n^2." After 24 seconds of inaction, Aznx cannot understand why Bwang has not replied, expressing agreement or disagreement with the first part of the proposal, for which Aznx expects a response. Because it is not a preferred move at this point for Aznx to reprimand Bwang for not responding, Aznx inquires if Bwang has disappeared, perhaps due to a technical software problem, which would not be anyone's fault. Two seconds later, we see that Bwang was typing a more involved response that implicitly accepted Aznx' proposal. Bwang then immediately explicitly accepts the proposal in line 1798, allowing Aznx to continue with the start of a new move with line 1802. Here we see Aznx and Bwang clearly orienting to the adjacency-pair structure of their discourse, in terms of their expectations and responses.

Aznx and Bwang co-constructed the longer (ten move) problem-solving sequence by engaging in the successive exchange of adjacency pairs. Sometimes one of the students would initiate the pair, sometimes the other. As soon as they completed one pair, they would start the next. This longer sequence also has a temporal structure. It is grounded in their present situation, trying to find a formula for the number of sticks in the diamond figure. It makes considerable use of resources from their shared (co-experienced) past during the previous four hours of online sessions. It is strongly driven forward into the future by the practices they have learned for engaging in problem solving, culminating teleologically in the presentation of a solution.

The problem-solving sequence analyzed in this essay—covering 100 lines of chat during 10 minutes—is not selected arbitrarily or imposed in accordance with criteria external to the interaction but is grounded in the discourse as structured by the participants. The excerpted sequence is defined as a coherent conversational topic by the discourse of Aznx and Bwang. They explicitly open (jointly decide upon) this topic with their interaction in Log 1 and they close it (wrap it up and move on) with the discourse move in Log 10 (Schegloff & Sacks, 1973).

This case study provides an unusually clear and simple example of group cognition in a virtual math team. In earlier sessions, the students encountered many difficulties, although they also achieved a variety of successes and learned much about both collaboration and mathematics. At the beginning of their first session, they did not know how to behave together and showed rather poor collaboration skills. Bwang said very little in English, often simply producing drawings or mathematical expressions. Aznx, at the other extreme, tried hard to engage the others, but seemed to display a weak mathematical understanding of what the others were discussing. At various points in the sessions, misunderstandings caused major detours and breakdowns in the group work. Moreover, from an analyst's perspective the interaction was often almost impossible to parse or interpret (Stahl, 2009b). By contrast, in the final segment that is here reviewed, the interaction is focused on two participants; they work well together; they seem to follow each other well; and their work goes quite smoothly. The structure of the interaction is also relatively easy to follow.

It seems that Aznx and Bwang have substantially increased their skills in online collaborative mathematics. The level of their excitement—especially in the excerpt of Log 8—shows they are highly motivated. Log 10 indicates that they would like to continue this kind of experience in the future.

Collaborative Mathematical Meaning Making

Shared meaning is co-constructed as the discourse moves (the log excerpts based around adjacency pairs) build on each other to form the longer sequence of the discourse topic. This is a key level of analysis for understanding the workings of group cognition. Because these discourse moves are founded upon adjacency pairs, they essentially involve more than one participant, and therefore lend themselves to being vehicles for cognitive phenomena at the group unit of analysis. Through their sequential positioning and subtle forms of mutual referencing, they contribute to problem solving and other cognitive accomplishments. As an example, we can see how Team B solved its mathematical problem across Logs 5, 6 and 7.

In Log 5, we see that collaborative problem solving of a math topic—like most group meaning making—is an intricate intertwining of individual interpretation and shared meaning (Stahl, 2009b). Bwang (line 1771) states the goal for the dyad of finding an equation to describe the pattern of twos. Aznx immediately announces that he knows the equation (1773) and wants to provide it (1772), to which Bwang acquiesces (1774). However, Aznx has trouble coming up with an equation: remembering it, explaining it, thinking of it or finding it. After a while, Bwang gradually announces that he will provide the equation (1784-1786). Then they both propose equations. Throughout the online session, mathematical proposals originate from the understanding of individual students. In this excerpt, they negotiate about who is to make the proposal, and end up both doing so.

Then it is necessary in Log 6 to decide whose proposal will be adopted by the group as a basis for future work. Interestingly, Aznx reconciles their proposals by algebraically transforming Bwang's equation to be the same as one of Aznx' own (1792-1794). This circumvents the possibility that Bwang will reject Aznx' proposal, which he in fact does (1797). It also establishes a group solution whose meaning (derivation, use, form) is likely to be mutually understood since the solution was proposed by both.

Finally, in Log 7, Aznx takes a further mathematical step, multiplying the n^2 by 4 to account for the 4 symmetrical sets of sticks. However, he presents this final formula in question format (1800-1802), soliciting Bwang's agreement in order to establish the formula within their joint problem space. Bwang implicitly accepts Aznx's step and reinterprets the question as requiring a next step of checking the formula for values of n. Bwang presumably checks several values and concludes that the formula works (1804-1805). Aznx summarizes, "So we got it!" Note his use of the pronoun, "we," attributing the solving to the group.

The formula, $4n^2$, is a particularly meaningful expression in this chat, the triumphal culmination of four hours of mathematical exploration. It is a highly meaningful expression for the group, summarizing their analysis of the diamond pattern of sticks at every level of n. The students understand its meaning as a consequence of their participation in the group processes of drawing and discussing together a rich set of related mathematical phenomena. The shared meaning of the math expression is publicly available in the discourse and through its traces in the log; it was co-constructed through the contributions of individuals and is interpreted by those individuals—and later by analysts.

The Structure of Group Cognition

The analysis of the case study in this essay provides a first analysis of the long-sequence-of-moves structure of collaborative mathematical problem solving in a virtual math team. This is a paradigmatic example of group cognition. The small group—here reduced to a dyad—solves a math problem whose solution had until then eluded them (and had escaped Team C as well).

The students accomplish the problem solving by successively completing a sequence of ten moves. Each of the moves seems almost trivial, but each takes place through an interaction that involves both students in its achievement. The moves are commonplace, taken-for-granted practices of mathematical problem solving. They are familiar from individual and classroom problem solving in algebra classrooms. They have also been encountered repeatedly by Team B in their previous four hours of collaborative problem solving (Medina et al., 2009).

Reviewing the sequence of the group's ten moves presented in this essay, we can follow the mathematical solution process. After opening the topic of the sticks problem (Log 1) and deciding to work on it together (Log 2), the team picked an approach of looking at the number of sticks as being countable with the series (1+3+5+7+...)*4 (Log 3). This series is generated by counting the sticks in a visual representation of the diamond pattern at different values of n (Figure 2). This uses the approach from previous sessions of separating the horizontal and vertical sticks (Figure 1) and then dividing each of those groups into two symmetrical groups (Figure 3). The group then articulates a verbal description of this visual series as being "a pattern of +2s" (Log 4). Both students try to symbolize the pattern of the verbal description as an equation (Log 5) and they come to agreement on the formula as n^2 (Log 6), presumably based on the formula for summing integer series, familiar to them from previous sessions. They then check that their equation works for a number of stages of the diamond pattern (Bwang does this off-line during Log 7). Having solved the mathematical challenge as a group they celebrate the group achievement: "WE DID IT!!!!!!" (Log 8), decide to present their solution publicly (Log 9) and close the discourse topic (Log 10).

It is this sequence of moves that accomplishes the problem solving. The sequence has an inner logic, with each move requiring the previous moves to have already been successfully completed (taking it up), and each move preparing the way for (anticipating, projecting) the following ones. Of course, in working on a problem, problem solvers—even professionals (Gowers & Nielsen, 2010; Polymath, 2010)—often make mistakes and explore deadends. Team B's wiki posting (Figure 4) documents that some of this had happened prior to the excerpt analyzed in this essay. Part of what contributes to the unusual clarity of our example is the simplicity of the sequence followed in the final segment.

The common assumption about mathematical problem solving is that information in the form of math facts and manipulations is what is most important. In our analysis of problem solving in a group context, math content and other information is simply, unproblematically included in individual postings. In fact, more often than not, it is implicitly used and understood "between the lines" of the text chat. Of course, this is only possible because the group had already co-constructed a joint problem space (Medina et al., 2009; Sarmiento & Stahl, 2008; Teasley & Roschelle, 1993) that included this math content as already meaningful for the group.

Rather, the important aspects of discourse engaged in collaborative math problem solving are matters of coordination, communication, explanation, decision making and perspective shifting (e.g., moving between visual, verbal and symbolic modes (Çakir, Zemel, et al., 2009) [Investigation 12]). To some extent, these are interactional moves required by most group activities; to some extent, these are adapted to the nature of mathematical discourse.

In conclusion, the group-cognitive achievement of the solution to the group's final problem was accomplished by a sequence of moves. Each move was mundane when considered by itself. The moves and their sequencing were common practices of mathematical problem solving. The group had adopted—implicitly or explicitly—the math practices as group practices [Investigation 16]. Each move was interactively achieved through the exchange of base adjacency pairs situated in the ongoing discourse. The problem solving was an act of group cognition structured as a sequence of these interactive moves.

While we cannot generalize from the analysis in this essay, it seems that this case study can serve as a perhaps unusually clear and simple model of the structure of group cognition in mathematical problem solving by a virtual math team. It shows the group cognition taking place through the co-construction of a temporal sequence of problem-solving moves. Each move is conducted on the basis of an interactional adjacency pair of chat utterances. While the fine structure adheres to the adjacency-pair system of interactional exchange, the larger problem-solving structure builds on these elements through a sequence defined by the topical moves of mathematical deduction.

More generally, this suggests a multi-layered hierarchical structure to discourse in virtual math teams, which we explored in Investigation 24. Each layer is oriented to by the participant activities:

a. **Group event**: E.g., Team B's participation in the VMT Spring Fest 2006. The team meets together and gradually starts to act as a collaborative group.

b. **Temporal session**: Session 4 of Team B on the afternoon of May 18, 2006. The participants agree when to break up a session, when to meet next, and then show up at the same time.

c. **Conversational topic**: Determining the number of sticks in a diamond pattern (lines 1734 to 1833 of Session 4). We saw how Bwang and Aznx open the topic and later close it.

d. **Discourse move**: A stage in the sequence of moves to accomplish discussing the conversational topic (e.g., lines 1767 to 1770). The team steps through the sequence of moves.

e. **Adjacency pair**: The base interaction involving two or three utterances, which drives a discourse move (lines 1767 and 1769). Each initial utterance elicits a response.

f. **Textual utterance**: A text chat posting by an individual participant, which may contribute to an adjacency pair (line 1767). The group members format their separate postings.

g. **Indexical reference**: An element of a textual utterance that points to a relevant resource. In VMT, actions and objects in the shared whiteboard are often referenced. Mathematical content and other resources from the joint problem space and from shared past experience are also brought into the discourse by explicit or implicit reference in an utterance.

The preceding analysis illustrates the applicability of the notion of a long sequence as suggested by both Sacks (1965/1995) and Schegloff (2007). The sequence consists of a coherent series of shorter sequences built on adjacency pairs. This multi-layered sequential structure is adapted from CA to the essentially different, but analogous, context of groupware-supported communication and group cognition. Having seen that this kind of sequential structure exists in the relatively simple case we analyzed, we can now look for longer sequences in the traces of other acts of groupware-mediated group cognition.

Addendum: Coding Scheme for Sequential Discourse

We have developed a coding scheme for the multi-layered hierarchical structure we have found in discourse in virtual math teams. The coding scheme was developed based on the analysis of adjacency pairs according to Schegloff (2007). It was applied to the entire log of Session 4 of Team B, conducted during VMT SpringFest 2006.

The basic idea is that discourse is built up hierarchically: from (g) various indexical references (e.g., "that") in (f) textual utterances (e.g., chat postings) contributing to (e) adjacency pairs (e.g., question/answer). Sequences of adjacency pairs (including extensions and recursive embeddings) form (d) discourse moves. The moves contribute to (c) conversational topics (that are opened and closed). Topics are included in larger (b) group events, which make up (a) the entire session (e.g., Session 4 of Team B).

In Table 1, examples of (c) through (f) are included under those headings. Schegloff's symbols are listed for use in coding utterances in adjacency pairs. For each symbol, its meaning is given. The list contains some common FPPs (first pair parts) of adjacency pairs, with their corresponding SPPs (second pair parts).

Table 1. Coding Scheme.

(c) Conversa-	(d) Discourse Move	(e) Adjacency Pair	(f) Textual Utterance	(g) Indexical	(b) Group Event	(a) Temporal

tional Topic				Reference			Session
transition	anticipate	announcement	announce; acknowledge; follow up				
opening	close	compliment	compliment; acknowledge	Schegloff symbols	meaning of symbol	FPP	SPP
technical	open	explanation	explain; acknowledge; follow up	F, Fbase	first pair part (base FPP)	question / ask	answer
feedback	return to	greeting	greet; return greeting; farewell; return farewell	S, Sbase	second pair part (base SPP)	request	grant
select	introduce new approach	joke	joke; laugh; respond to joke; return laughter	Fpre	pre-sequence FPP	offer *	reject
review	terminate use of approach	proposal	propose; acknowledge; ratify; reject; follow up	Spre	pre-sequence SPP	invite *	accept
wiki		question	question; answer; agree; disagree; follow up	Fins	insert sequence FPP	announce	decline
equation		request	request; acknowledge; accept; reject; follow up	Sins	insert sequence SPP	greet	agree
indexing		suggestion	suggest; acknowledge; ratify; reject; follow up	SCT	sequence closing third	farewell	disagree
compare		directive	direct; acknowledge; receive; reject; follow up; report	Fpost	post sequence FPP	notice *	acknowledge
strategy		evaluation	evaluate; acknowledge; agree; disagree	Spost	post sequence SPP	promise *	contest
wrong		commentary	comment; acknowledge; agree; disagree	+S	preferred SPP	tell *	tease
celebrate		clarification	clarify; acknowledge	PCM	post completion musing	complain *	finess
facilitator		repair	self-correct; question; clarify; acknowledge			propose	comply
follow-up		failed X escalated X		ni	non-interactive or system message	suggest	perform

closing		+ (continuation)	+ (continue)	+	continuation	request	ratify
Construc-tion						direct	follow up
narrative						joke	receive
reflection						laugh	report
						Compli-ment	assess
						explain	return
						clarify	clarify
						repair	
						evaluate	
						comment	

In Table 2, the excerpt (lines 1734-1829), which was analyzed in this Investigation, is coded in accordance with the coding scheme. The coding process involved considerable back-and-forth influence between the coding of the threading, the code, the utterance category, the adjacency pair and the discourse move.

Table 2. Coding of an excerpt from SpringFest 2013.

Line #	Time Posting	Bwang8	Aznx	Quicksilver	Gerry	Threading	Code	Utterance Category	Adjacency Pair	Dis-course Move	Con-versa-tional Topic
1734	20:17:20	i think we are very close to solving the problem here				1709	Fb	proposal	proposal	open topic	sticks
1735	20:17:35			Oh great...I have to leave		--	Fpre	announce	announcement	open topic	sticks
1736	20:17:39		We can solve on			1734	Sb	ratify		open topic	sticks

			that topic.								
1737	20:17:42			Sorry guys		1735	+	+		open topic	sticks
1738	20:17:45	oh				1735	Spre	acknowledge		open topic	sticks
1739	20:17:46		It shouldn't take much time.			1736	+	+		open topic	sticks
1740	20:17:47	ok				1738	SCT	acknowledge		open topic	sticks
1741	20:17:50		k, bye aditya			1737	F	farewell	farewell	open topic	sticks
1742	20:17:52			Just tell me the name of the room		1737	Fpost	question	question	open topic	sticks
1743	20:17:52	bye				1737	F	farewell	farewell	open topic	sticks
1744	20:18:14				The new room is in the lobby under Open Rooms	1742	Spost	answer		open topic	sticks
1745	20:18:44				It is under The Grid World. It has your names on it	1744	+	+		open topic	sticks

1746	20:18:49			leaves the room			ni			open topic	sticks
1747	20:19:00		Alright found it.			1745	SCT	acknowledge		open topic	sticks
1748	20:19:04		Thanks.			1747	+	+		open topic	sticks
1749	20:19:12		I guess we should leave then.			1748	F	question	question	decide to start	sticks
1750	20:19:34	well do you want to solve the problem				1749	Fb	question	question	decide to start	sticks
1751	20:19:36	i mean				1750	+	+		decide to start	sticks
1752	20:19:39	we are close				1751	+	+		decide to start	sticks
1753	20:19:48		Alright.			1752	Sb	agree		decide to start	sticks
1754	20:19:51	i don't want to wait til tomorrow				1752	+	+		decide to start	sticks
1755	20:19:53	ok				1753	SCT	agree		decide to start	sticks
1756	20:19:55		How do you want to approach it?			1755	Fb	question	question	pick an approach	sticks

1757	20:20:14	1st level have 1*4				1756	Sb	follow up		pick an approach	sticks
1758	20:20:20				You can put something on the wiki to summarize what you found today	1757	F	direct	directive	pick an approach	sticks
1759	20:20:29	2st level have (1+3)*4				1757	+	+	proposal	pick an approach	sticks
1760	20:20:32		bwang you put it.			1758	F	direct	directive	pick an approach	sticks
1761	20:20:35		for the wiki			1760	+	+		pick an approach	sticks
1762	20:20:37	ok				1761	S	agree		pick an approach	sticks
1763	20:20:42		we actually did quite a lot today			1761	F	proposal	proposal	pick an approach	sticks
1764	20:20:53	3rd level have (1+3+5)*4				1759	+	+		pick an approach	sticks
1765	20:21:05	4th level have (1+3+5+7)*4				1764	+	+		pick an approach	sticks

1766	20:21:10				This is a nice way to solve it	1764	F	compliment	compliment	pick an approach	sticks
1767	20:21:12		So it's a pattern of +2s?			1765	Fb	question	question	identify pattern	sticks
1768	20:21:15		Ah ha!			1767	+	+		identify pattern	sticks
1769	20:21:15	yes				1767	Sb	agree		identify pattern	sticks
1770	20:21:20		There's the pattern!			1769	Fpost	proposal	proposal	identify pattern	sticks
1771	20:21:39	now we have to find a equation that describe be that pattern				1770	F	proposal	proposal	seek equation	sticks
1772	20:21:49		Hold on.			1771	Fpre	request	request	seek equation	sticks
1773	20:21:51		I know it.			1772	+	+		seek equation	sticks
1774	20:21:57	what is it				1773	Fb	question	question	seek equation	sticks
1775	20:21:58		But I'm trying to remember it. =P			1773	+	+		seek equation	sticks
1776	20:22:04		and explain it as well.			1775	+	+		seek equation	sticks

1777	20:22:17		try and think of it			1776	+	+		seek equation	sticks
1778	20:22:53				Maybe Quicksilver can come back here tomorrow or next week to finish it with you	1777	F	proposal	failed proposal	seek equation	sticks
1779	20:23:01				I have to go now	1778	Fpre	announce	announcement	seek equation	sticks
1780	20:23:05				Bye!	1779	F	farewell	farewell	seek equation	sticks
1781	20:23:06	ok				1780	Spre	acknowledge		seek equation	sticks
1782	20:23:07	bye				1781	S	farewell		seek equation	sticks
1783	20:23:23				leaves the room		ni			seek equation	sticks
1784	20:23:29	ok				1771	SCT	acknowledge		seek equation	sticks
1785	20:23:32	so				1784	+	+		seek equation	sticks
1786	20:23:37	i think it is this				1785	Fpre	proposal	proposal	seek equation	sticks
1787	20:23:53		ok			1786	SCT	announce		seek equation	sticks
1788	20:23:55		i found it			1787	Fpre	announce	announcement	seek equation	sticks

1789	20:24:00		n^2			1788	Sb	answer		seek equation	sticks
1790	20:24:01	(2*n)*n/2				1786	F	proposal	proposal	seek equation	sticks
1791	20:24:09		or (n/2)^2			1789	+	+		seek equation	sticks
1792	20:24:14		I'm simplifying			1791	Fpre	explain		negotiate solution	sticks
1793	20:24:30		if u simplify urs			1790	+	+		negotiate solution	sticks
1794	20:24:35		its n^2			1793	Fb	proposal	proposal	negotiate solution	sticks
1795	20:24:59		bwang			1794	F	question	question	negotiate solution	sticks
1796	20:25:01		you there?			1795	+	+		negotiate solution	sticks
1797	20:25:03	so that's wrong				1796	Sb	reject		negotiate solution	sticks
1798	20:25:07	yeah				1796	S	answer		negotiate solution	sticks
1799	20:25:08	i am here				1798	+	+		negotiate solution	sticks
1800	20:25:11		so			1799	Fpre	announce	announcement	check cases	sticks
1801	20:25:13		the formula			1800	+	+		check cases	sticks

1802	20:25:22		would be 4n^2 ?			1801	Fb	question	question	check cases	sticks
1803	20:25:28	let's check				1802	S	follow up		check cases	sticks
1804	20:25:55	yes				1803	Spre	agree		check cases	sticks
1805	20:26:00	it actually is				1804	Sb	answer		check cases	sticks
1806	20:26:02		So we got it!			1804	SCT	ratify		check cases	sticks
1807	20:26:02	omg					F	laugh	joke	celebrate	sticks
1808	20:26:04		yay!			1807	S	laugh		celebrate	sticks
1809	20:26:08	i think we got it!!!!!!!!!!!!				1806	Fb	announce	announcement	celebrate	sticks
1810	20:26:12		WE DID IT!!!!!!			1809	Sb	ratify		celebrate	sticks
1811	20:26:12	and it is so simple				1809	Fpost	proposal	proposal	celebrate	sticks
1812	20:26:14		YAY!!!!			1812	S	laugh		celebrate	sticks
1813	20:26:16		i know			1811	Spost	agree		celebrate	sticks
1814	20:26:17	lol				1813	S	laugh		celebrate	sticks
1815	20:26:18		lol			1814	S	laugh		celebrate	sticks
1816	20:26:34		So you'r e puttin g it in the wiki, right?			1815	Fb	question	question	present solution	sticks

1817	20:26:37	yes				1816	Sb	agree		present solution	sticks
1818	20:26:41		Alright then.			1817	SCT	follow up		present solution	sticks
1819	20:26:43	ok				1817	SCT	agree		present solution	sticks
1820	20:26:53		Give an email to Gery, telling him that we got it. =)			1819	F	direct	directive	present solution	sticks
1821	20:26:57	ok				1820	S	agree		present solution	sticks
1822	20:26:59		I meant Gerry			1820	S	repair		present solution	sticks
1823	20:27:04	are you going to do it				1820	F	question	question	present solution	sticks
1824	20:27:07	or am i				1823	+	+		present solution	sticks
1825	20:27:12		You do it.			1824	S	answer		present solution	sticks
1826	20:27:14	ok				1825	S	agree		present solution	sticks
1827	20:27:19		Tell him that we both dervied n^2			1826	F	direct	directive	present solution	sticks

1828	20:27:29		And then we saw that pattern			1827	+	+		present solution	sticks
1829	20:27:37		and we got the formula			1828	+	+		present solution	sticks

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Notes on the Investigations

Introduction

Written for this volume in 2018.

Investigation 1

Written for this volume in 2018.

Investigation 2

Written for this volume in 2018, with ideas and excerpts from the following *ijCSCL* editorial introductions:

- Conceptualizing the intersubjective group (Stahl, 2015b)
- Ethnomethodologically informed (Stahl, 2012b)
- Cognizing mediating: Unpacking the entanglement of artifacts with collective minds (Stahl, 2012a)
- Traversing planes of learning (Stahl, 2012c)
- Learning across levels (Stahl, 2013)

Investigation 3

Reprint of (Jones, Dirckinck-Holmfeld & Lindstrom, 2006).

Investigation 4

Reprint of (Suthers, 2006).

Investigation 5

Reprint of (Zemel & Koschmann, 2013).

Investigation 6

Reprint of (Overdijk, van Diggelen, Andriessen & Kirschner, 2014).

Investigation 7

Reprint of (Ritella & Hakkarainen, 2012).

Investigation 8

Reprint of (Stahl, 2008).

Investigation 9

Reprint of (Öner, 2016).

Investigation 10

Reprint of (Reimann, 2009).

Investigation 11

Reprint of (Damsa, 2014).

Investigation 12

Reprint of (Çakir, Zemel & Stahl, 2009).

Investigation 13

Reprint of (Looi, So, Toh & Chen, 2011).

Investigation 14

Reprint of (Chan, 2011).

Investigation 15

Edited and extended version of (Stahl, 2016b).

Investigation 16

Edited and extended version of (Stahl, 2017).

Thanks to everyone who participated in the VMT project and who collaborated on the analysis of its data. Also, to the anonymous reviewers of the original article, who prompted me to elaborate several aspects and implications of this view of CSCL.

Investigation 17

Edited and extended version of (Stahl, Zhou, Çakir & Sarmiento-Klapper, 2011).

Investigation 18

Edited and extended version of (Stahl, 2016a).

Investigation 19

Edited and extended version of constitution (Stahl, 2014).

Investigation 20

Edited and extended version of (Stahl, 2005).

Investigation 21

Edited and extended version of (Stahl, 2015a).

Investigation 22

Edited and extended version of (Stahl, 2006a).

Investigation 23

Edited and extended version of (Stahl, 2006b) and (Stahl, 2011a).

An earlier version of the first part of this paper won a Best Paper Award at ICCE 2005 in Singapore (Stahl, 2005). The Virtual Math Teams Project is a collaborative effort at Drexel University. The Principal Investigators are Gerry Stahl, Stephen Weimar and Wesley Shumar. A number of Math Forum staff work on the project, especially Stephen Weimar, Annie Fetter and Ian Underwood. The graduate research assistants are Murat Cakir, Johann Sarmiento, Ramon Toledo and Nan Zhou. Alan Zemel is the post-doc; he facilitates weekly conversation analysis data sessions. The following visiting researchers have spent 3 to 6 months on the project: Jan-Willem Strijbos (Netherlands), Fatos Xhafa (Spain), Stefan Trausan-Matu (Romania), Martin Wessner (Germany), Elizabeth Charles (Canada). The VMT-Chat software was developed in collaboration with Martin Wessner, Martin Mühlpfordt and colleagues at the Fraunhofer Institute IPSI in Darmstadt, Germany. The VMT project is supported by grants from the NSDL, IERI and SoL programs of the US National Science Foundation.

Investigation 24

Edited and extended version of (Stahl, 2011c).

Investigation 25

Edited and extended version of (Stahl, 2011b).

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Computers have transformed how we think, discuss and learn—as individuals, in groups, within cultures and globally. However, social media are problematic, fostering flaming, culture wars and fake news.

This volume presents an alternative paradigm for computer support of group thinking, collaborative learning and joint knowledge construction. This requires expanding concepts of cognition to collectivities, like collaborative groups of networked students.

Theoretical Investigations explores the conditions for group cognition, supplying a philosophical foundation for new models of pedagogy and methods to analyze group interaction. Twenty-five self-contained investigations document progress in research on computer-supported collaborative learning (CSCL)—both in Stahl's own research and during the first decade of the CSCL journal.

The volume begins with two new reflections on the vision and theory that result from this research. Representing both ethnomethodological and social-constructivist research paradigms, the investigations within this volume comprise a selection of seminal and influential articles and critical commentaries that contribute to an understanding of concepts and themes central to the CSCL field. The book elaborates an innovative theory of group cognition and substantiates the pedagogical potential of CSCL.

Theoretical Investigations: Philosophical Foundations of Group Cognition is essential as a graduate text for courses in educational theory, instructional design, learning and networked technologies. The investigations will also appeal to researchers and practitioners in those areas.